North Sydney Boys
HIGH SCHOOL

MATHEMATICS (EXTENSION 1)

2015 Preliminary Course Assessment Task 1
Wednesday April 1, 2015

General instructions

- Working time – 70 minutes. (plus 5 minutes reading time)
- Commence each new question on a new page.
- Write using blue or black pen. Where diagrams are to be sketched, these may be done in pencil.
- Board approved calculators may be used.
- All necessary working should be shown in every question.
- Attempt **all** questions.
- At the conclusion of the examination, bundle the booklets used in the correct order within this paper and hand to examination supervisors.

Class (please ✔)

- ◯ 11M3A – Mr Hwang
- ◯ 11M3B – Mr Lin
- ◯ 11M3C – Mr Weiss
- ◯ 11M3D – Ms Ziaziaris/Mr Weiss
- ◯ 11M3E – Mr Ireland
- ◯ 11M3F – Miss Lee

STUDENT #: (4xxxxxxxx) .... **# SHEETS USED**: .....

Marker's use only.

QUESTION	1	2	3	4	5	6	7	**Total**	%
MARKS	$\frac{\quad}{15}$	$\frac{\quad}{16}$	$\frac{\quad}{13}$	$\frac{\quad}{10}$	$\frac{\quad}{11}$	$\frac{\quad}{12}$	$\frac{\quad}{11}$	$\frac{\quad}{88}$	

Question 1 (15 Marks) Commence a NEW page. **Marks**

(a) Consider the function $f(x) = \dfrac{2}{x^2 - 4}$

i. Show that $f(x)$ is an even function **2**

ii. What is the geometric significance of part (i)? **1**

(b) State the domain and range of

i. $y = -2x^2$ **2**

ii. $y = \sqrt{16 - x^2}$ **2**

iii. $y = -\sqrt{x + 2}$ **2**

(c) Sketch the graph of $y = 1 + \dfrac{1}{x+3}$, showing the x-intercept, the y-intercept, and asymptotes. **3**

(d) A baseball is projected vertically upwards and its height in metres after t seconds is given by $h = 30t - 5t^2$. Find the maximum height reached by the ball. **3**

Question 2 (16 Marks) Commence a NEW page. **Marks**

(a) Sketch: $f(x) = \begin{cases} -1 & \text{for } x < -1 \\ 1 & \text{for } -1 \leq x \leq 1 \\ x^2 + 1 & \text{for } x > 1 \end{cases}$ **3**

(b) Sketch each of the following on separate number planes.

i. $y = -2x^2$ **3**

ii. $xy = 2$ **3**

iii. $y = |6 - 3x|$ **2**

(c) Given that, for all values of x, $f(x) = ax^2 + bx + c$ and that $f(0) = 6, f(1) = 12$ and $f(-1) = -4$, determine the values of a, b and c **3**

(d) Find $f(x)$ if $f(x + 2) = 3x + 5$ **2**

Question 3 (13 Marks) Commence a NEW page. **Marks**

(a) Find the value of a if $\sqrt{a} = 6\sqrt{3}$. **2**

(b) Fully simplify: $\dfrac{a^2 - 2ab + b^2}{a^2 - b^2} - \dfrac{a^2 - b^2}{a^2 + 2ab + b^2}$ **3**

(c) Fully simplify: $\dfrac{3^{n+2} + 3^n}{3^n}$ **2**

(d) Solve for x:

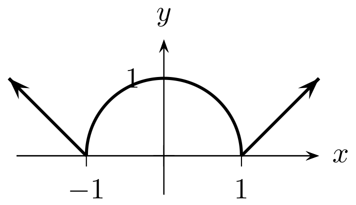
i. $x^4 = 4\left(x^2 + 3\right)$ **3**

ii. $3^{5x+7} = \dfrac{1}{81}$ **3**

Question 4 (10 Marks) Commence a NEW page. **Marks**

(a) Sketch $y = 1 - 3^x$, showing all important features. **3**

(b) The following diagram shows a sketch of $y = g(x)$.



Sketch, on separate diagrams (each being one third of a page):

i. $y - 1 = g(x)$ **2**

ii. $y = g(x-1)$ **2**

(c) Solve for x: $(x-1)^2(3-x) > 0$ **3**

Question 5 (11 Marks) Commence a NEW page. **Marks**

(a) Find the period and amplitude of the curve $y = 3\sin(2x)$. **2**

(b) i. Prove that $\left(\operatorname{cosec}^2 x - 1\right)\sin^2 x = \cos^2 x$. **2**

ii. Hence or otherwise, solve **3**

$$\left(\operatorname{cosec}^2 x - 1\right)\sin^2 x = \frac{3}{4}$$

for $0^\circ \le x \le 360^\circ$.

(c) A ship is sailing from on a bearing of 205° from port A to port B. Port B is 70 nm west of port A. **2**

Find how far the ship has sailed.

(d) If $\tan\theta = -\dfrac{4}{3}$ and $90^\circ \le \theta \le 180^\circ$, find the exact value of $\sin\theta$. **2**

Examination continues overleaf...

Question 6 (12 Marks) Commence a NEW page. **Marks**

(a) i. Given that $f(x) = x^2 + 2$, find the largest possible domain consisting of positive real numbers such that $f(x)$ has an inverse fuction $f^{-1}(x)$. **2**

ii. Find the equation of $f^{-1}(x)$. **2**

iii. State the domain and range of both $f(x)$ and $f^{-1}(x)$. **2**

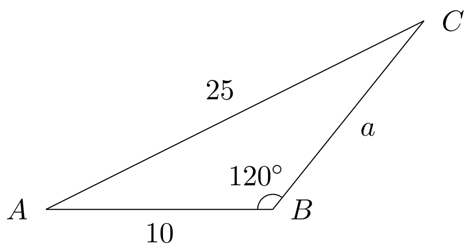
iv. Sketch $f(x)$ and $f^{-1}(x)$ on the same set of axes, using the same scale for the x and y axes. **3**

(b) Solve: $\dfrac{x^2 - 5x}{x - 4} \le 3.$ **3**

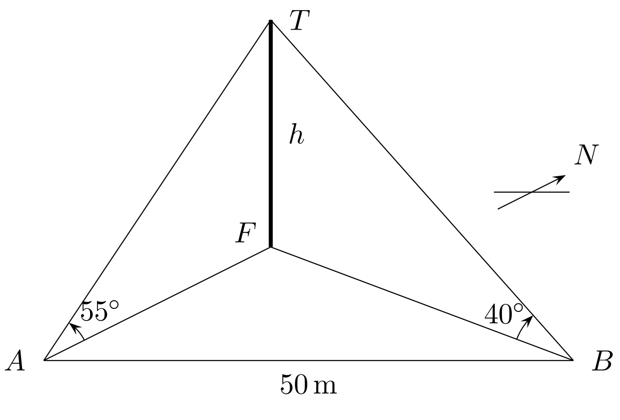
Question 7 (11 Marks) Commence a NEW page. **Marks**

(a) Sketch the graph of $y = 3 - 2\cos 4x$ for $0° \le x \le 180°$, showing all important features. **3**

(b) Find the *exact* value of a: **3**



(c) The diagram shows a tower TF of height h metres standing due north of A on level ground. The angles of elevation of the top T of the tower from two points A and B (due east of A) on the ground are 55° and 40° respectively. The distance AB is 50 m.



i. Find the size of $\angle FAB$. **1**

ii. Find AF and BF in terms of h. **2**

iii. Hence find the height of the tower to the nearest metre. **3**

End of paper.

Suggested Solutions

Suggested solutions

Question 8 (Weiss)

(a)

Question 9 (Weiss)

(a)

Question 10(Weiss)

(a)

Q1) a) i/ $f(x) = \dfrac{2}{x^2-4}$

$f(-x) = \dfrac{2}{x^2-4}$ 1

$= f(x)$

$\therefore$ even function 1

ii/ symmetry about y-axis. 1

b) i/ $\{x:$ all real $x\}$ 1

$\{y: \ y \leq 0\}$ 1

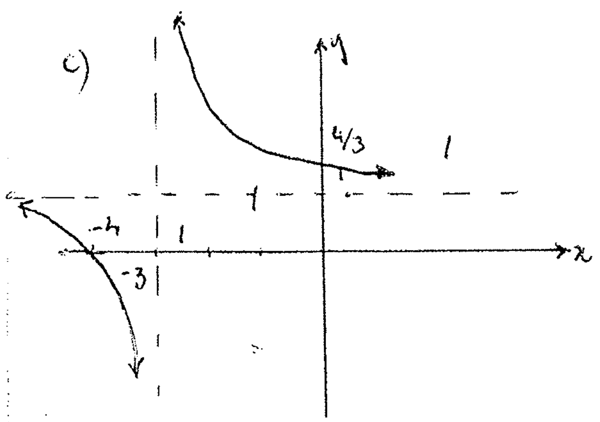
ii/ $\{x: \ -4 \leq x \leq 4\}$ 1

$\{y: \ 0 \leq y \leq 4\}$ 1

iii/ $\{x: \ x \geq -2\}$ 1

$\{y: \ y \leq 0\}$ 1

c) 1



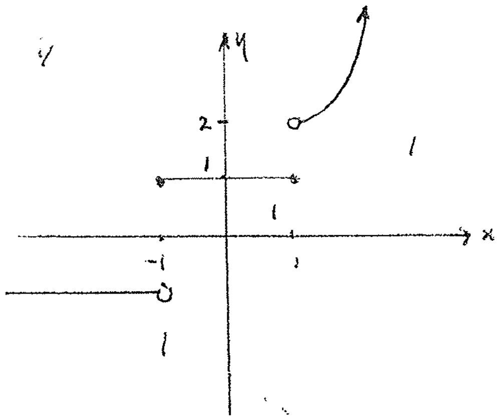
d) $h = 5t(6-t)$ 1

when $t = 3$

$h = 15(3)$ 1

$= 45$ m 1

Q2) a) i/

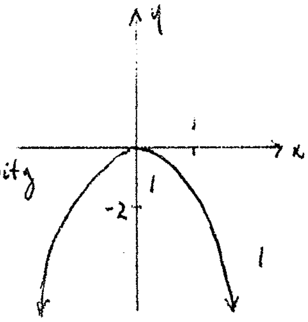


1

1

1

ii/

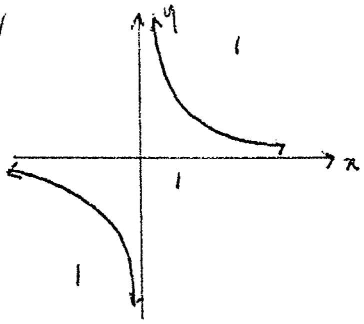


1 concavity

1

1

iii/

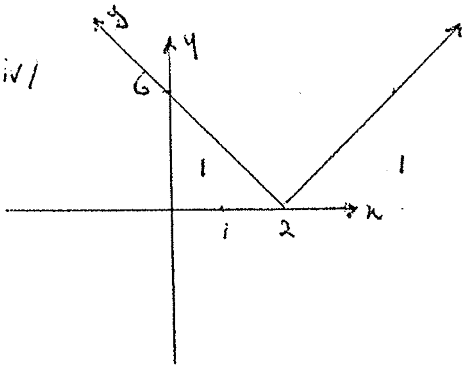


1

1

1

iv/



1

1

b) $f(x) = ax^2 + bx + c$

$f(0) = c = 6$ 1

$f(1) = a + b + 6 = 12$

$a + b = 6$ ①

$f(-1) = a - b + 6 = -4$ 1

$a - b = -10$ ②

① + ②

$2a = -4$

$\therefore$ $a = -2$ 1

$b = 8$

$c = 6$

c) $f(x+2) = 3x + 5$

$= 3(x+2) - 1$ 1

$f(x) = 3x - 1$ 1

Q3) a)

$$\sqrt{a} = 6\sqrt{3}$$
$$= \sqrt{36.3} \quad 1$$
$$\therefore a = 108 \quad 1$$

b)

$$\frac{(a-b)^2}{(a+b)(a-b)} - \frac{(a-b)(a+b)}{(a+b)^2} \quad 1$$
$$= \frac{(a-b)^2(a+b) - (a-b)^2(a+b)}{(a+b)^2(a-b)} \quad 1$$
$$= 0 \quad 1$$

c)

$$\frac{3^n(3^3+1)}{3^n} \quad 1$$
$$= 10 \quad 1$$

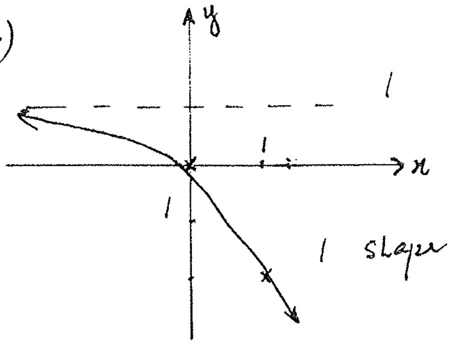
d) i/

$$x^4 - 4x^2 - 12 = 0$$
$$(x^2+2)(x^2-6) = 0 \quad 1$$
$$\therefore x^2 = 6 \text{ or } -2 \quad 1$$
$$\therefore x = \pm\sqrt{6} \quad \text{no real soln} \quad 1$$

ii/

$$3^{5x+7} = 3^{-4} \quad 1$$
$$5x + 7 = -4$$
$$5x = -11 \quad 1$$
$$x = \frac{-11}{5} \quad 1$$

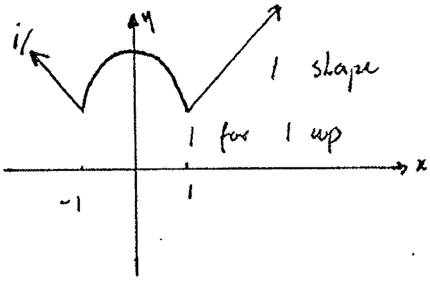
Q4) a)

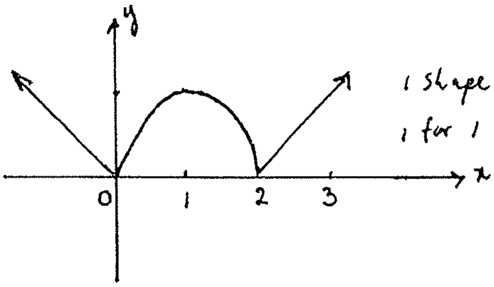


1

1 shape

b)

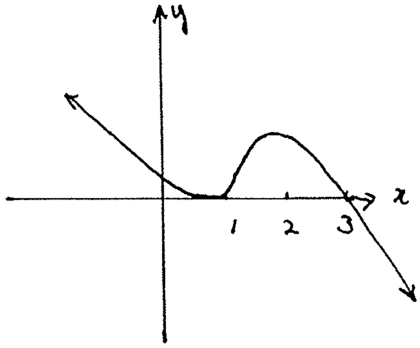




1 shape

1 for 1 across

c)



1

$\therefore\ f(x) > 0$

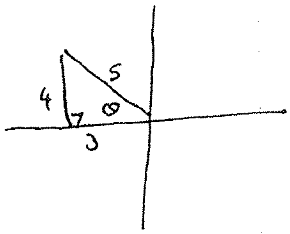
$x \leq 3$ but $x \neq 1$

1 1

Q5) a) amplitude is 3 1

period $180°$ 1

d)



$\sin\theta = \frac{4}{5}$ 1

+ve 1

b) i/ L.H.S $= (\operatorname{cosec}^2 x - 1)\sin^2 x$

$= \cot^2 x \cdot \sin^2 x$ 1

$= \cos^2 x$ 1

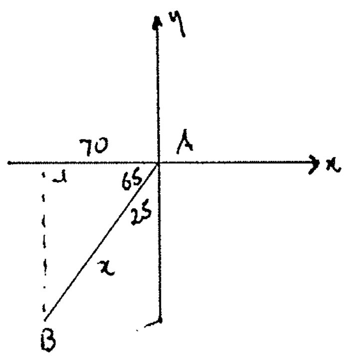
$=$ RHS

ii/ $\cos^2 x = \frac{3}{4}$ 1

$\cos x = \pm \frac{\sqrt{3}}{2}$ 1

$\therefore\ x = 30, 150, 210°, 330°$ 1

c)



1

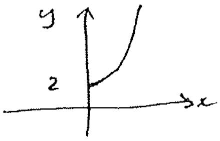
$\frac{70}{x} = \cos 65°$

$x = \frac{70}{\cos 65°}$

$= 165.6$ nm to 1D 1

Q6

(a) (i) $f(x) = x^2 + 2$



$\therefore D: x \geq 0$

(ii) For inverse, $x = y^2 + 2$

$\therefore y^2 = x - 2$

$y = +\sqrt{x-2}$ or $-\sqrt{x-2}$

Since the domain of $f(x)$ is $x \geq 0$

$\therefore$ the range of $f^{-1}(x)$ is $y \geq 0$

$\therefore f^{-1}(x) = \sqrt{x-2}$

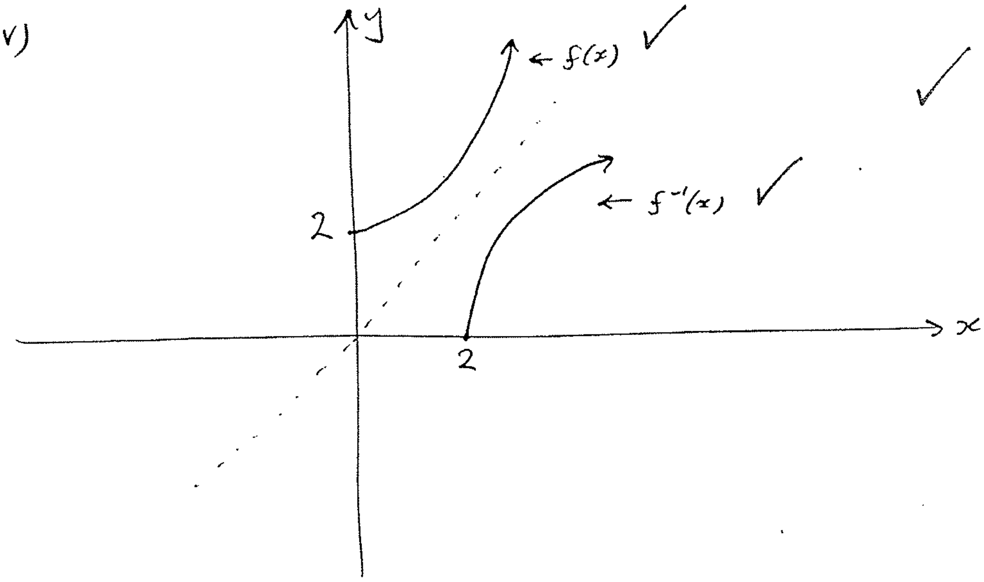
(iii) For $f(x)$, $D: x \geq 0$

$R: y \geq 2$

For $f^{-1}(x)$, $D: x \geq 2$

$R: y \geq 0$.

(iv)



needs:
- symmetry
- intercepts
- shape.

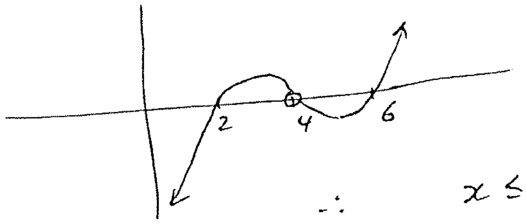
(b) $\dfrac{x^2-5x}{x-4} \le 3$, $x \neq 4$.

$(x^2-5x)(x-4) \le 3(x-4)^2$ ✓

$(x-4)\left(x^2-5x-3(x-4)\right) \le 0$

$(x-4)(x^2-8x+12) \le 0$

$(x-4)(x-6)(x-2) \le 0$ ✓



$\therefore$ $x \le 2$ or $4 < x \le 6$. ✓

ALTERNATIVES

(1) Critical points: $x = 4$ and $\dfrac{x^2-5x}{x-4} = 3$

$\hookrightarrow$ $x = 2, 6$. ✓

Test several points:

eg $x=0 \rightarrow$ $LHS = \dfrac{0-0}{-4} = 0 \le 3$ $\therefore$ ✓

$x = 3 \rightarrow$ $LHS = \dfrac{9-15}{3-4} = 6 \nleq 3$ $\therefore$ ✗ ✓

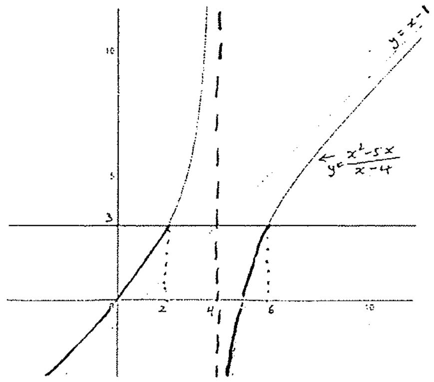
$x = 5 \rightarrow$ $LHS = \dfrac{25-25}{1} = 0 \le 3$ $\therefore$ ✓

$x = 10 \rightarrow$ $LHS = \dfrac{50}{6} \nleq 3$ $\therefore$ ✗

Also, $x=2$ & $x=6$ give $LHS \le 3$.

$\therefore$ $x \le 2$ or $4 < x \le 6$. ✓

(2) Graph:

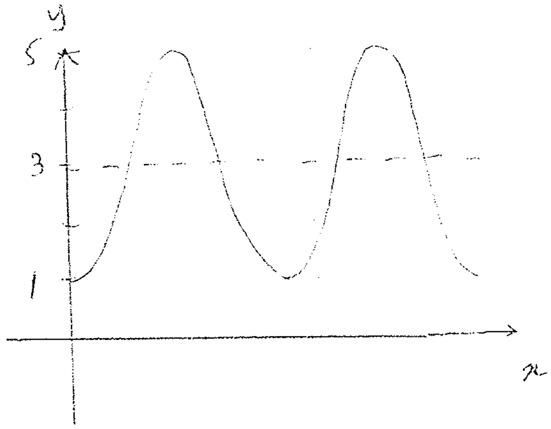


$\rightarrow$ $\therefore$ $x \le 2$ or $4 < x \le 6$. ✓

/12

Question 7

a)



b) From the cosine rule

$25^2 = a^2 + 10^2 - 2(a)(10)\cos 120°$

$25^2 - 10^2 = a^2 - 20a\cos 120°$

$a^2 + 10a - 525 = 0$.

$a = \frac{-10 \pm \sqrt{2200}}{2}$

as $a > 0$, $a = 5\sqrt{22} - 5$.

c) i) $\angle FAB = 90°$

ii) $\tan 55° = \frac{h}{AF}$

$\therefore AF = h\cot 55°$

$\tan 40° = \frac{h}{BF}$

$BF = h\cot 40°$

iii) $50^2 + (h\cot 55)^2 = (h\cot 40)^2$

$h^2(\cot^2 40 - \cot^2 55) = 50^2$

$h^2 = \frac{50^2}{\cot^2 40 - \cot^2 55}$

$\therefore h \doteqdot 52m$, $h > 0$.