Ext 1

NORTH SYDNEY BOYS HIGH SCHOOL

2017 HSC ASSESSMENT TASK 1

Mathematics
Extension 1

General Instructions

- Working time – 60 minutes
- Write on the lined paper in the booklet provided
- Write using blue or black pen
- Board approved calculators may be used
- All necessary working should be shown in every question
- Each new question is to be started on a **new page.**
- Attempt all questions

Class Teacher:
(Please tick or highlight)

- O Mr Berry
- O Mr Hwang
- O Mr Ireland
- O Dr Jomaa
- O Ms Lee
- O Mr Lin
- O Ms Ziaziaris

Student Name/Number: ___________________________

(To be used by the exam markers only.)

Question No	1	2	3	4	Total	Total %
Mark	$\frac{\quad}{13}$	$\frac{\quad}{3}$	$\frac{\quad}{11}$	$\frac{\quad}{13}$	$\frac{\quad}{40}$	$\frac{\quad}{100}$

Question 1 – Integration *(13 Marks)*

(a) Find:

(i) $\int (5x+2)dx$ **1**

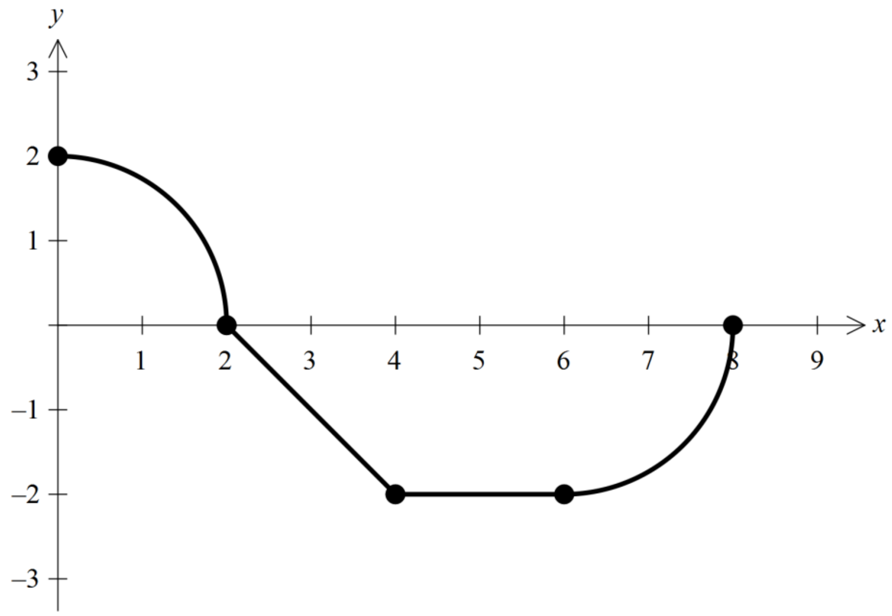
(ii) $\int \frac{x^3+1}{x^2}\, dx$ **2**

(b) Evaluate:

(i) $\int_{-1}^{1} (x^3+1)\, dx$ **2**

(ii) $\int_{1}^{4} \frac{dx}{\sqrt{x}}$ **2**

(c)

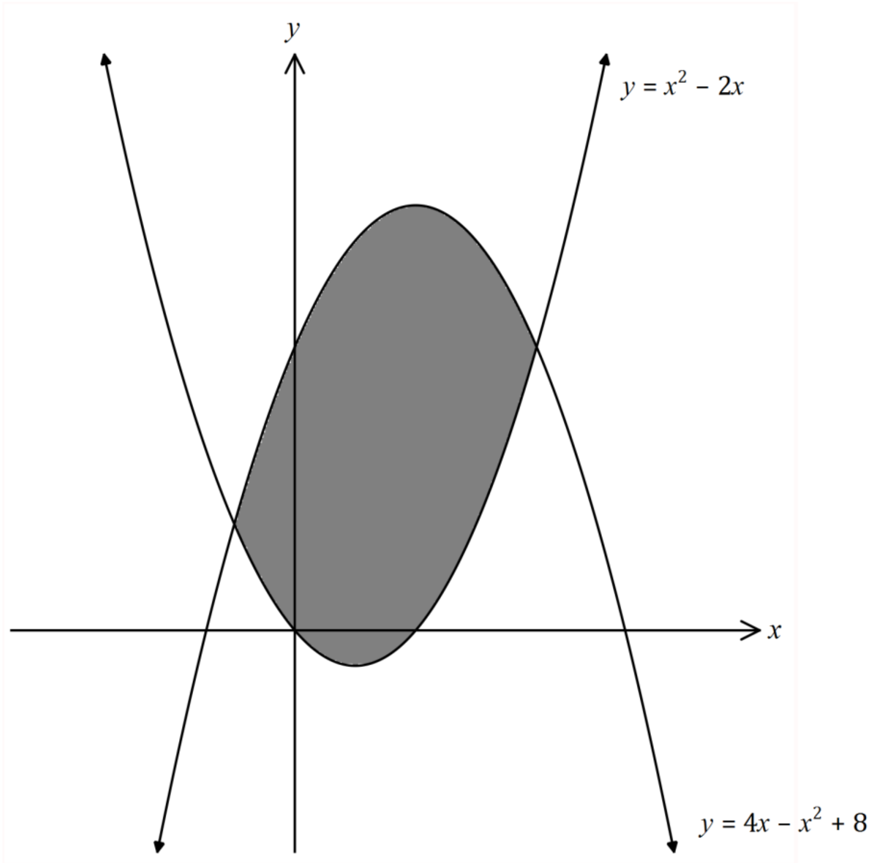


With reference to the above diagram, evaluate $\int_{0}^{8} f(x)dx$ **2**

Question 1 continues over page

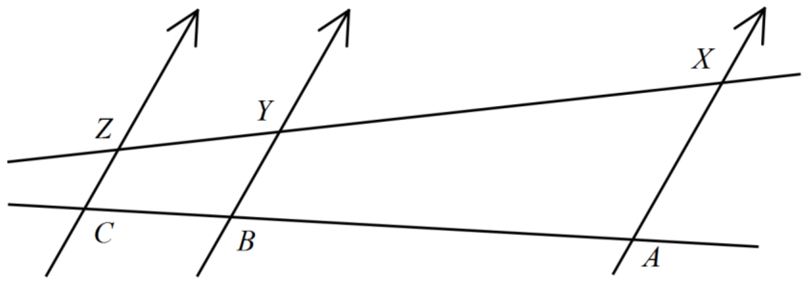
(d) (i) Find the coordinates where $y = 4x - x^2 + 8$ and $y = x^2 - 2x$ intersect. **2**

(ii)Calculate the area bounded by the two curves shown below. **2**



End of Question 1

Question 2 – Euclidean Geometry *(3 Marks)*



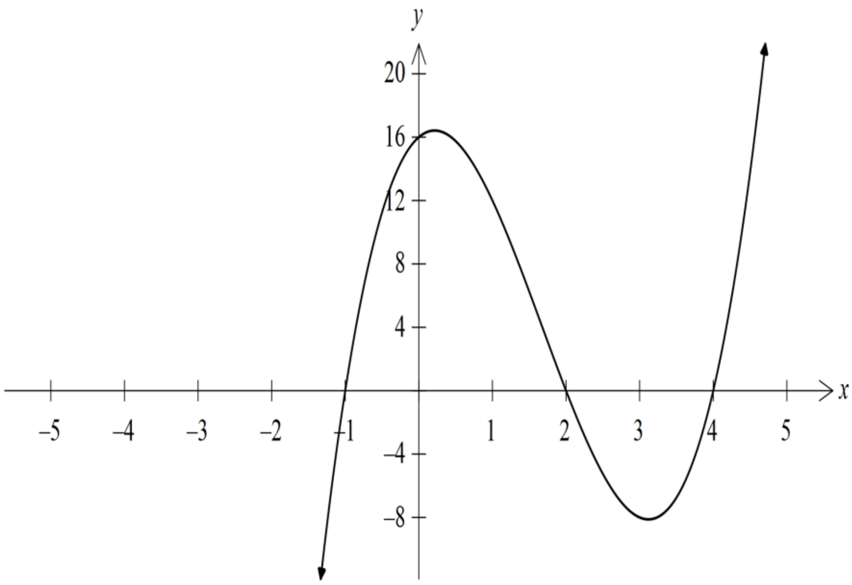
In the diagram *ABC* and *XYZ* are two transversals which cross a family of the parallel lines. $AB = 3BC$ and $XY = YZ + 3$, where all the measurements are in centimetres.

(i) State a reason why $\dfrac{AB}{BC} = \dfrac{XY}{YZ}$. **1**

(ii) Hence find the lengths of XY and YZ. **2**

Question 3 – Polynomials *(11 Marks)*

(a)



(i) Find the equation of the polynomial shown above. **2**

(ii) With reference to the above polynomial or otherwise, solve the following:

$$x^3 > 5x^2 - 2x - 8$$ **2**

(b) $P(x) = x^3 + ax^2 + 2x + b$ has a factor of $(x + 3)$ and a remainder of 4 when it is divided by $(x - 1)$. **3**

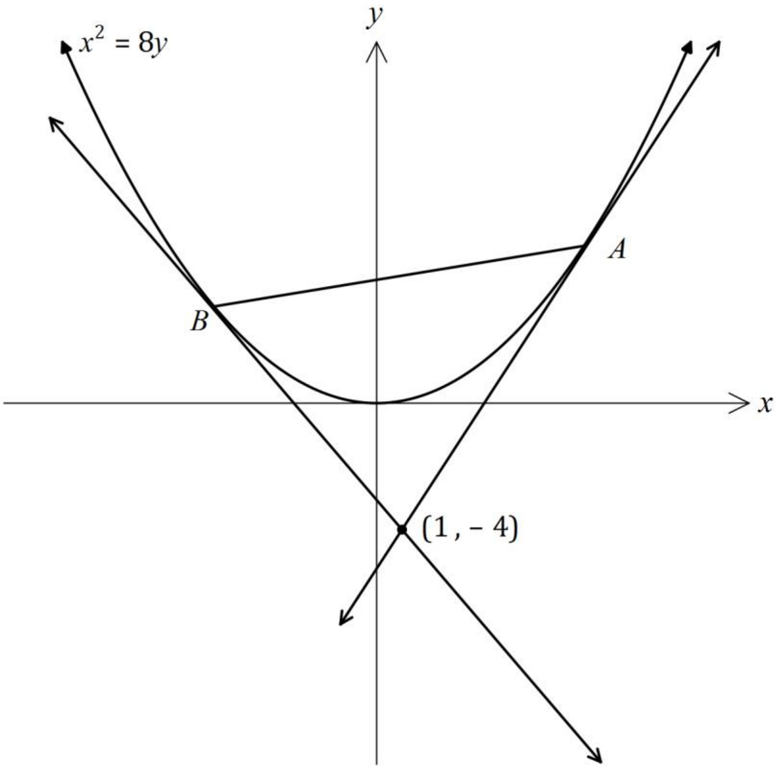
Find the values of *a* and *b*.

(c) Solve the equation $8x^3 - 14x^2 - 7x + 6 = 0$ given that two of the roots are reciprocals of each other. **4**

Question 4 – Parametrics *(13 Marks)*

(a) If $x = 1 + \cos\theta$ and $y = 2 - \sin\theta$ find a relationship between x and y only. **2**

(b)

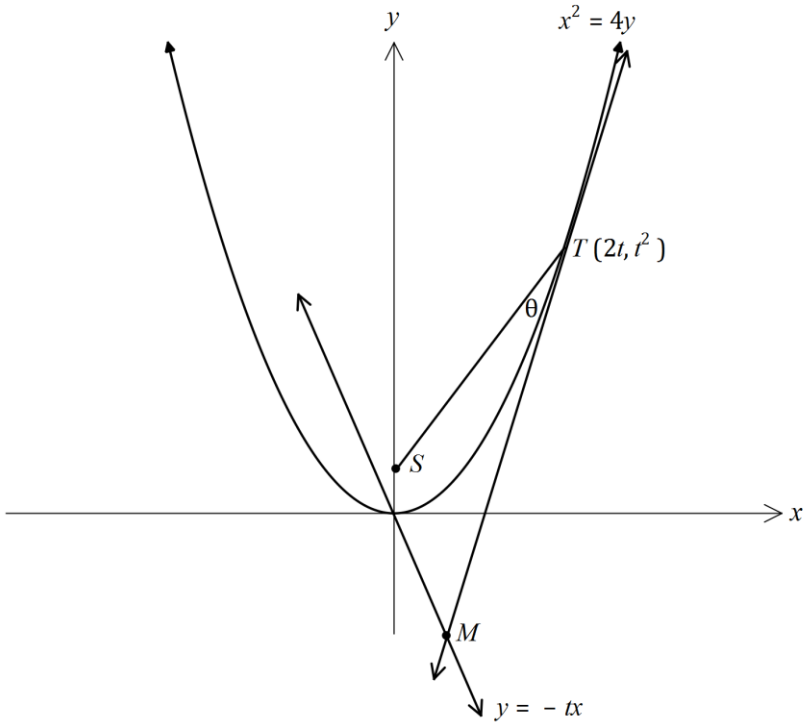


The diagram above shows the parabola $x^2 = 8y$. Tangents are drawn from points *A* and *B* on the parabola such that they intersect at $(1 - 4)$. Find the equation of *AB*. **2**

(c) The horizontal line $y = k$, where $k > 0$, cuts the parabola $x^2 = 4ay$ at the points $P(2at, at^2)$ and $Q(-2at, at^2)$. If O is the origin and ΔOPQ is equilateral, express *k* in terms of *a*. **3**

Question 4 continues over page

(d)



The parabola $x^2 = 4y$, with focus S, has a tangent at $T(2t, t^2)$.

The acute angle θ is the angle between *ST* and the tangent at *T*.

(i) Show that the tangent at *T* has a gradient of *t*. **1**

(ii) The tangent at *T* intersects the line $y = -tx$ at M. Find the locus of *M* as *T* moves on the parabola. **3**

(iii) Find $\tan\theta$ in terms of *t*. **2**

End of examination

Question 1

(a) (i) $\int (5x + 2)\, dx = \dfrac{5x^2}{2} + 2x + C$

(a) (ii)

$$\int \frac{x^3 + 1}{x^2} dx = \int (x + x^{-2})\, dx$$

$$= \frac{x^2}{2} - \frac{1}{x} + C$$

(b)(i)

$$\int_{-1}^{1} (x^3 + 1)\, dx = \left[\frac{x^4}{4} + x \right]_{-1}^{1}$$

$$= \left(\frac{1^4}{4} + 1 \right) - \left(\frac{(-1)^4}{4} - 1 \right)$$

$$= 2$$

(b)(ii)

$$\int_{1}^{4} \frac{dx}{\sqrt{x}} = \int_{1}^{4} x^{-\frac{1}{2}} dx$$

$$= \left[2x^{\frac{1}{2}} \right]_{1}^{4}$$

$$= 2\sqrt{4} - 2\sqrt{1}$$

$$= 2$$

(c)

$$\int_{0}^{8} f(x)dx = \cancel{\left(\frac{1}{4}\right)(\pi)(2^2)} - \left(\frac{1}{2}\right)(2)(2) - (2)(2) - \cancel{\left(\frac{1}{4}\right)(\pi)(2^2)}$$

$$= -6$$

(d) (i)

$$4x - x^2 + 8 = x^2 - 2x$$

$$2x^2 - 6x - 8 = 0$$

$$2(x - 4)(x + 1) = 0$$

$$x = -1 \text{ or } x = 4$$

$\therefore$ intersections are $(-1, 3)$ and $(4, 8)$

Question 1 – Continued

$$A = \int_{-1}^{4} ((4x - x^2 + 8) - (x^2 - 2x))dx$$

$$= \int_{-1}^{4} (-2x^2 + 6x + 8)\, dx$$

$$= \left[\frac{-2x^3}{3} + 3x^2 + 8x \right]_{-1}^{4}$$

$$= \left(\frac{-2(4)^3}{3} + 3(4)^2 + 8(4) \right) - \left(\frac{-2(-1)^3}{3} + 3(-1)^2 + 8(-1) \right)$$

$$= \frac{112}{3} - \left(-\frac{13}{3} \right)$$

$$= \frac{125}{3}\, units^2$$

Question 2

(i) Parallel lines preserve ratios of intercepts on transversals.

(ii)

$$\frac{AB}{BC} = \frac{XY}{YZ} \text{ (as above)}$$

$$\frac{3BC}{BC} = \frac{YZ + 3}{YZ}$$

$$3 = 1 + \frac{3}{YZ}$$

$$2 = \frac{3}{YZ}$$

$$YZ = \frac{3}{2} \text{ cm}$$

$$XY = \frac{3}{2} + 3$$

$$= \frac{9}{2} \text{ cm}$$

Question 3

(a)(i)

$$y = 2(x+1)(x-2)(x-4)$$

(a)(ii)

$$x^3 > 5x^2 - 2x - 8$$
$$x^3 - 5x^2 + 2x + 8 > 0$$
$$(x+1)(x-2)(x-4) > 0$$
$$\therefore \quad -1 < x < 2 \text{ or } x > 4$$

(b)

$$P(-3) = (-3)^3 + a(-3)^2 + 2(-3) + b = 0$$
$$9a + b - 33 = 0$$
$$b = 33 - 9a \ldots 1$$

$$P(1) = (1)^3 + a(1)^2 + 2(1) + b = 4$$
$$a + b + 3 = 4$$
$$a + b = 1 \ldots 2$$
$$1 \rightarrow 2$$

$$a + 33 - 9a = 1$$
$$8a = 32$$
$$a = 4$$
$$\rightarrow 1$$
$$b = 33 - 9(4)$$
$$b = -3$$

Question 3 - continued

(c)

let the roots be α , $\frac{1}{\alpha}$ & β

$$\cancel{\alpha} \times \frac{1}{\cancel{\alpha}} \times \beta = -\frac{6}{8}$$
$$\beta = -\frac{3}{4}$$
$$\alpha + \frac{1}{\alpha} - \frac{3}{4} = \frac{14}{8}$$
$$\alpha + \frac{1}{\alpha} = \frac{20}{8}$$
$$\frac{\alpha^2 + 1}{\alpha} = \frac{5}{2}$$
$$2\alpha^2 + 2 = 5\alpha$$
$$2\alpha^2 - 5\alpha + 2 = 0$$
$$(\alpha - 2)(2\alpha - 1) = 0$$
$$\alpha = 2 \text{ or } \alpha = \frac{1}{2}$$

$\therefore$ the roots are $-\frac{3}{4}, \frac{1}{2}$ & 2

Question 4

(a)

$$x = 1 + \cos\theta$$
$$\cos\theta = x - 1$$
$$y = 2 - \sin\theta$$
$$\sin\theta = 2 - y$$
$$(x-1)^2 + (2-y)^2 = \cos^2\theta + \sin^2\theta$$
$$(x-1)^2 + (y-2)^2 = 1$$

(b)

$$x\,x_0 = 2a(y + y_0)$$
$$x = 2(2)(y - 4)$$
$$x = 4y - 16$$
$$x - 4y + 16 = 0$$

(c)

$$PO = PQ \text{ (sides of equilateral triangle)}$$
$$\sqrt{(2at)^2 + (at^2)^2} = 4at$$
$$4a^2t^2 + a^2t^4 = \ = 16a^2t^2$$
$$a^2t^4 = 12a^2t^2$$
$$t^2 = 12$$
$$k = at^2 \text{ (since } k = y)$$
$$\therefore \quad k = 12a$$

(d)(i)

$$x^2 = 4y$$
$$y = \frac{x^2}{4}$$
$$y' = \frac{2x}{4}$$
when $x = 2t$
$$y' = \frac{\cancel{2}(2t)}{\cancel{4}}$$
$$= t$$

Question 4 - continued

(d) (ii)

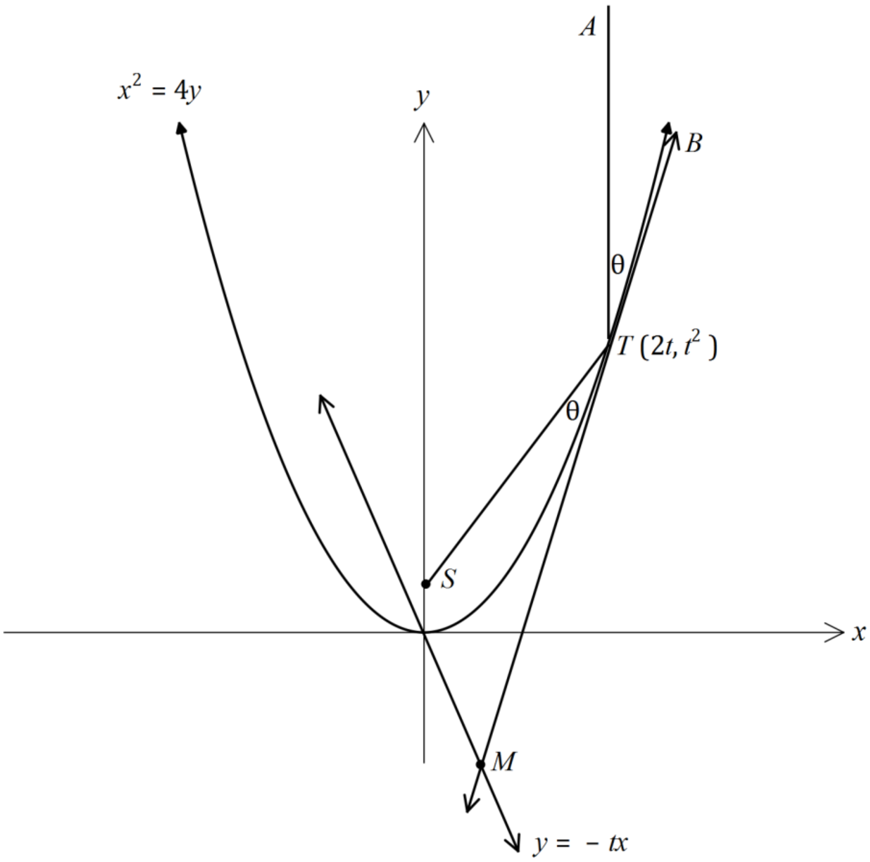
$$tx - (1)t^2 = -tx$$
$$2tx = t^2$$
$$t = 2x$$
$$y = -tx$$
$$y = -(2x)(x)$$
$$y = -2x^2$$

this is a concave down parabola with a vertex at (0,0) and a focal length of $\frac{1}{8}$

(d) (iii)

$$m_{TS} = \frac{t^2 - 1}{2t - 0}$$
$$m_{TM} = t$$
$$\tan\theta = \left|\frac{m_{TS} - m_{TM}}{1 + m_{TS}\,m_{TM}}\right|$$
$$= \left|\frac{t - \frac{t^2 - 1}{2t}}{1 + (t)\left(\frac{t^2 - 1}{2t}\right)}\right|$$
$$= \left|\frac{\left(\frac{2t^2 - (t^2 - 1)}{\cancel{2t}}\right)}{\left(\frac{2t + t^3 - t}{\cancel{2t}}\right)}\right|$$
$$= \left|\frac{t^2 + 1}{t^3 + t}\right|$$
$$= \left|\frac{t^2 + 1}{t(t^2 + 1)}\right|$$
$$= \left|\frac{1}{t}\right|$$

Alternative solution to 4 (d) (iii)



Let point A be as shown such that AT is parallel to the axis of the parabola

Let point B be a point on the tangent at T as shown

$\angle ATB = \theta$ (reflection property of the parabola)

$\therefore \tan\theta = \dfrac{1}{t}$

but since θ is acute

$\tan\theta = |\dfrac{1}{t}|$