Ext 1

NORTH SYDNEY BOYS HIGH SCHOOL

2017 Year 11 Task 2

Mathematics
Extension 1

General Instructions

- Working time – 50 minutes
- Write on the lined paper in the booklet provided
- Write using blue or black pen
- Board approved calculators may be used
- All necessary working should be shown in every question
- Each new question is to be started on a **new page.**
- Attempt all questions

Class Teacher:
(Please tick or highlight)

- ○ Mr Berry
- ○ Mr Hwang
- ○ Mr Ireland
- ○ Dr Jomaa
- ○ Ms Lee
- ○ Mr Lin
- ○ Ms Ziaziaris

Student Number:_____

(To be used by the exam markers only.)

Question No	1	2	3	4	5	6	7	Total	Total %
Mark	/7	/7	/7	/6	/4	/6	/5	/42	/100

Question 1

Differentiate with respect to x.

(a) $5x^4 - x$ 1

(b) $\dfrac{1}{x^2+2}$ 1

(c) $2(5-2x^2)^{-3}$ 1

(d) $(2x+1)(3x-2)^4$ 2

(e) $\dfrac{\sqrt[3]{x}}{x}$ 2

Question 2

(a) Find the acute angle between the two lines with equations $y = 4x - 4$ and

$3x - 6y + 1 = 0$ 2

(b) Find the coordinates of the point P, that divides externally the interval AB in the ratio $3:4$ where $A(-5,7)$ and $B(9,0)$. 2

(c) The distance between the point $P(1+k, 2-k)$ and the line $3x + 4y - 1 = 0$ is 5units. Find the possible values of k. 3

examination continues overleaf……

Question 3

(a) Prove that the point $(2,3)$ lies on the line whose equation is

$$x - y + 1 + k(2x + y - 7) = 0$$ 1

(b) Sketch the region which represents the **intersection** of

$y > x^2 - 4$ and $y \le 2x - 1$ showing points of intersection. 3

(c) Sketch the region $x < -\sqrt{9 - y^2}$ 3

Question 4

(a) Find a simple expression for

$$\cos 2x \cos 3x - \sin 2x \sin 3x$$ 1

(b) Solve $\cos \frac{\theta}{2} = \frac{1}{2}$ for $-\pi \le \theta \le \pi$ 2

(c) Solve $\sin 2\theta = \tan \theta$ for $0^\circ \le \theta \le 360^\circ$ 3

Question 5

(a) Prove $\frac{\sin 4A + \sin 2A}{1 + \cos 4A + \cos 2A} = \tan 2A$ 2

(b) Find the possible values of $\sin \frac{A}{2}$ if $\cos A = \frac{3}{5}$. 2

examination continues overleaf……→

Question 6

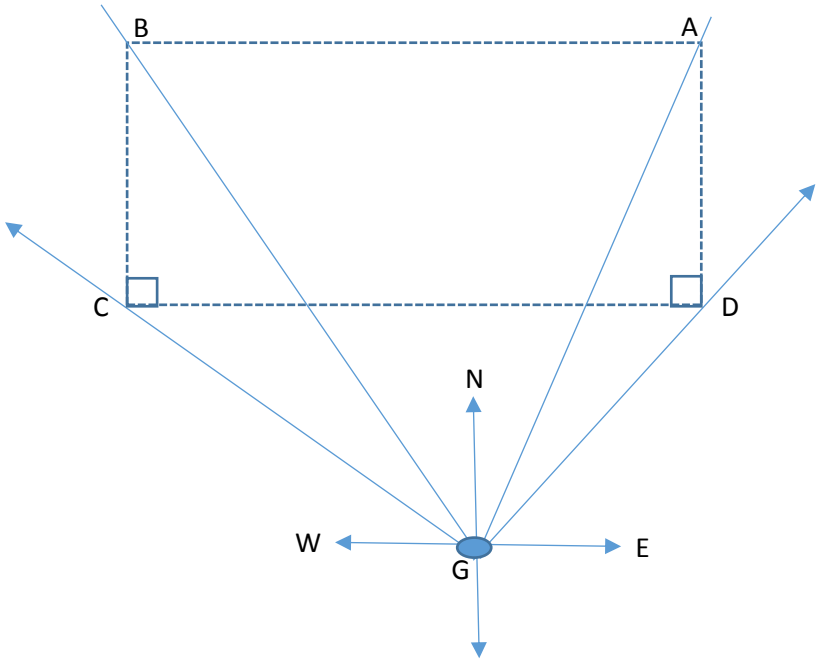
(a) For what values of x is the gradient of the curve $y = x - \frac{4}{x}$ greater than 5? 3

(b) If the tangent to the curve $y = \frac{ax+b}{(x-1)(x-4)}$ at the point $(2,-1)$ is parallel to the x-axis, find the values of a and b. 3

Question 7

A helicopter flies due west from A to B at a constant speed of 420km/h. From a point G on the ground the bearing of the helicopter when it is at A is $079^\circ T$ with an angle of elevation β. Four minutes later the helicopter is at B with a bearing from G being $302^\circ T$ and an angle of elevation 32°. The altitude of the helicopter is h metres.

(a) Copy and complete the sketch below showing the above information. 1
(b) Calculate the height of the plane to the nearest metre. 2
(c) Calculate the value of β to the nearest degree. 2



END OF EXAMINATION

2017 – YR 11 – EXT1 – TASK 2

Question 1

① a) $5x^4 - x$

$\frac{dy}{dx} = 20x^3 - 1$

b) $(x^2+2)^{-1}$

$\frac{dy}{dx} = -1(x^2+2)^{-2} \cdot 2x$

$= \frac{-2x}{(x^2+2)^2}$

c) $2(5-2x^2)^{-3}$

$\frac{dy}{dx} = -6(-4x)(5-2x^2)^{-4}$

$= 24x(5-2x^2)^{-4}$

d) $(2x+1)(3x-2)^4$

$\frac{dy}{dx} = 2(3x-2)^4 + (2x+1) \cdot 4 \cdot 3(3x-2)^3$

$= 2(3x-2)^4 + 12(2x+1)(3x-2)^3$

e) $\frac{\sqrt[3]{x}}{x} = \frac{x^{\frac{1}{3}}}{x}$

$= x^{-2/3}$

$\frac{dy}{dx} = -\frac{2}{3}x^{-5/3}$

$= \frac{-2}{3x^{5/3}}$

$= \frac{-2}{3\sqrt[3]{x^5}}$

Question 2

(a) $m_1 = 4$

$m_2 = \frac{1}{2}$

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \left|\frac{4 - \frac{1}{2}}{1 + 2}\right|$$

$$= \left|\frac{\frac{7}{2}}{3}\right|$$

$$= \frac{7}{6}$$

$\theta = 49°24'$

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b) $\overset{m}{-3} : \overset{n}{4}$ $(\overset{x_1 y_1}{-5, 7})$ $(\overset{x_2 y_2}{9, 0})$

$$(x, y) = \left(\frac{nx_1 + mx_2}{m+n}, \frac{ny_1 + my_2}{m+n}\right)$$

$$= \left(\frac{4(-5) - 3(9)}{-3+4}, \frac{4(7) - 3(0)}{-3+4}\right)$$

$$= \left(\frac{-47}{1}, \frac{28}{1}\right)$$

$$= (-47, 28)$$

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c) $d = \dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

$$5 = \frac{|3(1+k) + 4(2-k) - 1|}{\sqrt{9+16}}$$

$$25 = |3 + 3k + 8 - 4k - 1|$$

$$25 = |10 - k|$$

$10 - k = 25$ or $10 - k = -25$

$k = -15$ or $k = 35$

Question 3

a) Sub (2,3)

LHS $= x - y + 1 + k(2x + y - 7)$

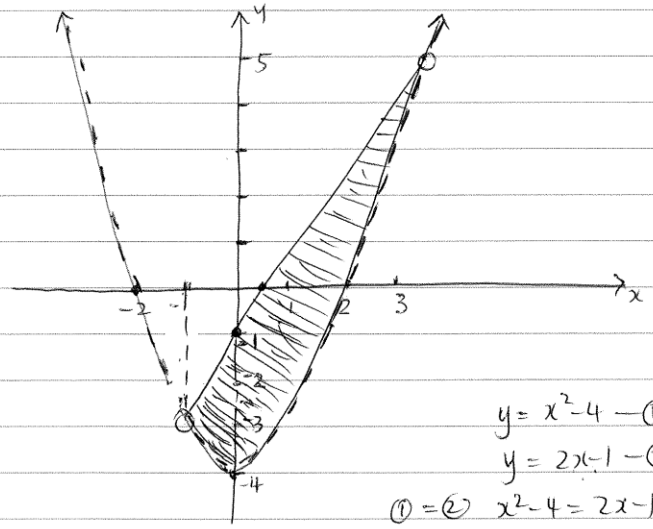
$= 2 - 3 + 1 + k(4 + 3 - 7)$

$= 0$

$= RHS$.

$\therefore (2,3)$ lies on line above.

b) $y > x^2 - 4 \cap y \leq 2x - 1$



$y = x^2 - 4$ — ①

$y = 2x - 1$ — ②

① = ② $x^2 - 4 = 2x - 1$

$x^2 - 2x - 3 = 0$

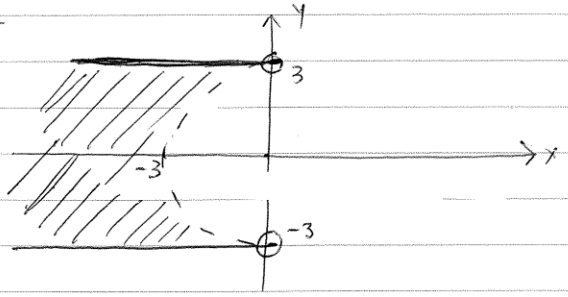
$(x - 3)(x + 1) = 0$.

$x = 3, y = 5$

$x = -1, y = -3$

c) $x < -\sqrt{9 - y^2}$

$-4 < -\sqrt{9 - 0}$



Question 4

(a) $\cos 2x \cos 3x - \sin 2x \sin 3x$

$= \cos(2x + 3x)$

$= \cos 5x$

b) $\cos\frac{\theta}{2} = \frac{1}{2}$ $\quad -\pi \leq \theta \leq \pi$

$-\frac{\pi}{2} \leq \frac{\theta}{2} \leq \frac{\pi}{2}$

$\frac{\theta}{2} = \frac{\pi}{3}, -\frac{\pi}{3}$.

$\theta = \frac{2\pi}{3}, \frac{-2\pi}{3}$

c) $\sin 2\theta = \tan\theta$ $\quad 0° \leq \theta \leq 360°$

$2\sin\theta\cos\theta = \frac{\sin\theta}{\cos\theta}$

$2\sin\theta\cos^2\theta - \sin\theta = 0$

$\sin\theta(2\cos^2\theta - 1) = 0$

$\sin\theta = 0$ OR $2\cos^2\theta - 1 = 0$

$\cos^2\theta = \frac{1}{2}$

$\theta = 0, 180°, 360°$ $\quad \cos\theta = \pm\frac{1}{\sqrt{2}}$

OR $\quad \theta = 45°, 135°, 225°, 315°$

Question 5

a) $\dfrac{\sin 4A + \sin 2A}{1 + \cos 4A + \cos 2A} = \tan 2A$

$$LHS = \frac{\sin 4A + \sin 2A}{1 + \cos 4A + \cos 2A}$$

$$= \frac{2\sin 2A \cos 2A + \sin 2A}{1 + 2\cos^2 2A - 1 + \cos 2A}$$

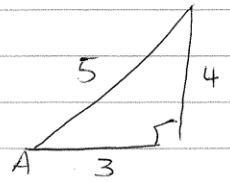
$$= \frac{\sin 2A\,(2\cos 2A + 1)}{2\cos^2 2A + \cos 2A}$$

$$= \frac{\sin 2A\,(2\cos 2A + 1)}{\cos 2A\,(2\cos 2A + 1)}$$

$$= \tan 2A$$

$$= RHS.$$

(b)



$$\cos A = 1 - 2\sin^2 \frac{A}{2}$$

$$\frac{3}{5} = 1 - 2\sin^2 \frac{A}{2}$$

$$\frac{3}{5} - 1 = -2\sin^2 \frac{A}{2}$$

$$\frac{1}{5} = \sin^2 \frac{A}{2}$$

$$\sin \frac{A}{2} = \pm \frac{1}{\sqrt{5}}$$

Question 6

$y = x - 4x^{-1}$

1a) $\frac{dy}{dx} = 1 + \frac{4}{x^2}$

$1 + \frac{4}{x^2} > 5$

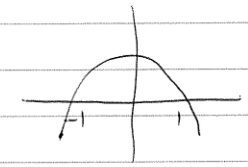
$\frac{4}{x^2} > 4$

$4 > 4x^2$

$4 - 4x^2 > 0$

$4(1 - x^2) > 0$

$4(1-x)(1+x) > 0$



$-1 < x < 1 \quad x \neq 0$

1b) $y = \frac{ax+b}{(x-1)(x-4)}$

$y = \frac{ax+b}{x^2-5x+4}$

$\frac{dy}{dx} = \frac{(x^2-5x+4).a - (ax+b).(2x-5)}{(x^2+5x+4)^2}$

When $x=2$, $m=0$

$0 = (4-10+4)a - (2a+b)(-1)$

$0 = -2a + 2a - b$

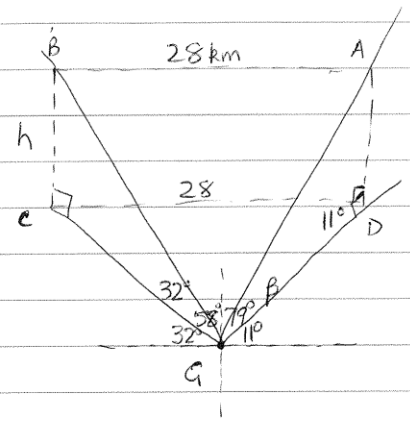
$\therefore b = 0.$

Sub $(2,-1)$ into y. and $b=0$.

$-1 = \frac{2a}{-2} \qquad a = 1 \qquad \therefore a=1, b=0$

Question 7

a)



b) 1h = 420km

$4\text{min} = \frac{420}{60} \times 4 \text{ km}$

$= 28\text{km}$

Find CG

$$\frac{CG}{\sin 11^\circ} = \frac{28}{\sin(58+79)}$$

$$CG = \frac{28\sin 11^\circ}{\sin 137^\circ}$$

Find h

$$\tan 32^\circ = \frac{h}{GC}$$

$$h = CG \tan 32$$

$$h = \frac{28\sin 11^\circ}{\sin 137^\circ} \tan 32^\circ$$

$$h = 4895\text{m}$$

Find GD

$$\frac{GD}{\sin 32^\circ} = \frac{28}{\sin 137^\circ}$$

$$GD = \frac{28\sin 32^\circ}{\sin 137^\circ}$$

$$\tan\beta = \frac{h}{GD}$$

$$\beta = 13^\circ$$