NORTH SYDNEY BOYS HIGH SCHOOL

MATHEMATICS EXTENSION 1

2016 Preliminary Course Assessment Task 2

9th of June, 2016

General instructions

- Working time – 50 minutes.
- Commence each new question on a new page.
- Write using blue or black pen. Where diagrams are to be sketched, these may be done in pencil.
- Board approved calculators may be used.
- All necessary working should be shown in every question.
- Attempt **all** questions.
- At the conclusion of the examination, bundle the booklets used in the correct order within this paper and hand to examination supervisors.

Teacher (please ✔)

- ◯ Mr Berry
- ◯ Mr Hwang
- ◯ Mr Ireland
- ◯ Dr Jomaa
- ◯ Miss Lee
- ◯ Mr Lin
- ◯ Ms Ziaziaris

NAME ..... **# BOOKLETS USED**: .....

Marker's use only.

QUESTION	1	2	3	4	5	**Total**	%
MARKS	/10	/10	/9	/11	/8	/48	

Question 1 (10 Marks) Commence a NEW page. **Marks**

(a) Express 165° in radians. **1**

(b) Solve $\sin 2\theta = -1$ where $0^\circ \leq \theta \leq 360^\circ$. **3**

(c) Show that $\dfrac{\sin x}{1+\cos x} + \dfrac{1+\cos x}{\sin x} = 2\operatorname{cosec} x$. **3**

(d) Find the exact value of $\cos \dfrac{7\pi}{12}$. **3**

Question 2 (10 Marks) Commence a NEW page. **Marks**

(a) Find the derivative of each of the following

i. $x^3 - 5x^2 + 3x - 1$ **1**

ii. $(1-3x^2)^5$ **2**

iii. $x^2\sqrt{4-x}$ **2**

iv. $\dfrac{x+2}{1-3x}$ **2**

(b) If $f(x) = x^2 - 2x$, use first principles to find $f'(x)$ **3**

Question 3 (9 Marks) Commence a NEW page. **Marks**

(a) The line m has equation $3x - 5y - 8 = 0$. **3**
The line p intersects line m at an acute angle of 45°. Find the exact gradient(s) of the line p.

(b) Find the coordinates of the point $P(x, y)$ which divides the interval AB externally in the ratio $1:3$ with $A(1, 4)$ and $B(5, 2)$. **3**

(c) The equation $(x - 2y + 2) + k(x - 4y) = 0$ represents a family of straight lines passing through a fixed point P. **3**
For what value of k is the line in the family parallel to the straight line $x + y = 2$?

Question 4 (11 Marks) Commence a NEW page. **Marks**

(a) Find the equation of the tangent to the curve $y = 3x^2 - 10x + 6$ at the point $P(2, -2)$. **2**

(b) At what point on the curve $y = (x - 2)(x + 3)$ is the tangent perpendicular to $3x - y + 5 = 0$? **3**

(c) Find the shortest distance between the lines $y = -\frac{1}{4}x + 1$ and $x + 4y + 12 = 0$. **3**

(d) Sketch the region $y < \sqrt{3 - x^2}$. **3**

Question 5 (8 Marks) Commence a NEW page. **Marks**

(a) The angle of elevation of a hill OT, at a point A due south of it is $20°$, and at point B which is on a bearing of $060°T$ from O, the angle of elevation is $40°$. The distance AB is 500m. Draw a diagram to represent all the given information. Find the height of the hill to the nearest metre. **4**

(b) i. Show that $\sin(A + B) + \sin(A - B) = 2 \sin A \cos B$. **1**

ii. Hence solve $\sin 3x + \sin 4x + \sin 5x = 0$ for $0° \leq x \leq 180°$. **3**

End of paper.

2016 Y11 AT2 Suggested Solutions

Marks

Question 1

1 a) $165 \times \frac{\pi}{180} = \frac{11\pi}{12}$

3 b) $\sin 2\theta = -1 \qquad 0 \leq \theta \leq 360$

$0 \leq 2\theta \leq 720$

$2\theta = 270, 630$

$\theta = 135, 315$

3 c) $LHS = \frac{\sin x}{1+\cos x} + \frac{1+\cos x}{\sin x}$

$= \frac{\sin^2 x + (1+\cos x)^2}{\sin x (1+\cos x)}$

$= \frac{\sin^2 x + 1 + 2\cos x + \cos^2 x}{\sin x (1+\cos x)}$

$= \frac{2 + 2\cos x}{\sin x (1+\cos x)}$

$= \frac{2(1+\cos x)}{\sin x (1+\cos x)}$

$= \frac{2}{\sin x}$

$= 2\operatorname{cosec} x$

$= RHS$

3 d) $\cos \frac{7\pi}{12} = \cos\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$

$= \cos\frac{\pi}{4}\cos\frac{\pi}{3} - \sin\frac{\pi}{4}\sin\frac{\pi}{3}$

$= \frac{1}{\sqrt{2}} \times \frac{1}{2} - \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}$

$= \frac{1-\sqrt{3}}{2\sqrt{2}}$

Marks

Question 2

1 a) i) $3x^2 - 10x + 3$

2 ii) $5(1-3x^2)^4 \cdot (-6x)$

$= -30x(1-3x^2)^4$

2 iii) $u = x^2 \qquad v = (4-x)^{\frac{1}{2}}$

$u' = 2x \qquad v' = \frac{1}{2}(4-x)^{-\frac{1}{2}} \cdot (-1)$

$$y' = 2x\sqrt{4-x} - \frac{x^2}{2\sqrt{4-x}}$$

2 iv) $u = x+2 \qquad v = 1-3x$

$u' = 1 \qquad v' = -3$

$$y' = \frac{1-3x+3(x+2)}{(1-3x)^2}$$

$$= \frac{7}{(1-3x)^2}$$

3 b) $f(x) = x^2 - 2x$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - 2(x+h) - x^2 + 2x}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 2x - 2h - x^2 + 2x}{h}$$

$$= \lim_{h \to 0} \frac{h(2x+h-2)}{h}$$

$$= \lim_{h \to 0} (2x+h-2)$$

$$= 2x - 2$$

Marks

Question 3

3 a) $3x - 5y - 8 = 0$

$m_2 = \frac{3}{5}$

$\tan\theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$

$\tan 45 = \left| \frac{m - \frac{3}{5}}{1 + \frac{3m}{5}} \right|$

$\frac{m - \frac{3}{5}}{1 + \frac{3m}{5}} = 1$ $\qquad$ $\frac{m - \frac{3}{5}}{1 + \frac{3m}{5}} = -1$

$\frac{5m - 3}{5 + 3m} = 1$ $\qquad$ $\frac{5m - 3}{5 + 3m} = -1$

$5m - 3 = 5 + 3m$ $\qquad$ $5m - 3 = -5 - 3m$

$2m = 8$ $\qquad$ $8m = -2$

$m = 4$ $\qquad$ $m = -\frac{1}{4}$

3 b) $-1 : 3$

$A(1, 4)$ $\qquad$ $B(5, 2)$

$P(x, y) = \left(\frac{-5 + 3}{-1 + 3}, \frac{-2 + 12}{-1 + 3} \right)$

$= (-1, 5)$

Marks

3 c) $(x-2y+2)+k(x-4y)=0$

$$m = -\frac{(-2-4k)}{(1+k)}$$

$$-1 = \frac{2+4k}{1+k}$$

$$-1-k = 2+4k$$

$$-3 = 5k$$

$$k = -\frac{3}{5}$$

Marks

Question 4

2 a) $y' = 6x - 10$

$m = 12 - 10$

$= 2$

Eqn of tangent:

$y + 2 = 2(x - 2)$

$y = 2x - 6$

3 b) $3x - y + 5 = 0$

$m = 3$

$m_\perp = -\frac{1}{3}$

$y = (x - 2)(x + 3)$

$= x^2 + x - 6$

$y' = 2x + 1$

$2x + 1 = -\frac{1}{3}$

$6x + 3 = -1$

$6x = -4$

$x = -\frac{2}{3}$

$y = \left(-\frac{2}{3}\right)^2 + \left(-\frac{2}{3}\right) - 6$

$= -\frac{56}{9}$

$\therefore$ point is $\left(-\frac{2}{3}, -\frac{56}{9}\right)$

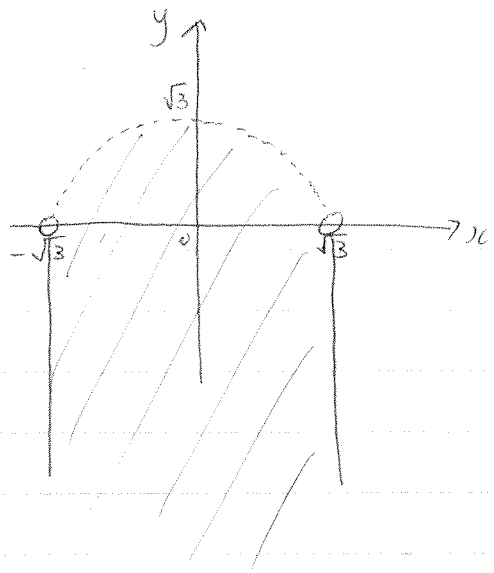
Marks

3 c) $(0, 1)$ lies on $y = -\frac{1}{4}x + 1$

$$d_{\perp} = \frac{|1(0) + 4(1) + 12|}{\sqrt{1^2 + 4^2}}$$

$$= \frac{16}{\sqrt{17}}$$

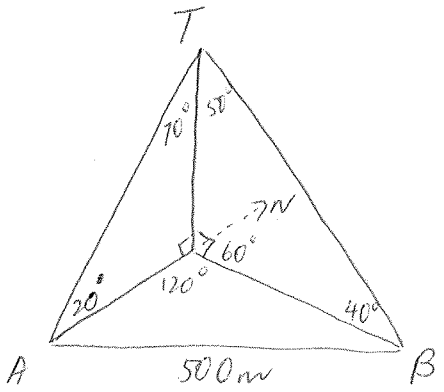
3 d)



Marks

Question 5

4 a)



$\tan 70° = \frac{AO}{h}$ $\qquad$ $\tan 50° = \frac{BO}{h}$

$AO = h\tan 70$ $\qquad$ $BO = h\tan 50$

$500^2 = (h\tan 70)^2 + (h\tan 50)^2 - 2(h\tan 70)(h\tan 50)\cos 120$

$= h^2[\tan^2 70 + \tan^2 50 - 2\tan 70 \tan 50 \cos 120]$

$h > 0$

$h = \sqrt{\frac{500^2}{\tan^2 70 + \tan^2 50 - 2\tan 70 \tan 50 \cos 120}}$

$= 143\,m$

1 b) i) $LHS = \sin(A+B) + \sin(A-B)$

$= \sin A\cos B + \cos A \sin B + \sin A \cos B - \cos A \sin B$

$= 2\sin A \cos B$

$= RHS$

3 ii) $\sin 3x + \sin 4x + \sin 5x = \sin(4x-x) + \sin 4x + \sin(4x+x)$

$\sin(4x-x) + \sin(4x+x) = 2\sin 4x \cos x$

$2\sin 4x \cos x + \sin 4x = 0$ $\qquad$ $0 \le x \le 180$

$\sin 4x (2\cos x + 1) = 0$ $\qquad$ $0 \le 4x \le 720$

$\sin 4x = 0$ $\qquad$ $2\cos x + 1 = 0$

$4x = 0, 180, 360, 540, 720$ $\qquad$ $\cos x = -\frac{1}{2}$

$x = 0, 45, 90, 135, 180$ $\qquad$ $x = 120$

$\therefore x = 0°, 45°, 90°, 120°, 135°, 180°$