NORTH SYDNEY BOYS HIGH SCHOOL

2018 HSC ASSESSMENT TASK 4

Mathematics
Extension 1

General Instructions

- Working time – 60 minutes (+ 5 minutes reading time)
- Write on the lined paper in the booklet provided
- Write using blue or black pen
- Board approved calculators may be used
- All necessary working should be shown in every question
- Each new question is to be started on a **new page.**

Class Teacher:
(Please tick or highlight)

- O Mr Berry
- O Mr Hwang
- O Mr Ireland
- O Dr Jomaa
- O Ms Lee
- O Ms Ziaziaris

Student Number:__________________________

(To be used by the exam markers only.)

Question No	1	2	3	4	Total	Total %
Mark	$\overline{12}$	$\overline{9}$	$\overline{10}$	$\overline{6}$	$\overline{37}$	$\overline{100}$

Question 1 – Binomial Theorem (12 Marks)

(a) Expand and simplify:

$(3x - 2y)^5$ **2**

(b) Find the coefficient of x^3 in.

$(x^2 + \frac{3}{x})^9$ **2**

(c) Find the term independent of *x*. Express your answer fully simplified in integer form.

$(3 - x)^4(1 + \frac{2}{x})^7$ **3**

(d) Find the value of *n* such that the coefficient of x^4 is twice the coefficient of x^3 in the expansion of $(1 + x)^n$ **2**

(e) In the expansion of $(1 + x)(a - bx)^{12}$ it is known that the coefficient of x^8 is 0.

Find $\frac{a}{b}$ in simplest form. **3**

Question 2 – Projectile Motion 1 (9 Marks)

(a) A golf ball is hit at angle α , where $0° < \alpha < 90°$. The initial velocity of the ball is $V \text{ ms}^{-1}$. Gravity is g and time is t.

Show that the horizontal and vertical displacement of the ball is given by:

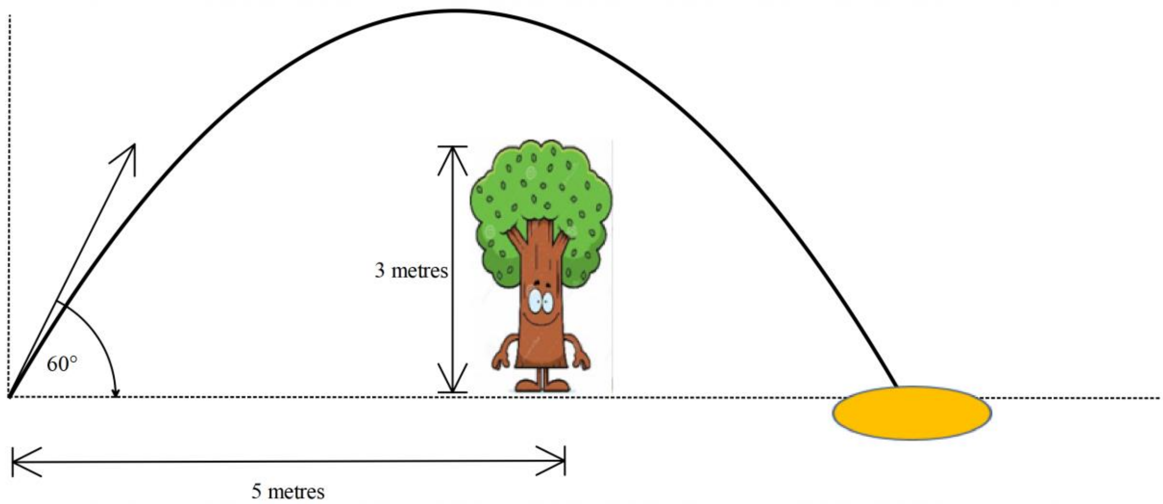
$$x = Vt\cos\alpha$$

$$y = -\frac{1}{2}gt^2 + Vt\sin\alpha$$ **2**

(b) Show that

$$y = x\tan\alpha - \frac{gx^2}{2V^2}(1 + \tan^2\alpha)$$ **2**

Mark hits his ball so that it has an initial velocity of 12 ms^{-1} and an angle of projection of $60°$. Acceleration due to gravity is $g = 9.8 \text{ ms}^{-2}$.



To succeed, Mark must clear a 3m shrub which is 5m away from his ball.

(c) How high over the shrub does the ball clear? *(i.e. how much higher than the top of the shrub is the ball when it passes directly over the shrub)* **2**

(d) What is the horizonal distance travelled by the ball? *(assume the ball lands in a bunker of sand and stops immediately)* **3**

Question 3 – Simple Harmonic Motion (10 Marks)

(a) A particle moving in simple harmonic motion about the origin has a period of $\frac{\pi}{3}$ seconds and a maximum velocity of 2 metres/second. Using the formula $v^2 = n^2(a^2 - x^2)$, or otherwise, find:

(i) The amplitude of motion **2**

(ii) The displacement after $\frac{\pi}{6}$ seconds *(assume the particle is initially at the origin)*. **1**

(b) The displacement *x* metres of a particle moving in simple harmonic motion is given by $x = 4\sin \pi t$ where *t* is the time expressed in seconds.

(i) Find the period of oscillation. **1**

(ii) Find the speed at the centre of motion. **2**

(iii) Show the acceleration is proportional to the displacement from the centre of motion **1**

(iv) What is the velocity when $x = 3$? **3**

Question 4 – Projectile Motion (6 Marks)

A projectile is fired from a point on horizontal ground with initial speed V ms^{-1} and an angle of projection θ. The acceleration due to gravity is g.

Let x and y be the horizontal and vertical displacements from O, the point of projection. They are given by the following equations: ***(you do <u>not</u> need to prove these)*****.**

$$x = Vt\cos\theta$$

$$y = -\frac{1}{2}gt^2 + Vt\sin\theta$$

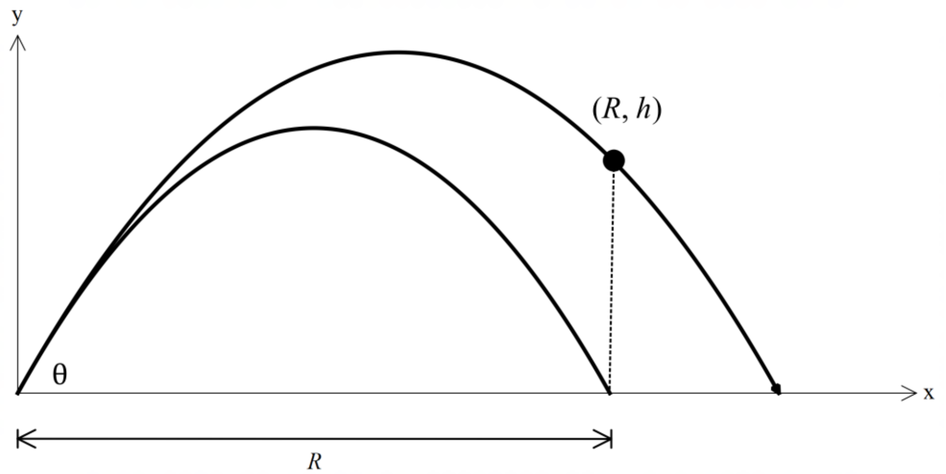
The cartesian equation of the path is: ***(you do <u>not</u> need to prove this).***

$$y = x\tan\theta - \frac{gx^2}{2V^2\cos^2\theta}$$

Show that the maximum range R on the horizontal plane is given by:

(a) **2**

$$R = \frac{V^2}{g}$$



(b) Show that to hit a target h metres above the ground at the same horizontal distance R using the same angle of projection θ, the speed of projection must be increased to $\dfrac{V^2}{\sqrt{V^2-gh}}$. **4**

Question 1

(a) $(3x-2y)^5$

$$= \binom{5}{0}(3x)^5 - \binom{5}{1}(3x)^4(2y) + \binom{5}{2}(3x)^3(2y)^2 - \binom{5}{3}(3x)^2(2y)^3 + \binom{5}{4}(3x)(2y)^4 - \binom{5}{5}(2y)^5 \checkmark$$

$$= 243x^5 - 810x^4y + 1080x^3y^2 - 720x^2y^3 + 240xy^4 - 32y^5 \checkmark$$

(b) A typical term in the expansion is:

$$\binom{9}{k}(x^2)^{9-k}(3x^{-1})^k \checkmark$$

$$= \binom{9}{k}3^k x^{18-2k} x^{-k}$$

$$= \binom{9}{k}3^k x^{18-3k}$$

$$18 - 3k = 3$$

$$3k = 15$$

$$k = 5$$

$$\therefore \binom{9}{5}3^5 \text{ or } 30618 \checkmark$$

(c) A typical term in the expansion is:

$$\binom{4}{k} 3^{4-k} (-x)^k \binom{7}{r} (1)^{7-r} (2x^{-1})^r$$ ✓

$$= \binom{4}{k}\binom{7}{r}\left(3^{4-k}\right)(-1)^k\, 2^r\, x^k (x^{-1})^r$$

$$= \binom{4}{k}\binom{7}{r} 3^{4-k} (-1)^k\, 2^r\, x^{k-r}$$

$k - r = 0$

$\therefore k = r$ ✓

$$\binom{4}{0}\binom{7}{0} 3^4 (-1)^0\, 2^0 = 81$$

$$\binom{4}{1}\binom{7}{1} 3^3 (-1)\, 2^1 = -1512$$

$$\binom{4}{2}\binom{7}{2} 3^2 (-1)^2\, 2^2 = 4536$$

$$\binom{4}{3}\binom{7}{3} 3^1 (-1)^3\, 2^3 = -3360$$

$$\binom{4}{4}\binom{7}{4} 3^0 (-1)^4\, 2^4 = 560$$

305 ✓

(d) $2\binom{n}{3} = \binom{n}{4}$

$$\frac{2\cancel{n!}}{3!\,(n-3)!} = \frac{\cancel{n!}}{4!\,(n-4)!} \quad \boxed{\checkmark}$$

$$\frac{2}{\cancel{3!}\,(n-3)\cancel{(n-4)!}} = \frac{1}{4\times\cancel{3!}\,\cancel{(n-4)!}}$$

$$\frac{2}{n-3} = \frac{1}{4}$$

$\therefore\ n-3 = 8$

$n = 11$ $\boxed{\checkmark}$

(e) $1(a-bx)^{12} + x(a-bx)^{12}$

x^8 is:

$$\binom{12}{8}a^4(-b)^8 + \binom{12}{7}a^5(-b)^7 = 0 \quad \boxed{\checkmark}$$

$$\therefore \binom{12}{8}a^4b^8 = \binom{12}{7}a^5b^7$$

$$\therefore \frac{a^5b^7}{a^4b^8} = \frac{\binom{12}{8}}{\binom{12}{7}} \quad \boxed{\checkmark}$$

$$\therefore \frac{a}{b} = \frac{5}{8} \quad \boxed{\checkmark}$$

Question 2

(a)

$\ddot{x} = 0$

$\dot{x} = C$

when $t = 0$

$\dot{x} = V\cos\alpha$

$x = Vt\cos\alpha + C$

when $t = 0 \quad x = 0$

$0 = V(0)\cos\alpha + C$

$\therefore C = 0$

$\therefore x = Vt\cos\alpha \quad \boxed{\checkmark}$

$\ddot{y} = -g$

$\dot{y} = -gt + C$

when $t = 0 \quad \dot{y} = V\sin\alpha$

$\therefore V\sin\alpha = -g(0) + C$

$\therefore C = V\sin\alpha$

$\dot{y} = -gt + V\sin\alpha$

$y = -\frac{1}{2}gt^2 + Vt\sin\alpha + C$

when $t = 0 \quad y = 0$

$0 = -\frac{1}{2}g(0)^2 + V(0)\sin\alpha + C$

$\therefore C = 0$

$\therefore y = -\frac{1}{2}gt^2 + Vt\sin\alpha \quad \boxed{\checkmark}$

(b) $x = Vt\cos\alpha$

$t = \dfrac{x}{V\cos\alpha}$

$y = -\frac{1}{2}g\left(\dfrac{x}{V\cos\alpha}\right)^2 + \cancel{V}\left(\dfrac{x}{\cancel{V}\cos\alpha}\right)\sin\alpha \quad \boxed{\checkmark}$

$= -\frac{1}{2}g\left(\dfrac{x^2}{V^2\cos^2\alpha}\right) + \dfrac{x\sin\alpha}{\cos\alpha}$

$= -\frac{1}{2}g\dfrac{x^2}{V^2}\sec^2\alpha + x\tan\alpha$

$= -\dfrac{gx^2}{2V^2}(1 + \tan^2\alpha) + x\tan\alpha$

$= x\tan\alpha - \dfrac{gx^2}{2V^2}(1 + \tan^2\alpha) \quad \boxed{\checkmark}$

(c) when $x = 5$, $\alpha = 60°$, $V = 12$, $g = 9.8$

$$y = 5\tan 60° - \frac{9.8 \times 5^2}{2 \times 12^2}(1 + \tan^2 60°)$$ ✓

$$= 5\sqrt{3} - \frac{245}{288}(4)$$

$$= 5.257476 \dots$$

$\therefore$ clearence $= 5.257\dots - 3$

$= 2.26$ m (3 s.f.) ✓

(d) $y = x\left(\tan\alpha - \frac{gx}{2v^2}(1 + \tan^2\alpha)\right)$

when $y = 0$

$x = 0$ or $\tan\alpha - \frac{gx}{2v^2}(1 + \tan^2\alpha) = 0$. ✓

as $x \neq 0$ at max distance

$$\tan\alpha - \frac{gx}{2v^2}(1 + \tan^2\alpha) = 0.$$

$$\therefore \frac{gx}{2v^2}(1 + \tan^2\alpha) = \tan\alpha$$

$$\therefore x = \frac{2v^2\tan\alpha}{g(1 + \tan^2\alpha)}$$ ✓

$$= \frac{2 \times 12^2 \times \sqrt{3}}{9.8(1 + 3)}$$

$= 12.7$ m (3 s.f.) ✓

Question 3

(a) (i) $\frac{2\pi}{n} = \frac{\pi}{3}$

$\therefore n = 6$ $\boxed{\checkmark}$

$v^2 = n^2(a^2 - x^2)$

when $x = 0$, $V = 2$

$2^2 = 6^2(a^2 - 0^2)$

$\therefore a^2 = \frac{2^2}{6^2}$

$\therefore a = \pm \frac{1}{3}$

$\therefore$ amplitude $= \frac{1}{3}$ $\boxed{\checkmark}$

(ii) Displacement is 0 $\boxed{\checkmark}$

($\frac{\pi}{6}$ " is half a period so it will be back at the origin) (b)(i) $\frac{2\pi}{\pi} = 2$ seconds $\boxed{\checkmark}$

(ii) $x = 4\sin \pi t$

$\dot{x} = 4\pi \cos \pi t$ $\boxed{\checkmark}$

at centre of motion $\cos \pi t = 1$

$\therefore$ speed $= 4\pi$ m/s $\boxed{\checkmark}$

(iii) $\ddot{x} = -4\pi^2 \sin \pi t$

$= -\pi^2 \, 4\sin \pi t$

$= -\pi^2 x$ $\boxed{\checkmark}$

$\therefore \ddot{x} \propto x$

(iv) $v^2 = n^2(a^2 - x^2)$

$x = 3$, $a = 4$, $n = \pi$ $\boxed{\checkmark}$

$v^2 = \pi^2(4^2 - 3^2)$

$= \pi^2 \times 7$

$\therefore v = \pm \pi\sqrt{7}$ m/s $\boxed{\checkmark}$ $\boxed{\checkmark}$

Question 4

(a) Max range occurs when $y = 0$

$$\therefore x\tan\theta - \frac{gx^2}{2v^2\cos^2\theta} = 0$$

$$\therefore x\left(\tan\theta - \frac{gx}{2v^2\cos^2\theta}\right) = 0$$

as $x \neq 0$ (assuming $\theta \neq 90°$)

$$\tan\theta - \frac{gx}{2v^2\cos^2\theta} = 0.$$

$\boxed{\checkmark}$

$$\therefore \frac{gx}{2v^2\cos^2\theta} = \tan\theta$$

$$\therefore x = \frac{2v^2\cos^2\theta\tan\theta}{g}$$

$$x = \frac{2v^2\cos^{\cancel{2}}\theta\,\frac{\sin\theta}{\cancel{\cos\theta}}}{g}$$

$$x = \frac{2v^2\sin\theta\cos\theta}{g}$$

$$x = \frac{v^2\sin 2\theta}{g}.$$

as the max value for $\sin 2\theta$ is the

max range is.

$$R = \frac{v^2}{g}.$$

$\boxed{\checkmark}$

(b) let the new velocity be w.

Sub $y = h$, $x = R$, $\theta = 45^\circ$ & $V = w$

into $y = x\tan\theta - \dfrac{gx^2}{2V^2\cos^2\theta}$ $\boxed{\checkmark}$

$$\therefore h = R\tan 45^\circ - \frac{gR^2}{2w^2(\cos 45^\circ)^2}$$

$$h = R(1) - \frac{gR^2}{\cancel{2}w^2\cancel{\left(\frac{1}{\sqrt{2}}\right)^2}}$$

$$h = R - \frac{gR^2}{w^2}$$

$$\therefore \frac{gR^2}{w^2} = R - h$$

$$\therefore w^2 = \frac{gR^2}{R-h} \quad \boxed{\checkmark}$$

but $R = \dfrac{V^2}{g}$ $\boxed{\checkmark}$

$$\therefore w^2 = \frac{g\left(\frac{V^2}{g}\right)^2}{\frac{V^2}{g} - h}$$

$$= \frac{\left(\frac{V^4\cancel{g}}{g^{\cancel{2}}}\right)}{\left(\frac{V^2 - gh}{g}\right)}$$

$$= \frac{\left(\frac{v^4}{\cancel{g}}\right)}{\left(\frac{v^2 - gh}{\cancel{g}}\right)}$$

$$= \frac{v^4}{v^2 - gh}$$

$$\therefore w = \pm\sqrt{\frac{v^4}{v^2 - gh}}$$

$$= \pm \frac{v^2}{\sqrt{v^2 - gh}}$$

but as $w > 0$.

$$w = \frac{v^2}{\sqrt{v^2 - gh}} \quad \boxed{\checkmark}$$