



NORTH SYDNEY BOYS HIGH SCHOOL

2017 Year 11 Task 1

Mathematics

Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 70 minutes
- Write on the lined paper in the booklet provided
- Write using blue or black pen
- Board approved calculators may be used
- All necessary working should be shown in every question
- Each new question is to be started on a **new page**.
- Attempt all questions

Class Teacher:

(Please tick or highlight)

- ☐ Mr Berry
- ☐ Mr Hwang
- ☐ Mr Ireland
- ☐ Dr Jomaa
- ☐ Ms Lee
- ☐ Mr Lin
- ☐ Ms Ziazaris

Student Name: _____

(To be used by the exam markers only.)

Question No	1	2	3	4	5	6	7	Total	Total %
Mark	$\frac{10}{10}$	$\frac{14}{14}$	$\frac{12}{12}$	$\frac{12}{12}$	$\frac{12}{12}$	$\frac{12}{12}$	$\frac{12}{12}$	$\frac{84}{84}$	$\frac{100}{100}$

Question 1

- (a) Evaluate correct to three decimal places.

$$\frac{\sqrt[3]{(4.23-5.7)^2}}{2.43\pi}$$

1

- (b) Express 5 987 456 in scientific notation correct to four significant figures **1**

- (c) Express $0.\overline{54}$ as a fraction in simplest terms **2**

- (d) Express with a rational denominator in simplest form

(i) $\frac{4}{3\sqrt{2}}$ **1**

(ii) $\frac{\sqrt{3}}{3-2\sqrt{2}}$ **2**

- (e) Simplify the following:

(i) $3\sqrt{2} + 2\sqrt{8}$ **1**

(ii) $5\sqrt{5} \times \sqrt{125}$ **2**

Question 2

- (a) Solve: 2
- i) $\frac{x-1}{2} + \frac{3x-1}{5} = 4$
- ii) $4x^2 + 9x + 2 = 11$ 2
- iii) $|4x + 2| = 10$ 2
- (b) Solve the following equations simultaneously
- $$\begin{cases} 2x + 3y - 2 = 0 \\ 5x - 2y + 1 = 0 \end{cases}$$
- 2
- (c) Solve the following inequalities:
- i) $5 - 3x > 11$ 2
- ii) $\left| \frac{x+2}{3} \right| < 4$ 2
- iii) $x^2 - 7x > 6$ 2

Question 3

(a) A function is defined as follows

$$f(x) = \begin{cases} 4 & x \leq -3 \\ |x + 1| & -3 < x < 0 \\ x^2 + 2 & x \geq 0 \end{cases}$$

(i) Sketch $f(x)$ 3

(ii) Find $f(-3) + f(0) - f(3)$ 2

(iii) Find the simplest expression for $f(a^2)$ 1

(b) For the function

$$f(x) = \frac{2}{3 + x^2}$$

i) What is the domain of $f(x)$? 1

ii) What is the range of $f(x)$? 1

iii) Hence, sketch $f(x)$. 2

(c) Find $f(x)$ if $f(x - 1) = 4x + 5$ 2

Question 4

Sketch each of the following, showing all relevant features such as intercepts and asymptotes:

(a) $y = 1 - \sqrt{1 - x^2}$ **3**

(b) $y = 4^{-x} - 2$ **3**

(c) $y = \frac{x - 1}{x + 2}$ **3**

(d) $y = \frac{x^3 + 1}{x^2}$ **3**

Question 5

- (a) Amy is standing 9m due East of Brenda. Charlene standing 15 m from Brenda. The bearing of Charlene from Brenda is $024^\circ T$. Find:

(i) The distance between Charlene and Amy (to the nearest metre) **2**

(ii) The bearing of Charlene from Amy (to the nearest degree). **2**

- (b) Solve for θ for the domain $0^\circ \leq x \leq 360^\circ$:

$$\sin^2 \theta - 3 \cos^2 \theta = 0 \quad \mathbf{3}$$

- (c) (i) Show that:

$$\frac{\cos^2 x}{1 - \sin x} = 1 + \sin x \quad \mathbf{1}$$

- (ii) Hence or otherwise, state the range of:

$$y = \frac{\cos^2 x}{1 - \sin x} \quad \mathbf{1}$$

- (iii) Sketch

$$y = \frac{\cos^2 x}{1 - \sin x} \quad \mathbf{3}$$

between $-180^\circ \leq x \leq 180^\circ$, showing all important features.

Question 6

- (a) Solve for x

$$\frac{2}{3-x} \geq 3$$

3

- (b)

- (i) Draw a neat sketch of $y = P(x)$ where

$$P(x) = \frac{1}{16}(x+2)^2(x-1)^3(x-4)$$

2

- (ii) Hence or otherwise solve $(x+2)^2(x-1)^3(x-4) \geq 0$

1

- (c) Consider the function $f(x) = 1 + \sqrt{1+x}$

- (i) State the domain and range of $f(x)$

2

- (ii) Sketch $y = f(x)$

1

- (iii) State the domain and range of $f^{-1}(x)$

1

- (iii) Sketch $y = f^{-1}(x)$

2

Question 7

- (a) Solve for x :

$$4^x + 2 \leq 3 \times 2^x$$

3

- (b) Fully factorise:

$$x^3 - 8y^6 + 2x^2y^2 - 4xy^4$$

3

- (c) Solve for x

$$9 - 3x = |x^2 - 6x + 5|$$

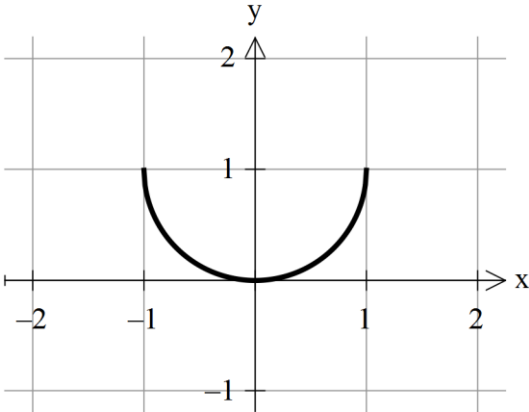
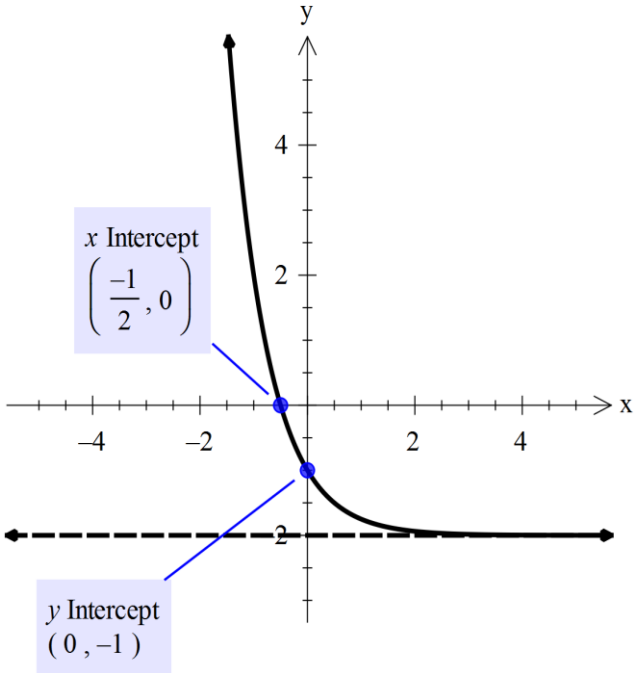
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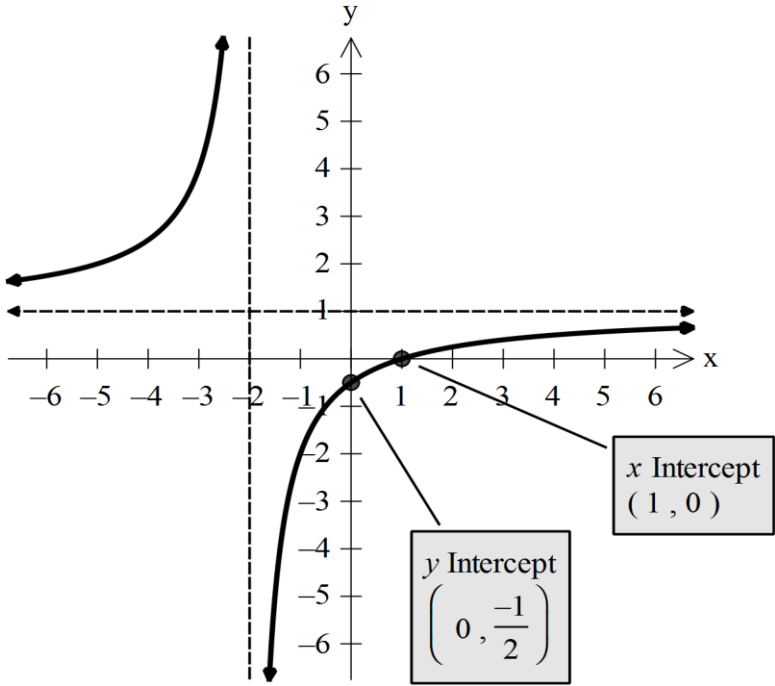
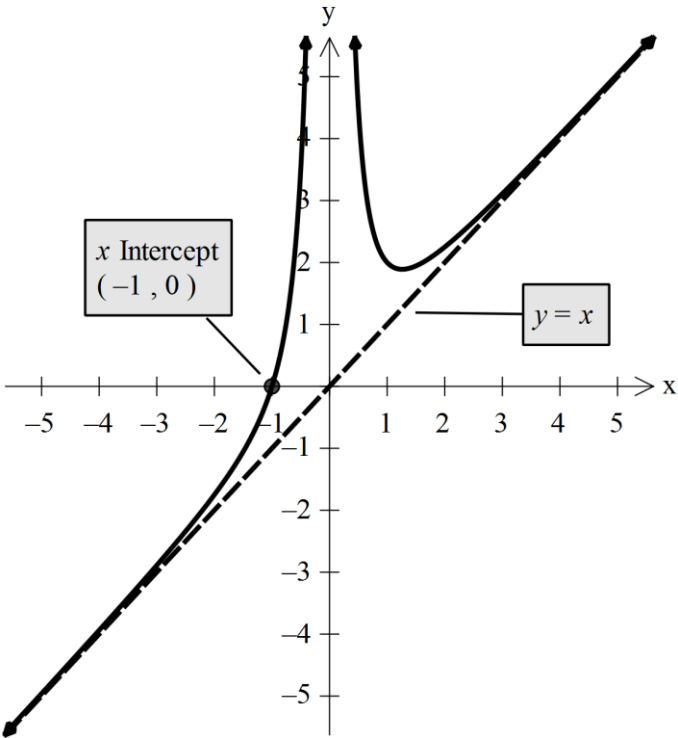
- (d) Consider the function $f(x)$ below:

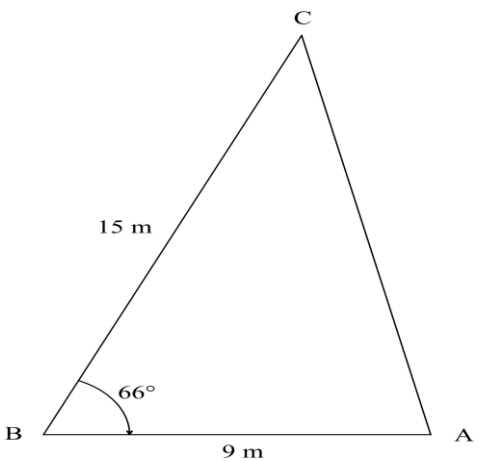
$$f(x) = \frac{\sin(x)}{\sqrt{1 - \cos^2 x}} + \frac{\cos(x)}{\sqrt{1 - \sin^2 x}} + \frac{\tan x}{\sqrt{\sec^2 x - 1}} + \frac{\cot x}{\sqrt{\operatorname{cosec}^2 x - 1}}$$

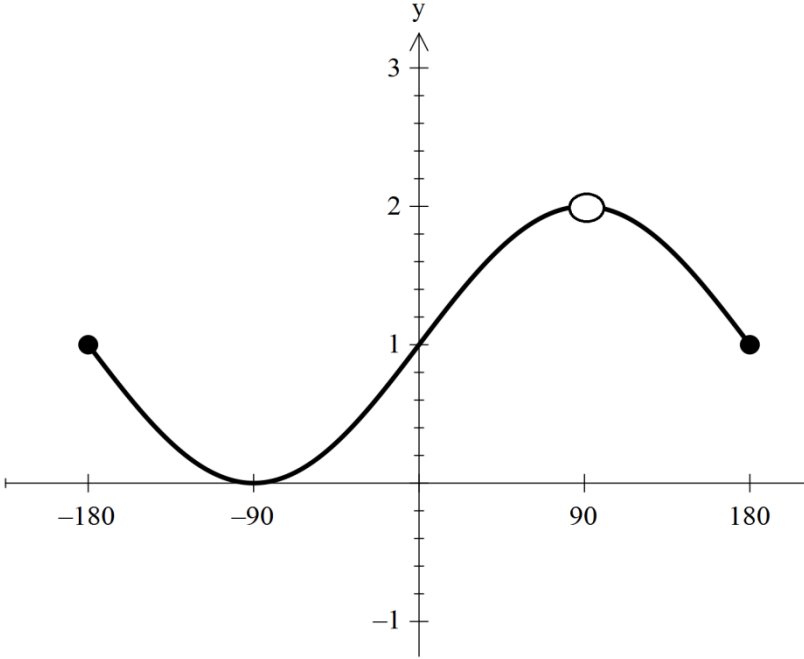
Sketch $y = f(x)$ for $0 \leq x \leq 360^\circ$

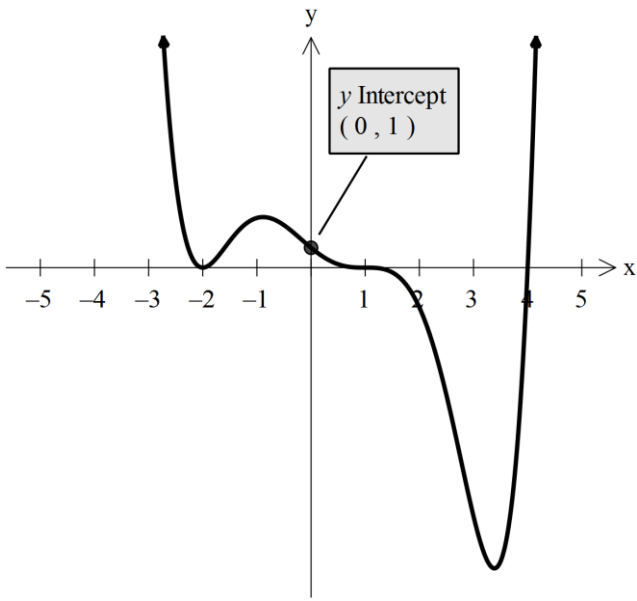
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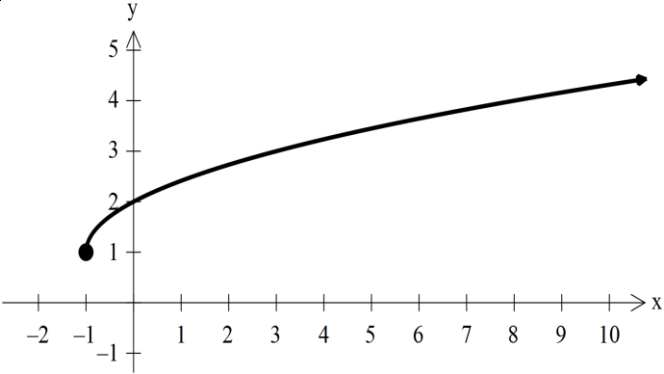
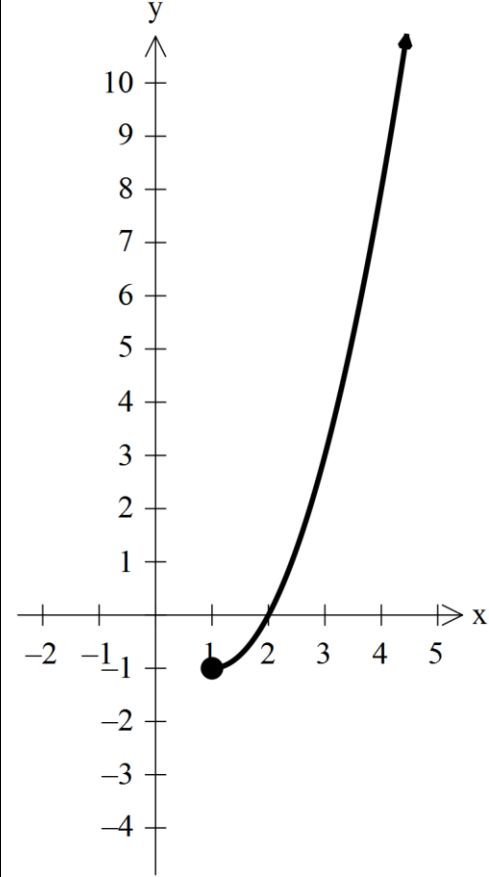
	Question 4	
(a)		✓ Semi-circle with correct radius ✓ Upside down ✓ Shifted up
(b)		✓ Exponential shape ✓ Correct asymptote ✓ At least 2 correct coordinates labelled

(c)	 The graph shows a hyperbola on a Cartesian coordinate system. The x-axis ranges from -6 to 6, and the y-axis ranges from -6 to 6. A vertical dashed line at $x = -2$ and a horizontal dashed line at $y = 1$ represent the asymptotes. The curve has two branches: one in the upper-left region relative to the asymptotes and another in the lower-right region. Two points are labeled with callouts: the x-intercept at $(1, 0)$ and the y-intercept at $(0, -\frac{1}{2})$.	✓ Recognising that the function is a hyperbola ✓ Correct asymptotes ✓ 2 points labelled
(d)	 The graph shows a function on a Cartesian coordinate system. The x-axis ranges from -5 to 5, and the y-axis ranges from -5 to 5. A vertical dashed line at $x = 1$ represents a vertical asymptote. A dashed line $y = x$ represents an oblique asymptote. The curve has two branches: one to the left of the vertical asymptote and one to the right. The left branch passes through the x-intercept at $(-1, 0)$. The right branch approaches the oblique asymptote $y = x$ as x increases.	✓ Addition of ordinates ✓ Vertical asymptote ✓ Oblique asymptote

	Question 5	
(a) (i)	 $AC^2 = 9^2 + 15^2 - 2 \times 9 \times 15 \times \cos(66^\circ)$ $\therefore AC = 14 \text{ m (to the nearest metre)}$	✓ Diagram and cosine rule ✓ Correct answer
(a) (ii)	$\cos A = \frac{9^2 + AC^2 - 15^2}{2 \times 9 \times AC}$ $\approx 0.20697...$ $\therefore A = 78^\circ \text{ (nearest degree)}$ $\therefore \text{bearing} = 270^\circ + 78^\circ$ $= 348^\circ \text{ (nearest degree)}$	✓ Angle A ✓ Correct bearing
(b)	$\begin{aligned} \sin^2(x) - 3\cos^2(x) &= \sin^2(x) - 3(1 - \sin^2(x)) \\ &= \sin^2(x) - 3 + 3\sin^2(x) \\ &= 4\sin^2(x) - 3 \end{aligned}$ $\therefore 4\sin^2(x) = 3$ $\sin^2(x) = \frac{3}{4}$ $\sin(x) = \pm \frac{\sqrt{3}}{2}$ $\therefore x = 60^\circ, 120^\circ, 240^\circ \text{ or } 300^\circ$	✓ In terms of a single trig ratio ✓ Any 2 correct answers ✓ All correct answers

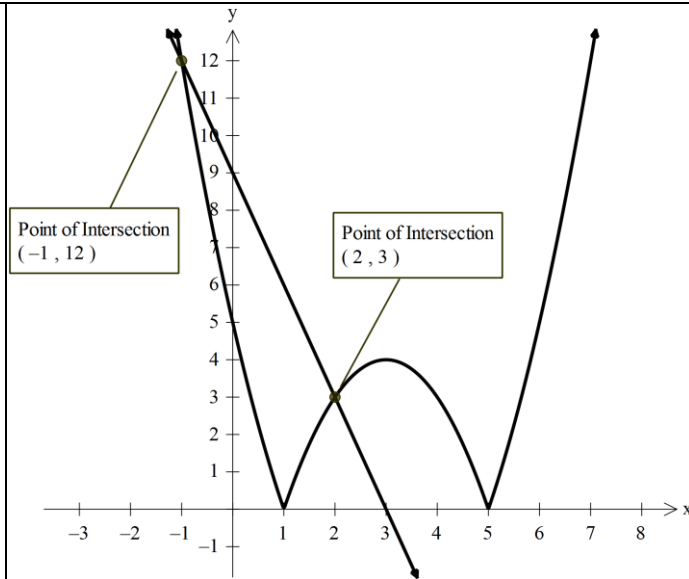
(c) (i)	$\begin{aligned} \text{LHS} &= \frac{\cos^2(x)}{1 - \sin(x)} \\ &= \frac{\cos^2(x)}{1 - \sin(x)} \times \frac{1 + \sin(x)}{1 + \sin(x)} \\ &= \frac{(\cos^2(x))(1 + \sin(x))}{1 - \sin^2(x)} \\ &= \frac{\cancel{\cos^2(x)}(1 + \sin(x))}{\cancel{\cos^2(x)}} \\ &= 1 + \sin(x) \\ &= \text{RHS} \end{aligned}$	✓ Correct solution
(c) (ii)	$0 \leq y < 2$	✓ Correct range
(c) (iii)		✓ correct shape in correct domain ✓ shifted up by 1 with proper x and y intercepts ✓ open circle at (90, 2)

	Question 6	
(a)	$\frac{2}{3-x} \geq 3$ $2(3-x) \geq 3(3-x)^2$ $2(3-x) - 3(3-x)^2 \geq 0$ $(3-x)(2-3(3-x)) \geq 0$ $(3-x)(2-9+3x) \geq 0$ $(3-x)(3x-7) \geq 0$ $\therefore \frac{7}{3} \leq x < 3$	✓ Correct method ✓ x between 7/3 and 3 ✓ Correct inequality signs
(b) (i)		✓ Correct shape (including y intercept) ✓ Correct single, double and triple roots
(b) (ii)	$x \leq 1 \text{ or } x \geq 4$	✓ Correct answer (must say "or")

(c) (i)	Domain: $x \geq -1$ Range: $y \geq 1$	✓ Domain ✓ Range
(c) (ii)		✓ Correct graph
(c) (iii)	Domain: $x \geq 1$ Range: $y \geq -1$	✓ Correct answer
(c) (iv)		✓ Correct shape of parabola ✓ Domain properly restricted

	Question 7	
(a)	$4^x + 2 \leq 3 \times 2^x$ $4^x - 3 \times 2^x + 2 \leq 0$ $(2^x)^2 - 3 \times 2^x + 2 \leq 0$ $(2^x - 1)(2^x - 2) \leq 0$ $\therefore 1 \leq 2^x \leq 2$ $\therefore 0 \leq x \leq 1$	✓ Recognising quadratic ✓ Solving 2^x ✓ Solving for x
(b)	$x^3 - 8y^6 + 2x^2y^2 - 4xy^4 = (x - 2y^2)(x^2 + 2xy^2 + 4y^4) + 2xy^2(x - 2y^2)$ $= (x - 2y^2)(x^2 + 2xy^2 + 4y^4 + 2xy^2)$ $= (x - 2y^2)(x^2 + 4xy^2 + 4y^4)$ $= (x - 2y^2)(x + 2y^2)^2$	✓ Difference of two cubes ✓ Reducing to 2 pairs of brackets ✓ Final answer

(c)



Case 1: $x < 1$

$$9 - 3x = x^2 - 6x + 5$$

$$x^2 - 3x - 4 = 0$$

$$(x + 1)(x - 4) = 0$$

$$\therefore x = -1 \text{ (since } x=4 \text{ is outside the domain)}$$

Case 2: $1 \leq x \leq 5$

$$9 - 3x = -(x^2 - 6x + 5)$$

$$9 - 3x = -x^2 + 6x - 5$$

$$x^2 - 9x + 14 = 0$$

$$(x - 7)(x - 2) = 0$$

$$x = 2 \text{ (since } x=7 \text{ cannot be a solution} \\ \text{since it is outside the domain)}$$

Case 3: $x > 5$

Clearly no solutions here either by inspection of the graph or from showing both the possible answers (-1 and 4) are outside the domain

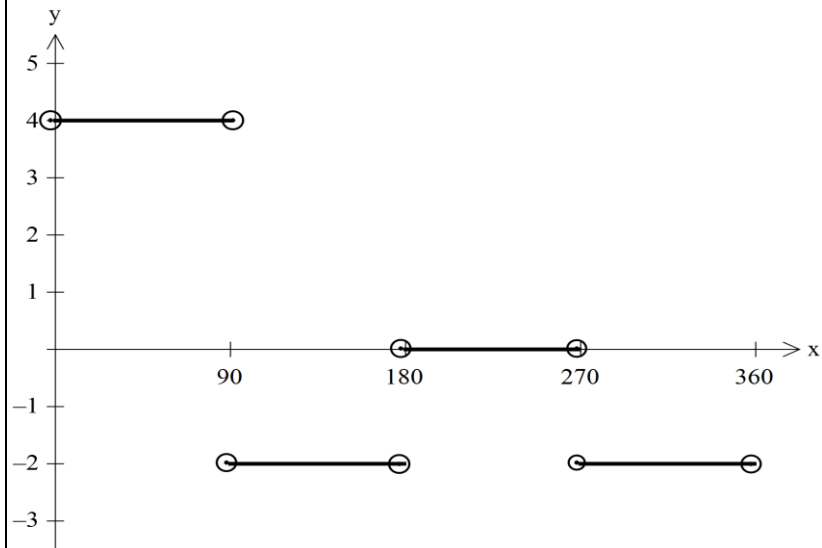
✓ Recognising there will be 3 regions/cases (either algebraically or on a diagram)

✓ Finding $x = -1$

✓ Finding $x = 2$

Note a mark will be deducted for each "false positive" solution

(d)



✓ Any horizontal line

✓ All horizontal lines

✓ Points of discontinuity

