NORTH SYDNEY BOYS HIGH SCHOOL

2015 YEAR 11 ASSESSMENT TASK 2

Mathematics Extension 1

General Instructions

- Working time – 50 minutes
- **Reading time – 5 minutes**
- Write on the lined paper in the booklet provided
- Write using blue or black pen
- Board approved calculators may be used
- All necessary working should be shown in every question
- Each new question is to be started on a **new page.**
- Attempt all questions

Class Teacher: Please colour

- O Mr Hwang
- O Mr Ireland
- O Ms Lee
- O Mr Lin
- O Mr Weiss
- O Mr Ziaziaris

Student Number:________________________

(To be used by the exam markers only.)

Question No	1	2	3	4	5	6	Total	Total
Mark	$\overline{8}$	$\overline{8}$	$\overline{8}$	$\overline{8}$	$\overline{8}$	$\overline{6}$	$\overline{46}$	$\overline{100}$

YEAR 11 ASSESSMENT TASK 2 2015

QUESTION 1 **START A NEW PAGE**

(a) Differentiate

(i) $(5x^2-4)^5$ 1

(ii) $\dfrac{2x^3+3x-4}{x}$ 2

(iii) $2x\sqrt{x}$ 2

(b) Find the equation of the normal to the curve $y=5x^2-3x+1$ at the point where $x=-1$. 3

QUESTION 2 **START A NEW PAGE**

(a) Find the equation of the straight line which passes through the point C(4, -1) and is parallel to the line through A(-1, 2) and B(2,7). 2

(b) Graph the intersection of the regions
$y<3x$ and $x-3y+1\le 0$ 3

(c) The point C (6, 2) divides the interval AB in the ratio 2:3. If A has coordinates (4, -2) find the coordinates of B.

3

QUESTION 3 **START A NEW PAGE**

(a) Express 140° in radians as an exact value. 1

(b) Given that $\operatorname{cosec}\alpha = 4$ where α is acute,
Find the value of $\cos 2\alpha$. 3

(c) Find the exact value of $\frac{1}{4}\sin\frac{\pi}{8}\cos\frac{\pi}{8}$ 2

(d) Solve for $0^\circ \le \theta \le 360^\circ$

$$\cos\frac{\theta}{2} = \frac{1}{2}$$ 2

QUESTION 4 **START A NEW PAGE**

(a) For what values of x is $f(x) = 3x^3 - 2x^2 - 5x - 4$ increasing. 3

(b) Find $\lim_{x \to 2} \frac{2x^2 - 8}{x - 2}$ 2

(c) A function is defined by

$$f(x) = \begin{cases} ax^2 + b & for x < 0 \\ ax^3 - 6 & for x > 0 \end{cases}$$

Given that $f'(1) - f'(-1) = -15$, show that $f(2) = -30$ 3

exam continues overleaf.......

QUESTION 5 **START A NEW PAGE**

(a) Find the acute angle between the lines $5x-6y=30$ and $4x+3y=12$ giving your answer to the nearest minute. 3

(b) Find the perpendicular distance between the parallel lines $3x+4y=-1$ and $3x+4y=4$. 2

(c) Solve for $-180^\circ \le \theta \le 270^\circ$

$2\cos(\theta-225)^\circ = \sqrt{3}$ 3

QUESTION 6 **START A NEW PAGE**

(i) Show that $\left(\frac{\sin 3x}{\sin x}\right)^2 - \left(\frac{\cos 3x}{\cos x}\right)^2 = 8\cos 2x$

for $\sin x \neq 0$ and $\cos x \neq 0$ 3

(ii) Hence or otherwise solve

$\left(\frac{\sin 3x}{\sin x}\right)^2 - \left(\frac{\cos 3x}{\cos x}\right)^2 = 8\sin x$ where $0^\circ \le \theta \le 360^\circ$ 3

END OF EXAMINATION

ASSESS TSK 2 YR 11 2015.

1.(a) (i) $5(5x^2-4)^4 \cdot 10x$

$= 50x(5x^2-4)^4$

(ii) $\frac{2x^3}{x} + \frac{3x}{x} - \frac{4}{x}$

$= 2x^2 + 3 - 4x^{-1}$

$\frac{dy}{dx} = 4x + \frac{4}{x^2}$

(iii) $2x^{3/2}$

$\frac{dy}{dx} = 3x^{1/2}$

(b) $y = 5x^2 - 3x + 1$

$\frac{dy}{dx} = 10x - 3$

$\therefore$ At $x = -1$

$\frac{dy}{dx} = -13$

$\therefore$ m of normal $= \frac{1}{13}$ at $(-1, 9)$

Equn of normal

$y - 9 = \frac{1}{13}(x+1)$

$13y - 117 = x + 1$

$x - 13y + 118 = 0$.

2. (a) $m = \frac{7-2}{2+1}$

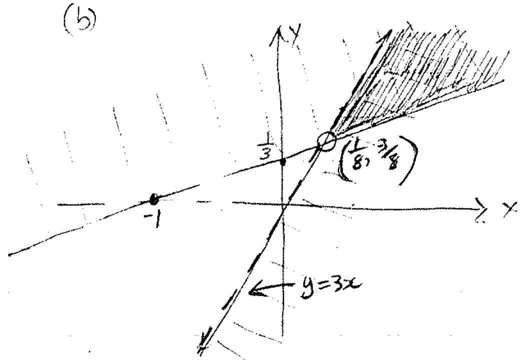
$m = \frac{5}{3}$

Equn: $y + 1 = \frac{5}{3}(x-4)$

$3y + 3 = 5x - 20$

$5x - 3y - 23 = 0$

(b)



c) $(6,2) = \frac{nx_1+mx_2}{m+n}, \frac{ny_1+my_2}{m+n}$

$(6,2) = \frac{3(4)+2(x_2)}{5}, \frac{3(-2)+2y_2}{5}$

$\therefore 6 = \frac{12+2x_2}{5}$, $2 = \frac{-6+2y_2}{5}$

$\frac{30-12}{2} = x_2$ $\qquad 10 = -6 + 2y_2$

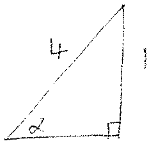
$x_2 = 9$ $\qquad y_2 = 8$

$\therefore B(9,8)$

OR you could use diagram form

3. a) $140 \times \frac{\pi}{180} = \frac{7\pi}{9}$

b) $\sin\alpha = \frac{1}{4}$



$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$
$= 1 - 2\sin^2\alpha$
$= 1 - 2\left(\frac{1}{16}\right)$
$= 1 - \frac{1}{8}$
$= \frac{7}{8}$

c) $\frac{1}{4}\sin\frac{\pi}{8}\cos\frac{\pi}{8}$

$= \frac{1}{4} \times \frac{1}{2}\sin 2\left(\frac{\pi}{8}\right)$

$= \frac{1}{8}\sin\frac{\pi}{4}$

$= \frac{1}{8} \times \frac{1}{\sqrt{2}}$

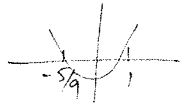
$= \frac{1}{8\sqrt{2}}$

$= \frac{\sqrt{2}}{16}$

d) $\cos\frac{\theta}{2} = \frac{1}{2}$ $\quad 0^\circ \leq \frac{\theta}{2} \leq 180^\circ$
$\frac{\theta}{2} = 60^\circ$
$\theta = 120^\circ$

4. a) $f(x) = 3x^3 - 2x^2 - 5x - 4$
$f'(x) = 9x^2 - 4x - 5$

Inc. when $f'(x) > 0$
$9x^2 - 4x - 5 > 0$
$9x \quad 5$
$x \quad -1$
$(9x+5)(x-1) > 0$



Inc when $x < -\frac{5}{9}$ OR $x > 1$

b) $\lim_{x \to 2} \frac{2(x^2-4)}{x-2}$
$= \lim_{x \to 2} 2(x+2)$
$= 8$

c) $f(2) = 8a - 6$.
$f'(x) = 3ax^2 \quad x > 0$
$f'(1) = 3a$
$f'(x) = 2ax \quad x < 0$
$f'(-1) = -2a$
$\therefore 3a - {}^{-}2a = -15$
$5a = -15$
$a = -3$
Now $f(2) = 8a - 6$
$= -24 - 6$
$= -30$.

5\. a) $m_1 = \frac{5}{6}$

$m_2 = \frac{-4}{3}$

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \left|\frac{\frac{5}{6} + \frac{4}{3}}{1 - \frac{5}{6}\times\frac{4}{3}}\right|$$

$$= \left|\frac{\frac{13}{6}}{1 - \frac{20}{18}}\right|$$

$$= \left|\frac{13}{6} \div -\frac{1}{9}\right|$$

$$= \left|\frac{13}{6} \times \frac{9}{-1}\right|$$

$$= \left|-\frac{39}{2}\right|$$

$$\theta = \tan^{-1}\frac{39}{2}$$

$$\theta = 87^\circ 4'$$

b) Point on Line $3x + 4y = 4$ is $(0, 1)$

$\overset{a}{3}x + \overset{b}{4}y \overset{c}{+1} = 0$ $\quad$ $\overset{x_1, y_1}{(0, 1)}$

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$$= \left|\frac{0 + 4 + 1}{\sqrt{9 + 16}}\right|$$

$$= \left|\frac{5}{5}\right|$$

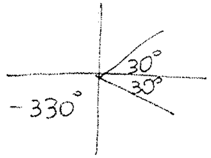
$= 1$ unit.

c) $\cos(\theta - 225)^\circ = \frac{\sqrt{3}}{2}$ $\qquad$ $-180^\circ \leq \theta \leq 270^\circ$

$-405^\circ \leq \theta - 225^\circ \leq 45^\circ$

$(\theta - 225)_b^\circ = 30^\circ$

Consider 1st + 4th Quads.



$\theta - 225^\circ = 30^\circ, -30^\circ, -390^\circ, -330^\circ$

$\therefore \theta = 255^\circ, 195^\circ, -165^\circ, -105^\circ$

6. (i) LHS $= \left(\frac{\sin 3x}{\sin x} - \frac{\cos 3x}{\cos x}\right)\left(\frac{\sin 3x}{\sin x} + \frac{\cos 3x}{\cos x}\right)$

$= \left(\frac{\cos x \sin 3x - \cos 3x \sin x}{\sin x \cos x}\right)\left(\frac{\cos x \sin 3x + \cos 3x \sin x}{\sin x \cos x}\right)$

$= \left(\frac{\sin(3x - x)}{\frac{1}{2}\sin 2x}\right)\left(\frac{\sin(3x+x)}{\frac{1}{2}\sin 2x}\right)$

$= \left(\frac{\sin 2x}{\frac{1}{2}\cancel{\sin 2x}}\right) \frac{2\sin 4x}{\sin 2x}$

$= \frac{4\sin 4x}{\sin 2x}$

$= \frac{4 \cdot 2\sin 2x \cos 2x}{\sin 2x}$

$= 8\cos 2x$

$=$ RHS.

(ii) $8\cos 2x = 8\sin x$

$\cos 2x = \sin x$

$1 - 2\sin^2 x = \sin x$

$2\sin^2 x + \sin x - 1 = 0$

$2\sin x \quad -1$

$\sin x \quad 1$

$(2\sin x - 1)(\sin x + 1) = 0.$

$\sin x = \frac{1}{2}$, $\sin x = -1$

$x = 30°, 150°, 270°$

But $x \neq 270$ as $\cos x$ in denom.

$\therefore$ $\boxed{\text{Only soln } x = 30°, 150°}$