NORTH SYDNEY BOYS HIGH SCHOOL

MATHEMATICS (EXTENSION 1)

2018 Year 11 Task 1

Thursday April 5, 2018

General instructions

- Working time – 70 minutes.
  (plus 5 minutes reading time)
- Write using blue or black pen. Where diagrams are to be sketched, these may be done in pencil.
- Board approved calculators may be used.
- Attempt **all** questions.
- At the conclusion of the examination, bundle the booklets used in the correct order within this paper and hand to examination supervisors.

SECTION I

- Mark your answers on the answer grid provided.

SECTION II

- Commence each new question on a new page. Write on both sides of the paper.
- All necessary working should be shown in every question. Marks may be deducted for illegible or incomplete working.

STUDENT NUMBER: ..... **# BOOKLETS USED**: .....

Class (please ✔)

◯ 11M3A – Mr Hwang	◯ 11M3D – Dr Jomaa	◯ 11M3G – Mr Lin
◯ 11M3B – Ms Lee	◯ 11M3E – Mr Berry	
◯ 11M3C – Mr Ireland	◯ 11M3F – Ms Ziazaris	

Marker's use only.

QUESTION	1	2	3	4	5	6	7	Total	Total %
MARKS	/14	/7	/11	/10	/8	/9	/7	/66	/100

Question 1 (14 Marks) Commence a NEW page. **Marks**

(a) Write $0.1\dot{3}\dot{2}$ as a fraction in lowest terms, showing all working. **2**

(b) Simplify $\sqrt[7]{x^2}.\sqrt[3]{x}$. **2**

(c) Factorise $6y^2 - 11y + 3$. **2**

(d) Simplify giving your answer with positive indices **3**

$$\left(\frac{a^{-2}b}{a^3b^{-4}}\right)^{-2} \times \left(\frac{a^{-3}b^2}{a^{-1}b}\right)^3$$

.

(e) Simplify $\dfrac{x^3-1}{ax^2-a} \div \dfrac{\left(x^2+1\right)^2 - x^2}{x^3+1}$. **3**

(f) Rationalise $\dfrac{a\sqrt{b}+b\sqrt{a}}{\sqrt{a}+\sqrt{b}} - \dfrac{a\sqrt{b}-b\sqrt{a}}{\sqrt{a}-\sqrt{b}}$. **2**

Question 2 (7 Marks) Commence a NEW page. **Marks**

Solve the following:

(a) $|2x-1| = 7$ **1**

(b) $\dfrac{3}{x+2} \leq x$ **2**

(c) $|3x+5| > 8$ **2**

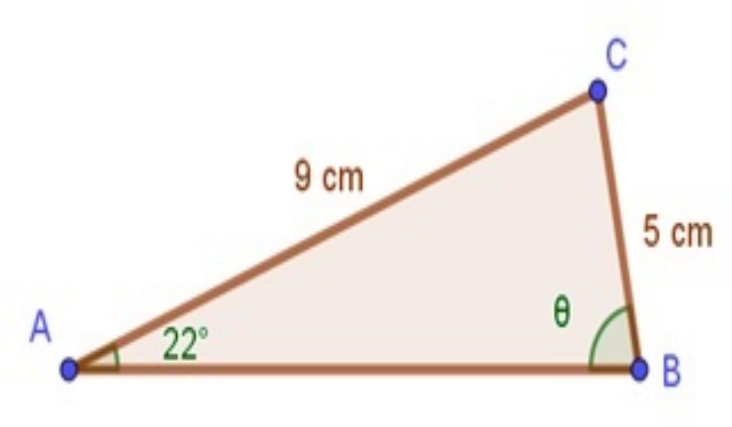
(d) $|x+4| = 2x-5$ **2**

Question 3 (11 Marks) Commence a NEW page. **Marks**

(a) Prove the identity: $\dfrac{1+\sin t}{\cos t} = \dfrac{\cos t}{1-\sin t}$ **1**

(b) Solve $2\tan(\theta - 60°) = 0.9, \quad 0 \leq \theta \leq 360°$ **2**

(c) Find all possible values of θ in triangle ABC (see diagram). **2**



(d) If $\cos\theta = \frac{1}{2}$ and $\tan\theta = -\sqrt{3}$, find $\sin\theta$ where $0 \leq \theta \leq 360°$ **2**

(e) i. Solve $3\cos(3x) = 2$ where $0 \leq \theta \leq 360°$ **2**

ii. Hence, sketch $y = 2 - 3\cos 3x$ for $0 \leq x \leq 360°$, showing all important features. **2**

Question 4 (10 Marks) Commence a NEW page. **Marks**

(a) Sketch graphs of the following curves indicating any intercepts and asymptotes.

i. $y = \sqrt{3-x}$ **2**

ii. $y = 2^{2+x}$ **2**

iii. $y = \dfrac{x+2}{x^2-5x+6}$ **2**

(b) Given that

$$f(x) = \begin{cases} x^2 - 2 & x < -1 \\ x^3 & -1 \leq x \leq 1 \\ -x^2 + 2 & x > 1 \end{cases}$$

i. Find $f(-n) + f(0) + f(n)$, $n > 1$ **1**

ii. Sketch the graph of $f(x)$, showing all the important features. **3**

Please Turn Over

Question 5 (8 Marks) Commence a NEW page. **Marks**

(a) Write down the domain and range of these functions:

i. $y = x^2 + 8x + 11$ **2**

ii. $y = \dfrac{1}{\sqrt{16 - x^2}}$ **2**

(b) Let $f(x) = 3x + 5$ and $h(x) = 3x^2 + 3x + 2$

i. Find $f^{-1}(x)$ the inverse function of $f(x)$. **2**

ii. Find a function g such that $f(g(x)) = h(x)$ for all x. **2**

Question 6 (9 Marks) Commence a NEW page. **Marks**

(a) A is 3 km North and 4.5 km East of P. The Bearing of B from A is 155°. Find the distance PB, if the distance AB is 15 km. **4**

(b) Consider a triangle ABC.

i. Use the sine rule and the cosine rule simultaneously to prove that **3**

$$\cos^2 A + \sin^2 B + \sin^2 C = 1 + 2\cos A \sin B \sin C$$

ii. Let $\angle A = 60°$ and $\angle B = 45°$. Use (i) to find the exact value of $\sin 75°$. **2**

Question 7 (7 Marks) Commence a NEW page. **Marks**

(a) A right angled triangle ABC has an area of 25 cm^2. P and h are the perimeter and the hypotenuse of ABC respectively.

i. Show that $h = \dfrac{P^2 - 100}{2P}$ **2**

ii. Sketch the graph of the function h. **2**

(b) Let $f(x)$ be a function defined as $f(0) = 0$ and $f(x) = f(x-1) + 2x - 1$ for $x \geq 1$. Find an explicit formula for $f(x)$. **3**

End of paper.

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Year 11 - 2018 - Ext1 - Task 1

Solutions

Question 1

2 a) $0.1\dot{3}\dot{2}$

Let $x = 0.1323232\ldots$

$10x = 1.323232\ldots$

$1000x = 132.323232\ldots$

$990x = 131$

$x = \frac{131}{990}$

2 b) $\sqrt[7]{x^2} \cdot \sqrt[3]{x} = x^{2/7} \cdot x^{1/3}$

$= x^{13/21}$

2 c) $6y^2 - 11y + 3 = (3y - 1)(2y - 3)$

3 d) $\left(\frac{a^{-2}b}{a^3 b^{-4}}\right)^{-2} \times \left(\frac{a^{-3}b^2}{a^{-1}b}\right)^3$

$= \left(\frac{b^5}{a^5}\right)^{-2} \times \left(\frac{b}{a^2}\right)^3$

$= \frac{a^{10}}{b^{10}} \times \frac{b^3}{a^6}$

$= \frac{a^4}{b^8} \quad \frac{a^4}{b^7}$

3 e) $\frac{x^3-1}{ax^2-a} \div \frac{(x^2+1)^2 - x^2}{x^3+1}$

$= \frac{(x-1)(x^2+x+1)}{a(x+1)(x-1)} \div \frac{(x^2+1+x)(x^3+1-x)}{(x+1)(x^3-x+1)}$

$= \frac{x^2+x+1}{a(x+1)} \times \frac{x+1}{x^2+x+1}$

$= \frac{1}{a}$

2 f) $\dfrac{a\sqrt{b}+b\sqrt{a}}{\sqrt{a}+\sqrt{b}} - \dfrac{a\sqrt{b}-b\sqrt{a}}{\sqrt{a}-\sqrt{b}}$

$$= \frac{(a\sqrt{b}+b\sqrt{a})(\sqrt{a}-\sqrt{b}) - (a\sqrt{b}-b\sqrt{a})(\sqrt{a}+\sqrt{b})}{a-b}$$

$$= \frac{a\sqrt{ab} + ab - ab - b\sqrt{ab} - (a\sqrt{ab} - ab + ab - b\sqrt{ab})}{a-b}$$

$$= 0$$

Question 2 :

(a) $|2x-1| = 7$

$\therefore (2x-1)^2 = 49$

$\therefore (2x-1+7)(2x-1-7) = 0$

$\therefore (2x+6)(2x-8) = 0$

$\therefore x = -3$ or $x = 4$

(b) $\frac{3}{x+2} \leq x$

$\therefore 3(x+2) \leq x(x+2)^2$

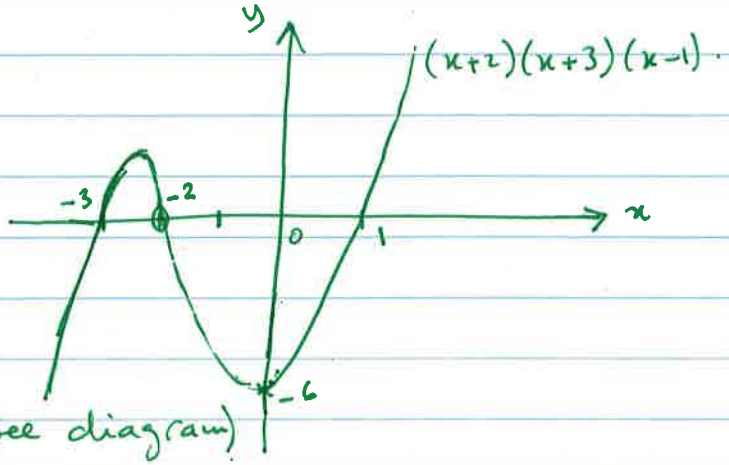
$\therefore (x+2)(3 - x(x+2)) \leq 0$

$\therefore (x+2)(-x^2-2x+3) \leq 0$

$\therefore (x+2)(x^2+2x-3) \geqslant 0$

$\therefore (x+2)(x+3)(x-1) \geqslant 0$

$\therefore -3 \leqslant x < -2$ or $x \geqslant 1$



(see diagram)

(c) $|3x+5| > 8$

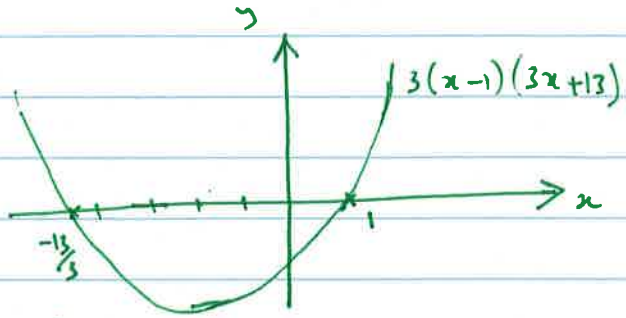
$\therefore (3x+5)^2 > 64$

$\therefore (3x+5+8)(3x+5-8) > 0$

$\therefore (3x+13)(3x-3) > 0$

$\therefore 3(x-1)(3x+13) > 0$

$\therefore x < \frac{-13}{3}$ or $x > 1$



(see diagram).

(d) $|x+4| = 2x-5$

$\therefore (x+4)^2 = (2x-5)^2$

$\therefore (x+4+2x-5)(x+4-2x+5) = 0$

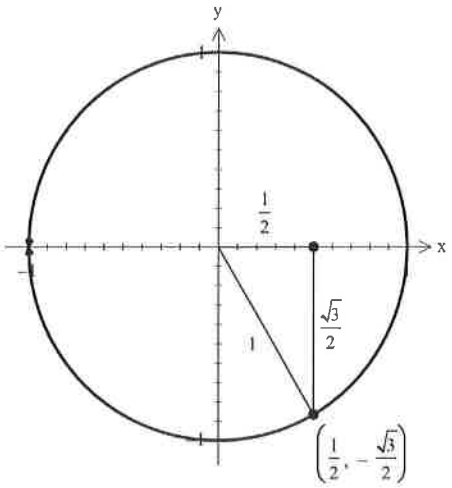
$\therefore (3x-1)(-x+9) = 0$

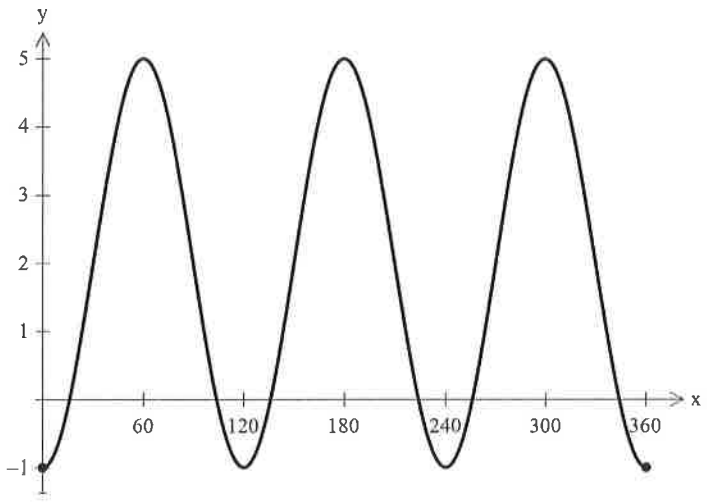
$\therefore x = 1/3$ or $x = 9$

check: $x = 9$, $|9+4| = 13$ and $2\times 9 - 5 = 13$ ✓

$x = 1/3$, $|1/3+4| = \frac{13}{3}$ and $2x-5 = \frac{2}{3} - 5 = \frac{-13}{3}$ rejected

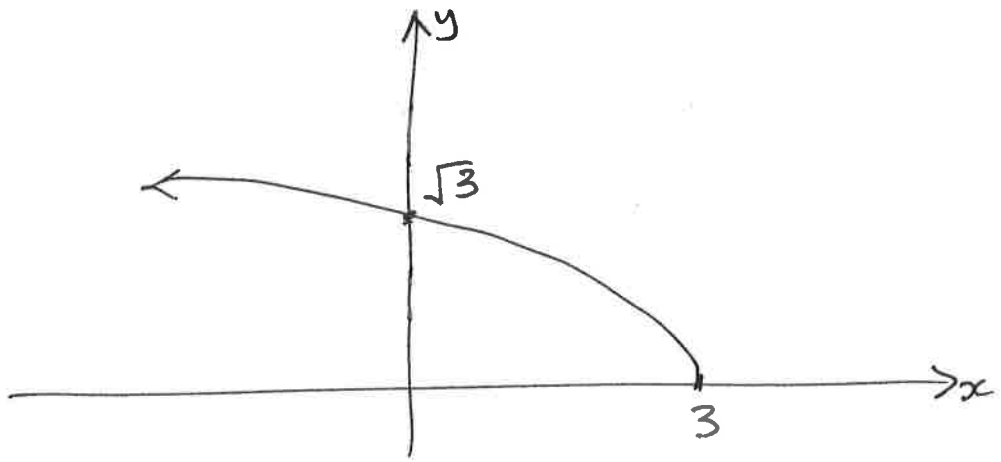
$\therefore x = 9$.

Q3	Solutions	Marks
(a)	$\text{LHS} = \dfrac{1+\sin t}{\cos t} \times \dfrac{1-\sin t}{1-\sin t}$ $= \dfrac{1-\sin^2 t}{\cos t\,(1-\sin t)}$ $= \dfrac{\cos^2 t}{\cos t(1-\sin t)}$ $= \dfrac{\cos t}{1-\sin t}$ $=$ RHS	✓ Correct proof clearly showing all steps
(b)	$0 \leq \theta \leq 360°$ $-60° \leq \theta - 60° \leq 300°$ $2\tan(\theta - 60°) = 0.9$ $\tan(\theta - 60°) = 0.45$ $\theta - 60° = 24°\ 13'\ 40''$ or $204°\ 13'\ 40''$ $\theta = 84°\ 13'\ 40''$ or $264°\ 13'\ 40''$ (to the nearest second)	✓ Acute answer ✓ Reflex answer
(c)	$\dfrac{\sin\theta}{9} = \dfrac{\sin(22°)}{5}$ $\sin\theta = \dfrac{9\sin(22°)}{5}$ $\theta = 42°\ 23'\ 57''$ or $137°\ 36'\ 3''$ (to the nearest second)	✓ Acute answer ✓ Obtuse answer
(d)	 $\sin\theta = -\dfrac{\sqrt{3}}{2}$	✓ Correct value ✓ Correct quadrant

(e) (i)	$0 \leq x \leq 360°$ $0 \leq 3x \leq 1080°$ $3\cos(3x) = 2$ $\cos(3x) = \frac{2}{3}$ $3x = 48° 11' 23"$, $311° 48' 37"$ $408° 11' 23"$, $671° 48' 37"$ $768° 11' 23"$ or $1031° 48' 37"$ $x = 16° 3' 48"$, $103° 56' 13"$ $136° 3' 48"$, $223° 56' 13"$ $256° 3' 48"$ or $343° 56' 13$ (to the nearest second)	✓ Any one correct answer ✓ All correct answers
(e) (ii)		✓ Amplitude and period ✓ Vertical translation with the x intercepts in reasonable positions

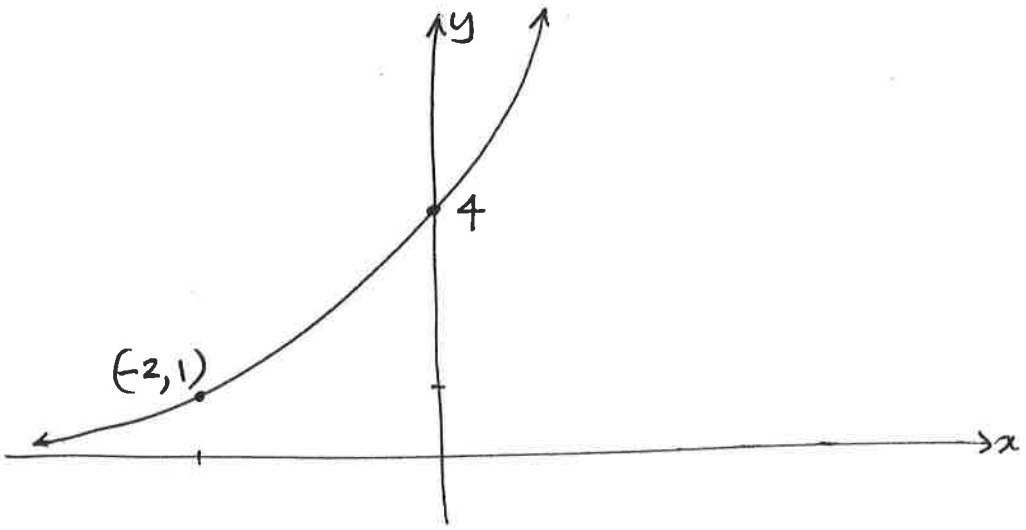
Q4

(a) (i) $y = \sqrt{3-x}$



✓✓

(ii) $y = 2^{2+x} = 4 \times 2^{x}$



✓✓

Q4 (a) (ctd)

(iii) $y = \dfrac{x+2}{x^2-5x+6} = \dfrac{x+2}{(x-2)(x-3)}$

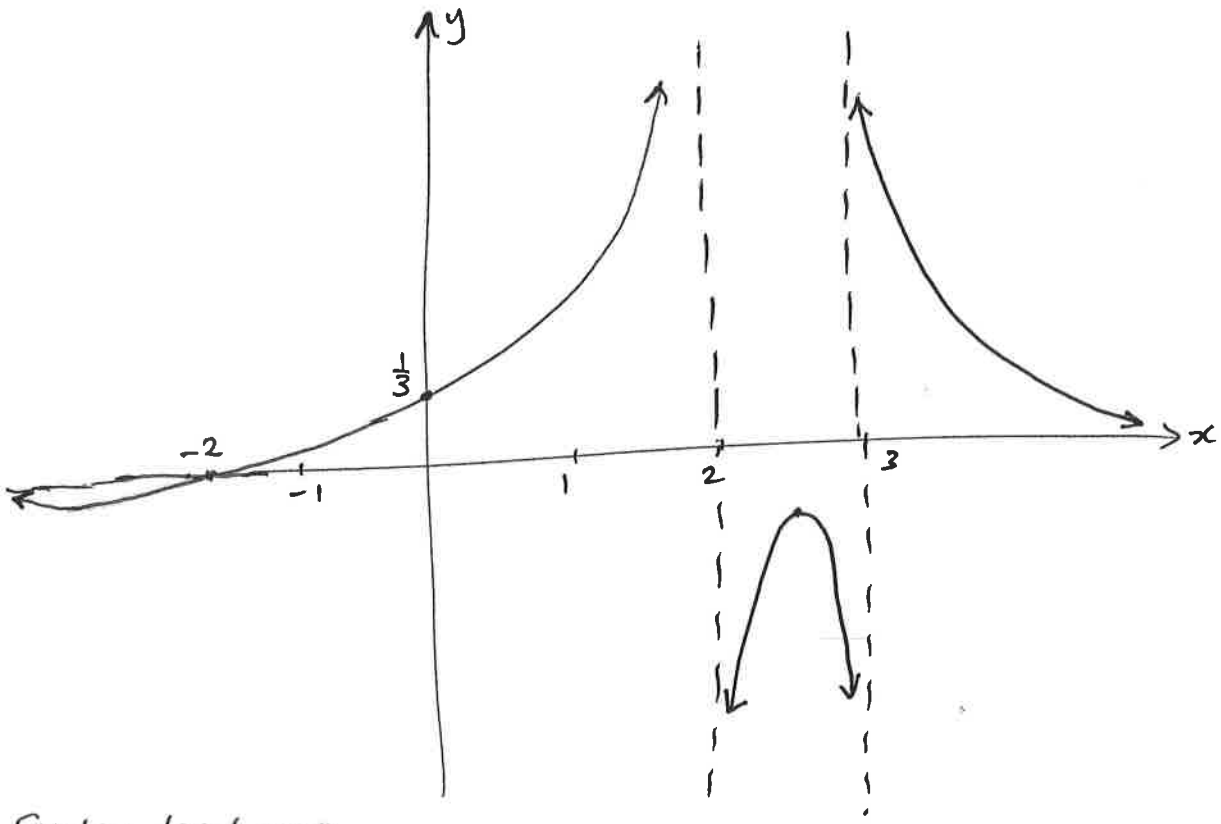
Vert. asymptotes : $x=2,\ x=3$

y-int. $= \frac{1}{3}$, x-int. $= -2$

as $x \to \infty$, $y \to 0^+$. as $x \to -\infty$, $y \to 0^-$.

x	-3	-2	0	2	$2\frac{1}{2}$	3	4
y	$-$	0	$+$	$*$	$-$	$*$	$+$

↑ Table of signs.



✓✓

[Note: local max between 2 & 3 is at $x = 2\sqrt{5}-2 \div 2.47$ not at 2.5]

Full marks needs all asymptotes (labelled) and y-intercept.

Q4 (b)

$$f(x) = \begin{cases} x^2 - 2, & x < -1 \\ x^3, & -1 \le x \le 1 \\ -x^2 + 2, & x > 1 \end{cases}$$

(i) $f(-n) = (-n)^2 - 2 = n^2 - 2 \quad (\text{as } n > 1)$

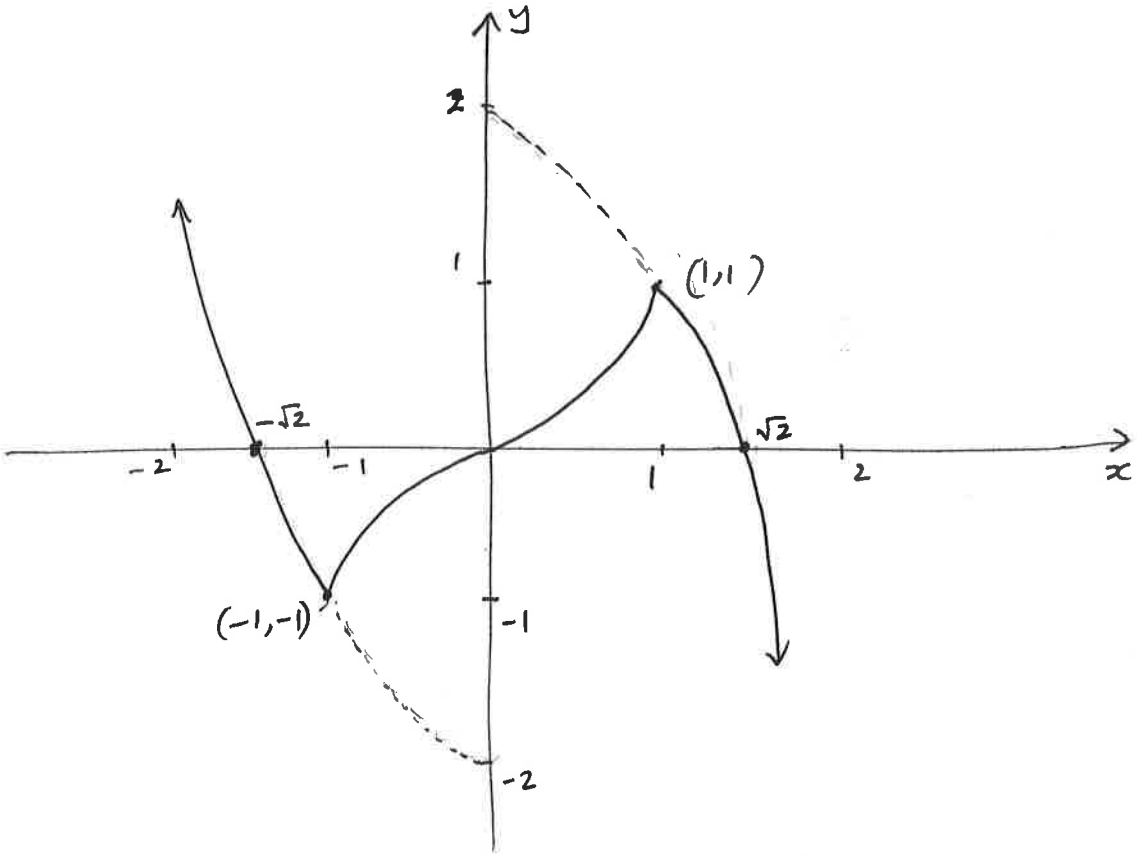
$f(0) = 0^3 = 0$

$f(n) = -n^2 + 2$

$\therefore f(-n) + f(0) + f(n)$

$= n^2 - 2 + 0 - n^2 + 2 = 0$ ✓

(ii)



✓✓✓

10

Question 5

a) i) $y = x^2 + 8x + 11$

vertex at $(-4, -5)$ and concave up

$\therefore$ D: x is all real

R: $y \geq -5$

ii) D: $-4 < x < 4$

R: $y \geq \frac{1}{4}$

b) i) Inverse relation:

$x = 3y + 5$

$\therefore 3y = x - 5$

$y = \frac{x-5}{3}$

$\therefore f^{-1}(x) = \frac{x-5}{3}$

ii) $f(g(x)) = 3x^2 + 3x + 2$

$3g(x) + 5 = 3x^2 + 3x + 2$

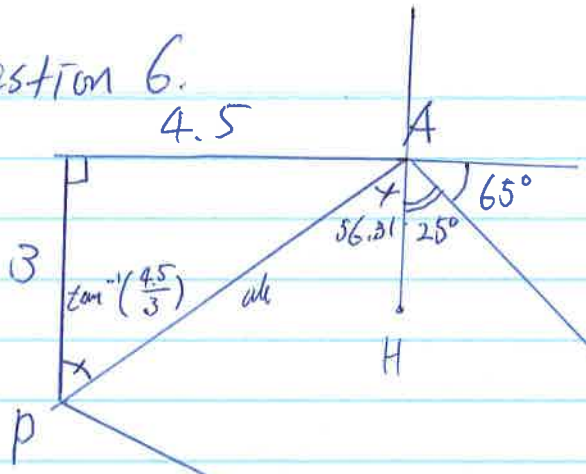
$3g(x) = 3x^2 + 3x - 3$

$g(x) = x^2 + x - 1$

Question 6.

4.

a)



$\angle BAH = 25°$ (from bearing of B from A)

$\angle PAH = \tan^{-1}(\frac{4.5}{3}) \approx 56.31°$

$\therefore \angle PAB = 81.31°$ ✓

15

B

$PA = \sqrt{4.5^2+3^2}$

$= \sqrt{29.25}$ ✓

≈ 5.408 (3 d.p)

$PB = \sqrt{15^2+29.25-2\times15\times\sqrt{29.25}\times\cos 81.31°}$ ✓

$= 15.16$ km (2 d.p) ✓.

3

b) i) $\cos^2A + \sin^2B + \sin^2C = 1 + 2\cos A \sin B \sin C$.

from sine rule. $a = b\times\frac{\sin A}{\sin B}$ $b = b$ $c = b\times\frac{\sin C}{\sin B}$ ✓.

sub into cos rule.

$a^2 = b^2 + c^2 - 2bc\cos A$ ✓.

$b^2\times\frac{\sin^2A}{\sin^2B} = b^2 + b^2\times\frac{\sin^2C}{\sin^2B} - 2b^2\frac{\sin C}{\sin B}\cos A$ | $\div b^2$ & $\times\cos^2B$.

$\sin^2A = \sin^2B + \sin^2C - 2\sin B\sin C\cos A$ | since $\sin^2A = 1-\cos^2A$

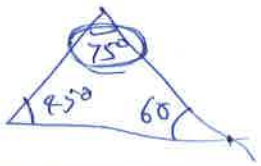
$1-\cos^2A = \sin^2B + \sin^2C - 2\cos A\sin B\sin C$

$1 + 2\cos A\sin B\sin C = \cos^2A + \sin^2B + \sin^2C$. ✓

as given.

~~1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11~~

~~12, 13, 14, 15~~



Q6.

(2)

b) ii) Q. $\cos^2 60 + \sin^2 45 + \sin^2 75 = 1 + 2\cos 60 \sin 45 \sin 75$

let $\sin 75 = x$.

$$\frac{1}{4} + \frac{1}{2} + x^2 = 1 + 2 \times \frac{1}{2} \times \frac{\sqrt{2}}{2} \times x.$$

$$x^2 - \frac{\sqrt{2}}{2}x - \frac{1}{4} = 0 \quad | \times 4$$

✓

$$4x^2 - 2\sqrt{2}x - 1 = 0.$$

Using quadratic formula

$$x = \frac{2\sqrt{2} \pm \sqrt{8+16}}{8} = \frac{2\sqrt{2} \pm 2\sqrt{6}}{8}$$

$$= \frac{\sqrt{2} \pm \sqrt{6}}{4}$$

$$\therefore \sin 75^\circ = \frac{\sqrt{2}+\sqrt{6}}{4} \quad \text{or} \quad \frac{\sqrt{2}-\sqrt{6}}{4}.$$

but sine 75° is acute $\sin 75^\circ$ has to be positive

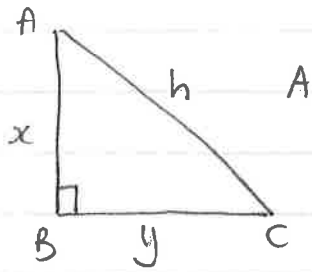
$$\therefore \sin 75^\circ = \frac{\sqrt{2}+\sqrt{6}}{4}$$

✓

$$= 0.9659$$

Question 7 Yr 11 2018 Task 1. Ext 1

a)



$A = 25cm^2$

(i) $P = x + y + h$ $\quad A = \frac{1}{2}xy$ $\quad x^2 + y^2 = h^2$ —②

$25 = \frac{1}{2}xy$

$xy = 50$ —①

$\therefore P - h = x + y$

$(P-h)^2 = (x+y)^2$

$(P-h)^2 = x^2 + 2xy + y^2$

Using ① & ②

$(P-h)^2 = h^2 + 100$

$P^2 - 2Ph + h^2 = h^2 + 100$

$P^2 - 100 = 2Ph$

$h = \frac{P^2 - 100}{2P}$

(ii) $h = \frac{P^2 - 100}{2P}$

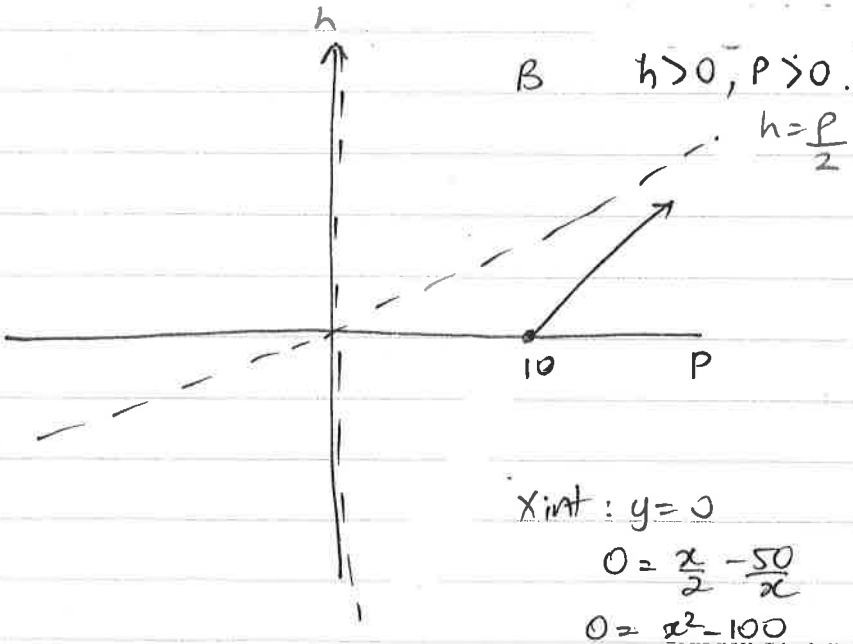
$h = \frac{P^2}{2P} - \frac{100}{2P}$

$h = \frac{P}{2} - \frac{50}{P}$

i.e. $y = \frac{x}{2} - \frac{50}{x}$

Asymptotes $y = \frac{x}{2}$

$x = 0$.



B $h > 0, P > 0$.

x int : $y = 0$

$0 = \frac{x}{2} - \frac{50}{x}$

$0 = x^2 - 100$

$x = 10$

b) $f(0) = 0$

$f(1) = f(0) + 1$

$= 0 + 1$

$= 1$

$f(2) = f(1) + 3$

$= 1 + 3$

$= 4$

$f(3) = f(2) + 5$

$= 4 + 5$

$= 9$

$f(4) = f(3) + 7$

$= 9 + 7$

$= 16.$

x	0	1	2	3
$f(x)$	0	1	4	9

$\therefore \boxed{f(x) = x^2}$

OR

$f(x) = f(x-1) + 2x - 1$

$(x-1)^2 = x^2 - 2x + 1$

$\therefore f(x) = (x-1)^2 + 2x - 1$

$= x^2 - 2x + 1 + 2x - 1$

$= x^2$

Check

$f(0) = 0$

f