Ext 1

# NORTH SYDNEY BOYS HIGH SCHOOL

**2016 Year 11 Task 1**

# Mathematics
## Extension 1

### General Instructions

- Reading time – **5 minutes**
- Working time – **80 minutes**
- Write on both sides of the lined paper in the booklet provided
- Write using blue or black **pen**
- Board approved calculators may be used
- All necessary working should be shown in every question
- Each new question is to be started on a **new page.**
- Marks may not be awarded for illegible or incomplete working.

**Class Teacher:**
(Please tick or highlight)

- O Mr Berry
- O Mr Hwang
- O Mr Ireland
- O Mr Jomaa
- O Ms Lee
- O Mr Lin
- O Mrs Ziaziaris

Student Name:______________________________

(To be used by the exam markers only.)

| Question No | 1 | 2 | 3 | 4 | 5 | 6 | 7 | Total | Total % |
|---|---|---|---|---|---|---|---|---|---|
| Mark | /15 | /14 | /10 | /16 | /12 | /11 | /12 | /90 | /100 |

**Question 1** (15 marks) Commence a NEW page **Mark**

(a) An atom of a certain metal weighs $7{\cdot}31\times10^{-26}$ grams. 2

How many atoms of this metal are in a 2 kilogram block?

Give your answer correct to three significant figures.

(b) If $x=\sqrt{3}+1$, find the value of $x^2-\dfrac{1}{x^2}$ as a single fraction with a rational denominator. 3

(c) Simplify $\dfrac{a^2-a-12}{a^2-9}\times\dfrac{6a-18}{18a}$ 2

(d) Simplify $\dfrac{3^n+6^n}{2+2^{n+1}}$ 2

(e) Factorise $6x^2+5xy-4y^2$ 2

(f) Factorise $x^5+32$ 2

(g) Factorise $a^2-b^2+2b-1$ 2

**Question 2** (14 marks) Commence a NEW page **Marks**

Sketch graphs of the following curves, **indicating any intercepts and asymptotes:**

(a) $y=\sqrt{2-x^2}$ 2

(b) $y=(x^2-1)(x-3)$ 2

(c) $y=3-\dfrac{1}{x-2}$ 3

(d) $y=2^{2-x}$ 2

(e) $y=\dfrac{x^2+3x-10}{x-2}$ 2

(f) Sketch $y=\sqrt{x+1}$ and hence sketch $y=\dfrac{1}{\sqrt{x+1}}$ showing any intercepts or asymptotes. 3

**Question 3** (10 marks) Commence a NEW page **Marks**

(a) Write down the Domain and Range of these functions:

(i) $y = x^2 + 4x + 9$ 2

(ii) $y = \dfrac{4}{\sqrt{25 - x^2}}$ 2

(b) If $f(x) = \dfrac{x+5}{x-3}$, then

(i) Find the equation of the *inverse* function $f^{-1}(x)$. 2

(ii) Find the coordinates of any points where the graphs of $y = f(x)$ and $y = f^{-1}(x)$ intersect. 2

(c) What is the natural domain of $f(x) = \sqrt{x-4} + \sqrt{10-x}$ ? 2

**Question 4** (16 marks) Commence a NEW page **Marks**

(a) Simplify $\tan(90° - \theta)\sin(360° - \theta)$, leaving your answer in terms of $\theta$. 2

(b) Prove that $\cot A(\operatorname{cosec} A - \cot A) = \dfrac{\cos A}{1 + \cos A}$ 3

(c) If $\sin\theta = \dfrac{1}{2}$ and $\tan\theta = \dfrac{-1}{\sqrt{3}}$, find $\theta$ for $0° \le \theta \le 360°$ 2

(d) Solve these trigonometric equations:

(i) $\cos\theta = \sec\theta$, $0° \le \theta \le 360°$ 2

(ii) $\sin 2\theta = -\dfrac{1}{\sqrt{2}}$, $-180° \le \theta \le 180°$ 3

(iii) $2\sin^2\theta + 3\cos\theta = 0$, $0° \le \theta \le 360°$ 2

(iv) $2\cos(\theta - 30°) = 0.7$, $0° \le \theta \le 360°$ 2

**Question 5** (12 marks) Commence a NEW page **Marks**

(a) Use about one-third of a page to sketch $y = 2 - 3\sin 2\theta$ for $0° \le \theta \le 360°$ 3

(b) Bernie sails 30km from Port Cruz on a bearing of 300°T. Due to Cyclone Hillary he is then forced to change course and sail 80km on a bearing of 020°T to Trumpville.

(i) Sketch a diagram to show this information (be sure to show North!) 2

(ii) Calculate how far Trumpville is from Port Cruz, to the nearest km. 3

(c) A triangle has angles of size 45°, 55°, and 80°, and its longest side is 10cm long.

(i) Sketch this situation. 1

(ii) Show that the area $A$ of this triangle is given by the formula 3

$$A = \frac{50 \sin 135° \cos 35°}{\cos 10°}$$

**Question 6** (11 marks) Commence a NEW page **Mark**

(a) Solve $(x-4)^2 = 11$ 2

(b) Solve $2x^2 + 3x - 5 > 0$ 2

(c) Solve $|2x-1| < 5$ 2

(d) Solve $|2x-1| = x-2$ 2

(e) Solve $\dfrac{4x}{x-3} \le 5$ 3

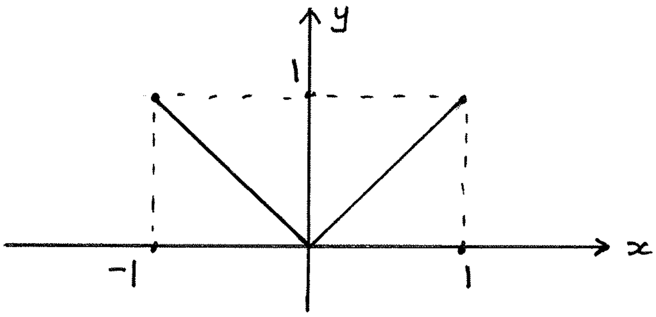
**Question 7** (12 marks) Commence a NEW page **Mark**

(a) Sketch $y = \sqrt{x^2 - 2}$ 2

(b) Solve $|x+1| + |x-3| = 7$ 2

(c) Given that $x + y = 1$ and $x^3 + y^3 = 16$, evaluate $x^2 + y^2$. 3

(d) Use the graph of $y = f(x)$ below to sketch the graph of $y = f(2x-4)$. 2




(e) You are given that $f(x-1) + f(1-x) = 2x^2 + 8$.

Clearly, $2x^2 + 8$ is an even expression, since $2x^2 + 8$ equals $2(-x)^2 + 8$ for any real number $x$. But is the function $f(x)$ itself ODD, EVEN or NEITHER? Justify your conclusion! 3

**END OF EXAM**

# 2016 Year 11 Ext. 1 Task 1

Suggested Solutions

**Q1**

(a) Number atoms $= \dfrac{2000}{7.31 \times 10^{-26}}$

$\doteqdot 2.73597811 \times 10^{28}$

$= 2.74 \times 10^{28}$ (3 sig. figs.) ✓✓

(b) $x^2 = (\sqrt{3}+1)^2 = 4 + 2\sqrt{3}$ ✓

$\therefore x^2 - \dfrac{1}{x^2} = 4 + 2\sqrt{3} - \dfrac{1}{4+2\sqrt{3}} \cdot \dfrac{4-2\sqrt{3}}{4-2\sqrt{3}}$ ✓

$= 4 + 2\sqrt{3} - \dfrac{4-2\sqrt{3}}{4}$

$= \dfrac{12+10\sqrt{3}}{4} \quad \left(= \dfrac{6+5\sqrt{3}}{2}\right)$ ✓

(c) $\dfrac{a^2-a-12}{a^2-9} \times \dfrac{6a-18}{18a} = \dfrac{(a-4)\cancel{(a+3)}}{\cancel{(a+3)}\cancel{(a-3)}} \cdot \dfrac{6\cancel{(a-3)}}{18a}$ ✓

$= \dfrac{a-4}{3a}$ ✓

(d) $\dfrac{3^n + 6^n}{2 + 2^{n+1}} = \dfrac{3^n \cancel{(1+2^n)}}{2\cancel{(1+2^n)}}$ ✓

$= \dfrac{3^n}{2}$ ✓

(e) $6x^2 + 5xy - 4y^2 = (3x+4y)(2x-y)$ ✓✓

(f) $x^5 + 32 = (x+2)(x^4 - 2x^3 + 4x^2 - 8x + 16)$ ✓✓

(g) $a^2 - b^2 + 2b - 1 = a^2 - (b^2 - 2b + 1)$

$= a^2 - (b-1)^2 = (a+b-1)(a-b+1)$ ✓✓

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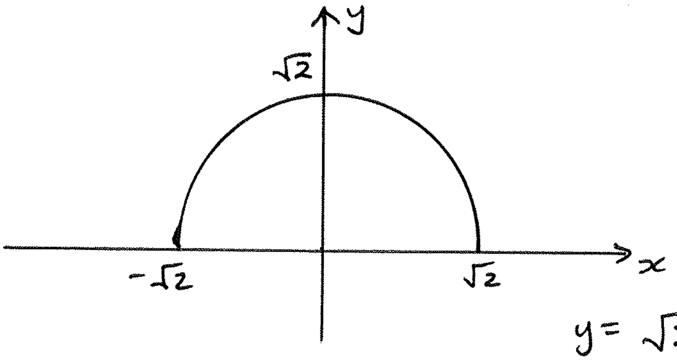
**Q2**

(a)

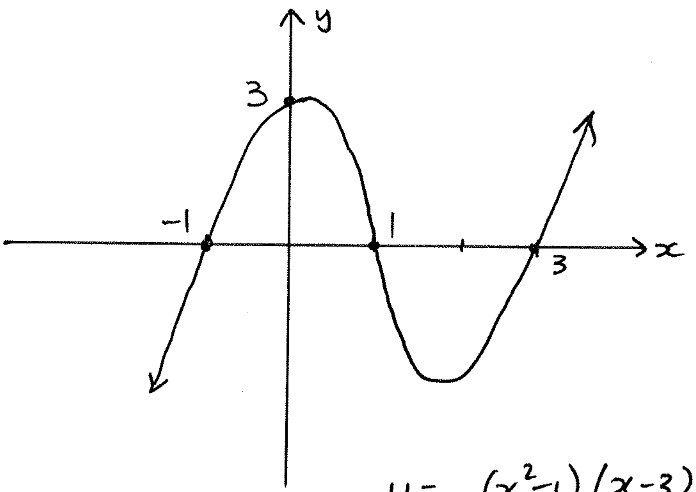



$y = \sqrt{2-x^2}$

(✱) Delete 1 mark for unlabelled axes (once).

✓ ✓

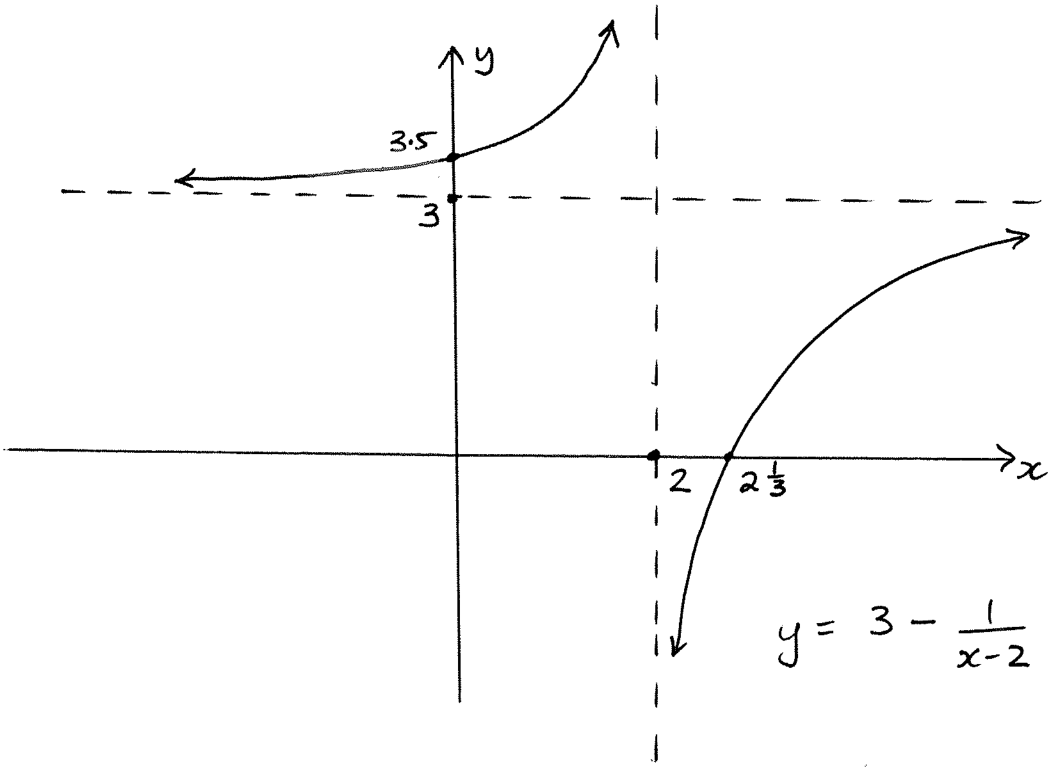
(b)




$y = (x^2-1)(x-3)$
$= (x-1)(x+1)(x-3)$

✓ ✓

(c)

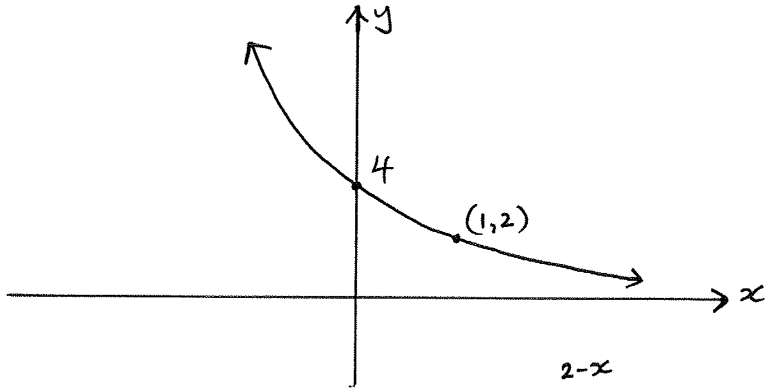



$y = 3 - \dfrac{1}{x-2}$

✓ ✓ ✓

$\boxed{Q2}$ ctd

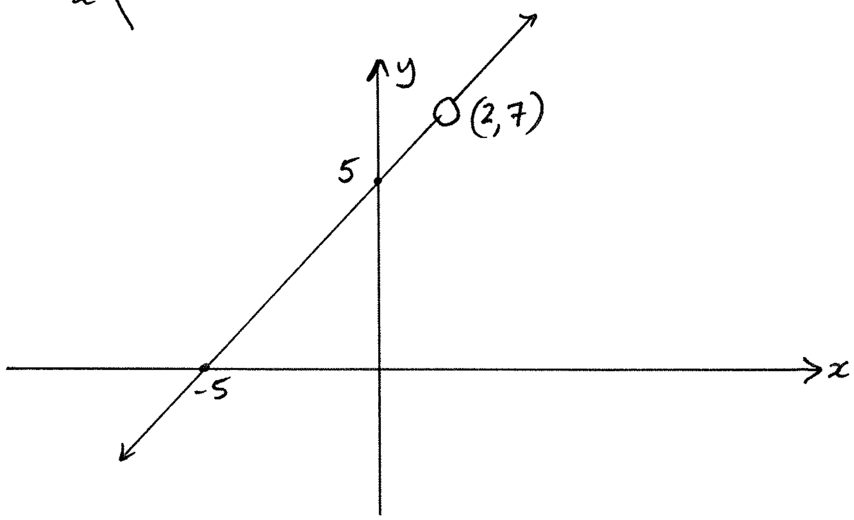
(d)




$$y = 2^{2-x}$$
$$= 2^2 \cdot 2^{-x}$$
$$= 4 \cdot 2^{-x}$$

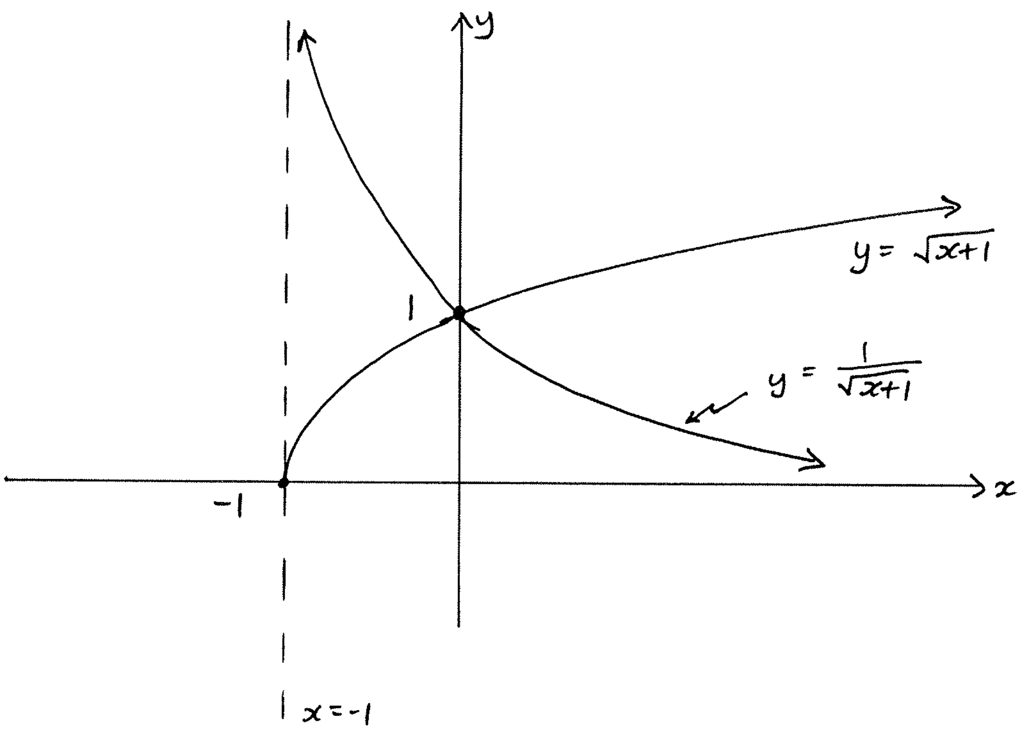
✓✓

(e) $y = \dfrac{(x+5)\cancel{(x-2)}}{\cancel{x-2}}$ $\therefore$ $y = x+5$ $(x \neq 2)$




✓✓

(f)




✓ $\sqrt{x+1}$

✓✓ $\frac{1}{\sqrt{x+1}}$

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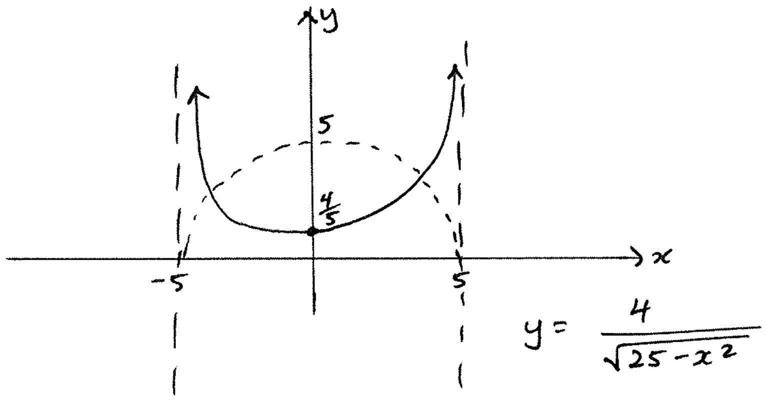
**Q3** (a) (i) $y = x^2 + 4x + 9 = (x+2)^2 + 5$

$\therefore$ D: all real $x$ ✓ D

R: $y \geq 5$ ✓ R

(ii)




$\therefore$ D: $-5 < x < 5$ ✓ D

R: $y \geq \frac{4}{5}$ ✓ R

(b) $y = f(x) = \dfrac{x+5}{x-3}$

(i) For inverse, $x = \dfrac{y+5}{y-3}$

$\therefore xy - 3x = y + 5$ ✓

$(x-1)y = 3x + 5$ $\quad \therefore y = \dfrac{3x+5}{x-1}$

i.e. $f^{-1}(x) = \dfrac{3x+5}{x-1}$ ✓

(ii) $f(x)$ & $f^{-1}(x)$ intersect on $y = x$.

$\therefore$ solve simultaneously $y = x$ & $y = \dfrac{x+5}{x-3}$

$\therefore \dfrac{x+5}{x-3} = x \quad \therefore x^2 - 3x = x + 5$
$x^2 - 4x - 5 = 0 \quad \therefore (x-5)(x+1) = 0$

$\therefore x = 5$ or $-1$ $\quad \therefore$ points are $(-1,-1)$, $(5,5)$. ✓✓

(c) For $y = \sqrt{x-4}$, D: $x \geq 4$ ✓

For $y = \sqrt{10-x}$, D: $x \leq 10$

$\therefore$ For $y = \sqrt{x-4} + \sqrt{10-x}$, D: $4 \leq x \leq 10$. ✓

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Q4

(a) $\tan(90-\theta)\cdot\sin(360-\theta) = \cot\theta \cdot -\sin\theta$

$= \dfrac{\cos\theta}{\cancel{\sin\theta}} \cdot -\cancel{\sin\theta}$

$= -\cos\theta$ ✓✓

(b) $LHS = \cot A\,(\operatorname{cosec} A - \cot A)$

$= \dfrac{\cos A}{\sin A}\left(\dfrac{1}{\sin A} - \dfrac{\cos A}{\sin A}\right)$ ✓

$= \dfrac{\cos A\,(1-\cos A)}{\sin^2 A}$

$= \dfrac{\cos A\,(1-\cos A)}{1-\cos^2 A} = \dfrac{\cos A\,\cancel{(1-\cos A)}}{\cancel{(1-\cos A)}(1+\cos A)}$ ✓✓

$= \dfrac{\cos A}{1+\cos A} = RHS$

(c) $\left\{\begin{array}{l}\sin > 0 \text{ and } \tan < 0 \quad \therefore \theta \text{ is in Q2} \\ \text{related angle is } 30^\circ\end{array}\right.$

$\therefore \theta = 150^\circ$ ✓✓

(d) (i) $\cos\theta = \sec\theta, \quad 0^\circ \le \theta \le 360^\circ.$

$= \dfrac{1}{\cos\theta}$

$\therefore \cos^2\theta = 1$

$\therefore \cos\theta = \pm 1$

$\therefore \theta = 0^\circ, 180^\circ, 360^\circ.$ ✓✓

$\boxed{Q4}$ – ctd

(d) (ii) $\sin 2\theta = -\frac{1}{\sqrt{2}}$ , $-180° \le \theta \le 180°$

$\therefore -360° \le 2\theta \le 360°$

Related angle is $45°$, Quadrants are 3 & 4.

$\therefore \quad 2\theta = 225°, 315°, -45°, -135°$

$\theta = 112.5°, 157.5°, -22.5°, -67.5°$ ✓✓✓

(iii) $2\sin^2\theta + 3\cos\theta = 0, \quad 0° \le \theta \le 360°$

$\therefore \quad 2(1-\cos^2\theta) + 3\cos\theta = 0, \quad 2 - 2\cos^2\theta + 3\cos\theta = 0$

$\therefore \quad 2\cos^2\theta - 3\cos\theta - 2 = 0$

$\therefore \quad (2\cos\theta + 1)(\cos\theta - 2) = 0$

$\therefore \quad \cos\theta = -\frac{1}{2}$ or $\cos\theta = 2$

not possible

$\therefore \quad \theta = 120°, 240°.$ ✓✓

(iv) $2\cos(\theta - 30°) = 0.7, \quad 0° \le \theta \le 360°$

$\therefore \quad \cos(\theta - 30°) = 0.35, \quad -30° \le \theta - 30° \le 330°$

Related angle is $69°31'$, in Q1 or Q4

$\therefore \quad \theta - 30° = 69°31'$ or $290°29'$

$\therefore \quad \theta = 99°31'$ or $320°29'$ ✓✓

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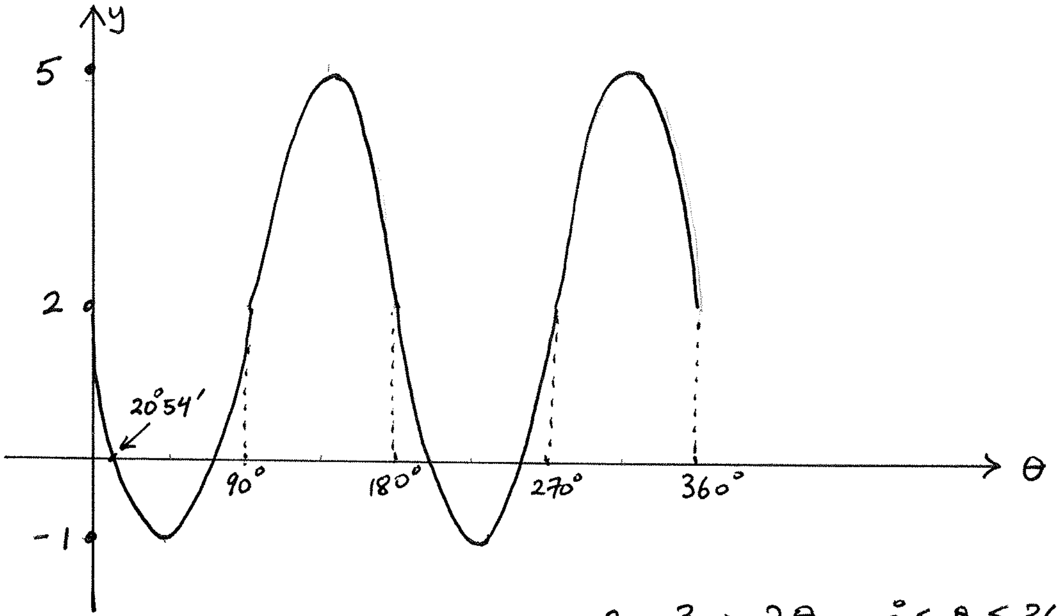
**Q5**

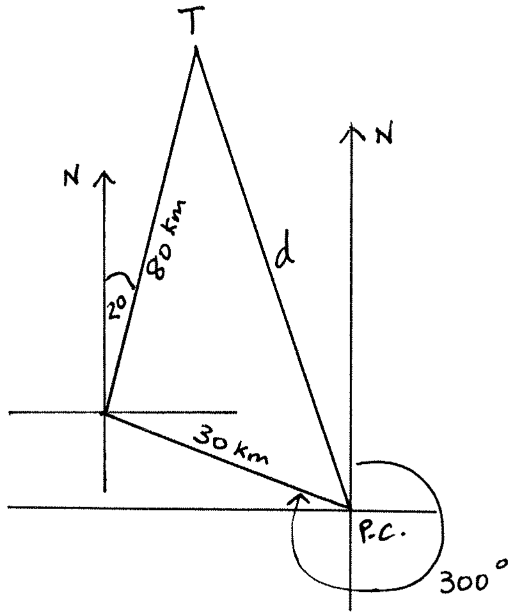
(a)




$y = 2 - 3\sin 2\theta, \quad 0^\circ \le \theta \le 360^\circ$ ✓✓✓

(b)

(i)




(* unlabelled North → deduct 1 mk.) ✓✓

(ii) Large angle in triangle $= 70^\circ + 30^\circ = 100^\circ$ ✓

$\therefore d^2 = 30^2 + 80^2 - 2(30)(80)\cos 100^\circ$ ✓

$\doteqdot 8133 \cdot 511253$

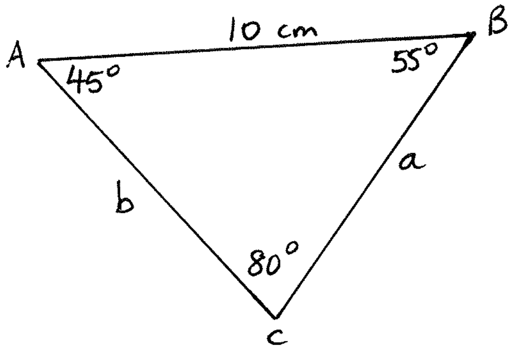
$\therefore d \doteqdot 90 \cdot 18598$

$\therefore$ distance $= 90$ km (nearest km) ✓

$\boxed{\text{Q5}}$-ctd

(c)

(i)




(ii) We use sine rule to find one other side, then use the area formula.

eg. $\dfrac{a}{\sin 45°} = \dfrac{10}{\sin 80°}$

$\therefore \quad a = \dfrac{10 \sin 45°}{\sin 80°}$

Thus area $= \frac{1}{2} \cdot 10 \cdot a \cdot \sin 55°$

$= \frac{1}{2} \cdot 10 \cdot \dfrac{10 \sin 45°}{\sin 80°} \cdot \sin 55°$

$= 50 \dfrac{\sin 45° \sin 55°}{\sin 80°}$

$\therefore$ area $= \dfrac{50 \sin 135° \cos 35°}{\cos 10°}$, as required.

(since $\sin\theta = \sin(180 - \theta)$ and $\sin\theta = \cos(90 - \theta)$).

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**Q6**

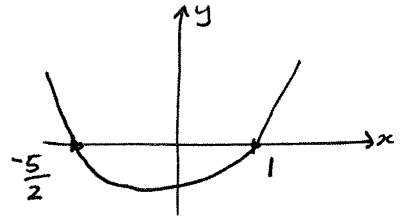
(a) $(x-4)^2 = 11$

$\therefore \ x-4 = \pm\sqrt{11}$

$\therefore \ x = 4+\sqrt{11}$ or $4-\sqrt{11}$.

(b) $2x^2 + 3x - 5 > 0$

$(2x+5)(x-1) > 0$




$\therefore \ x < -\frac{5}{2}$ or $x > 1$.

(c) $|2x-1| < 5$

$\therefore \ -5 < 2x-1 < 5$

$\therefore \ -4 < 2x < 6$

$\therefore \ -2 < x < 3$

(d) $|2x-1| = x-2$

$\therefore \ 2x-1 = x-2$ or $2x-1 = -x+2$

$x = -1$      $3x = 3$

$x = 1$

$\therefore \ x=-1$ or $x=1$

Test: $x=-1 \rightarrow$ LHS $= |-2-1| = 3$, RHS $= -1-2 = -3$   $\times$ not a solution

$x=1 \rightarrow$ LHS $= |2-1| = 1$, RHS $= 1-2 = -1$   $\times$ not a solution

$\therefore$ no solutions.

$\boxed{Q6}$-ctd

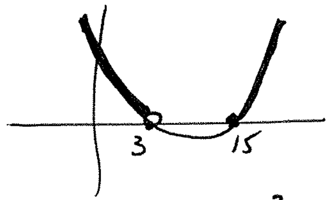
(e) $\dfrac{4x}{x-3} \le 5 \qquad (\therefore\ x \ne 3)$

$\therefore\ 4x(x-3) \le 5(x-3)^2$

$\therefore\ 5(x-3)^2 - 4x(x-3) \ge 0$

$(x-3)\left[5(x-3) - 4x\right] \ge 0$

$(x-3)(x-15) \ge 0$

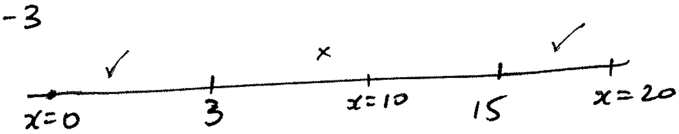



$\therefore\ x < 3$ or $x \ge 15$

[ALT. 1 (Critical point method)

$x \ne 3 \quad \therefore \quad x = 3$ is critical point.

$\dfrac{4x}{x-3} = 5 \Rightarrow 4x = 5x - 15 \quad \therefore \quad x = 15$ is critical point.




Testing a point in each region,

eg. at $x=0 \rightarrow \dfrac{0}{-3} = 0 < 5 \quad \therefore$ ✓

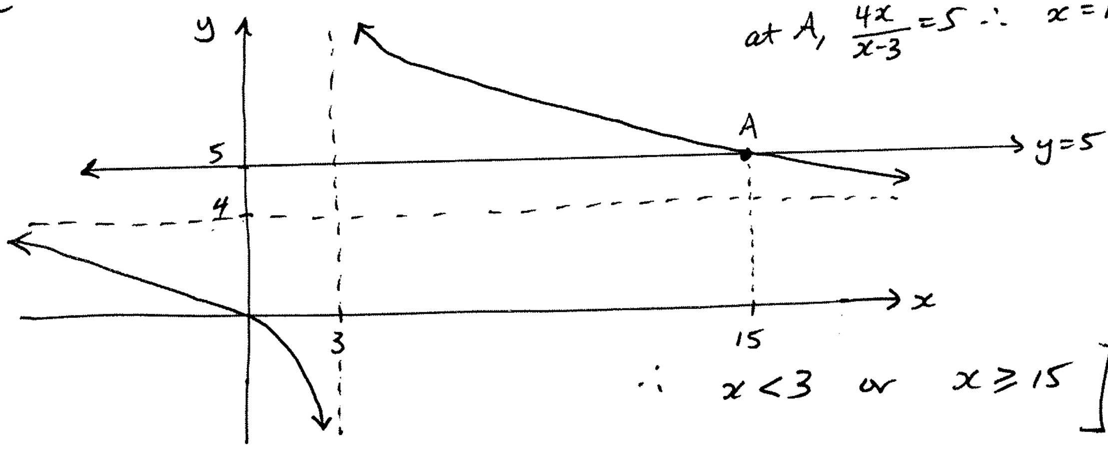
at $x=10 \rightarrow \dfrac{40}{7} > 5 \quad \therefore$ ×

at $x=20 \rightarrow \dfrac{80}{17} < 5 \quad \therefore$ ✓

Thus $x < 3$ or $x \ge 15$ ]

[ALT. 2 (Graphical) $\quad y = \dfrac{4x}{x-3} = \dfrac{4(x-3)+12}{x-3} = 4 + \dfrac{12}{x-3}$

at A, $\dfrac{4x}{x-3} = 5 \quad \therefore \quad x = 15$




$\therefore\ x < 3$ or $x \ge 15$ ]

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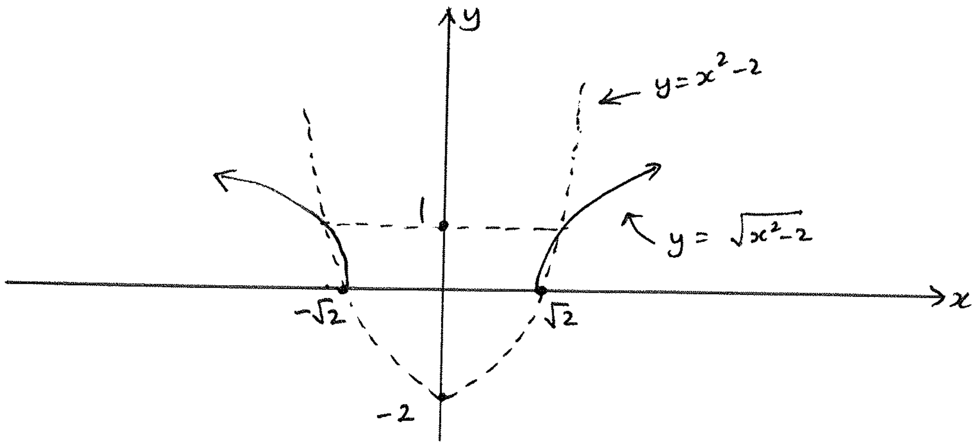
**Q7**

(a)




✓✓

(b) $|x+1| + |x-3| = 7$

On a numberline, the distance of $x$ from $-1$ plus the distance of $x$ from 3 equals 7:

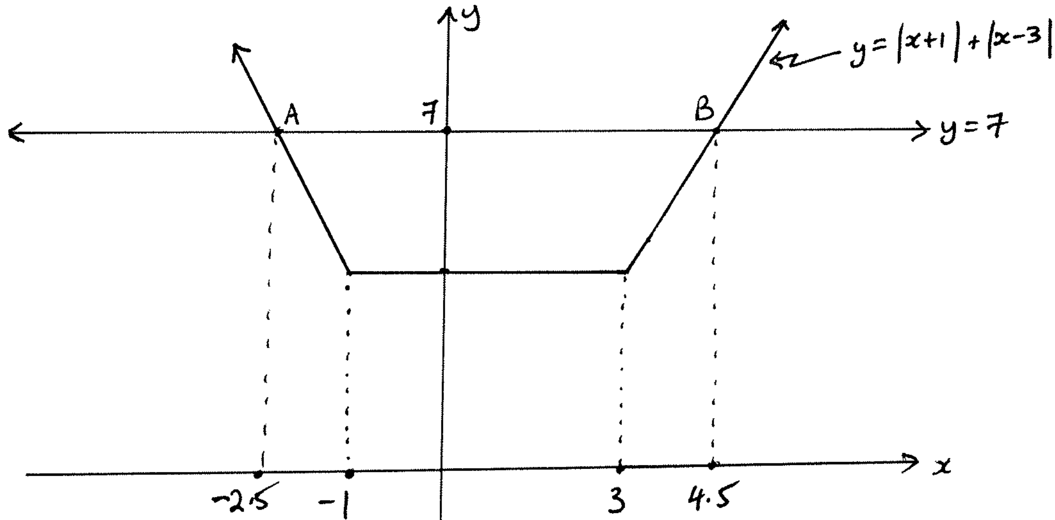
$$\underbrace{\quad}_{1.5}\ -1\ \underbrace{\quad\quad}_{4}\ 3\ \underbrace{\quad}_{1.5}$$

This will be satisfied by points 1.5 units to the right of $x=3$ or 1.5 units to the left of $x=-1$.

$\therefore\ x = 4.5$ or $x = -2.5$.

✓✓

[ ALT.




at A, $-x-1 + -x+3 = 7$

$-2x = 5$

$x = -2.5$

at B, $x+1 + x-3 = 7$

$2x = 9$

$x = 4.5$

$\therefore\ x = -2.5$ or $4.5$. ]

$\boxed{Q7}$-ctd

(c) $x+y=1$ and $x^3+y^3=16$. Find $x^2+y^2$.

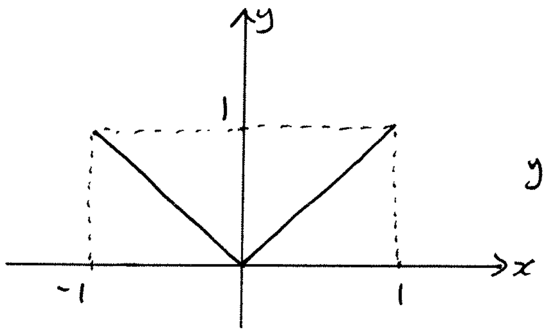
$$x^3+y^3 = (x+y)(x^2-xy+y^2)$$
$$\therefore\ 16 = 1\,(x^2+y^2-xy)$$
$$\therefore\ x^2+y^2 = 16+xy$$

But $(x+1)^2 = x^2+2xy+y^2$

$$1^2 = x^2+y^2+2xy$$
$$\therefore\ x^2+y^2 = 1-2xy$$
$$\therefore\ 16+xy = 1-2xy \quad \therefore\ xy=-5$$

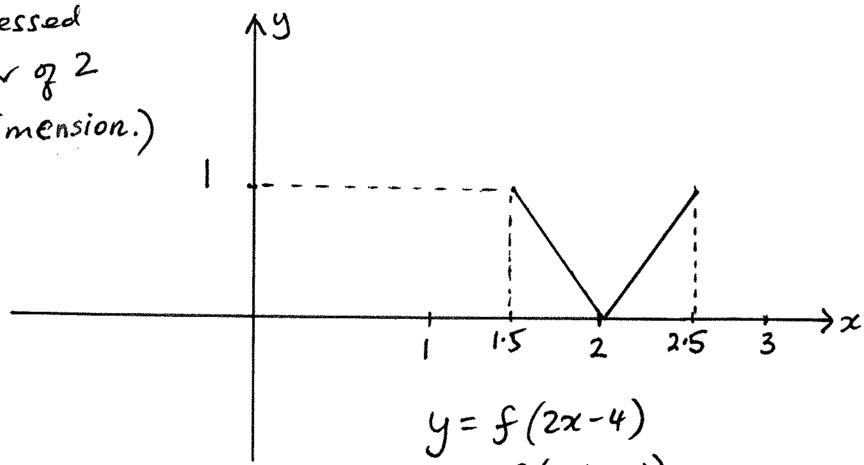
Thus $x^2+y^2 = 16-5 = 11$

(d)




$y=f(x)$

(The graph has been shifted 2 units to the right, and compressed by a factor of 2 in the $x$ dimension.)




$$y=f(2x-4)$$
$$=f(2(x-2))$$

Q7 ctd

(e) $f(x-1) + f(1-x) = 2x^2 + 8$

$f(x)$ has to be odd, or even, or neither.
(no other possibilities!)

IF $f(x)$ is odd, then $f(-x) = -f(x)$.

$\therefore \; f(x-1) + f(1-x) = f(x-1) - f(x-1)$
$= 0$, for all $x$

But $2x^2+8 \neq 0$ for <u>any</u> $x$.
$\therefore$ $f(x)$ is not odd.

IF $f(x)$ is even, then

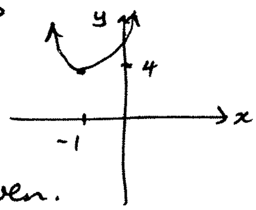
$f(x-1) + f(1-x) = 2 \cdot f(x-1)$

$\therefore \; f(x-1) = x^2 + 4$

$\therefore \; f(x) = (x+1)^2 + 4$

$= x^2 + 2x + 5$

But this is not even! viz:




Contradiction! $\therefore$ $f(x)$ is not even.

So if $f(x)$ cannot be odd, and cannot be even,
$\therefore$ it must be <u>neither</u>.

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