NORTH SYDNEY BOYS HIGH SCHOOL

MATHEMATICS EXTENSION 1

2018 Preliminary Course Assessment Task 2
Tuesday, 12 June 2018

General instructions

- Working time – 60 minutes.
- Write using blue or black pen. Where diagrams are to be sketched, these may be done in pencil.
- Board approved calculators may be used.
- Attempt all questions.
- At the conclusion of the examination, bundle the booklets used in the correct order **within** this paper and hand to examination supervisors.
- Commence each **new question** on a **NEW PAGE.** Write on both sides of the paper.
- All necessary working should be shown in every question. Marks may be deducted for illegible or incomplete working.

STUDENT NUMBER: ..... **# EXTRA BOOKLETS USED: .....**

Class: (please ✓)

- ○ Mr Berry
- ○ Mr Hwang
- ○ Mr Ireland
- ○ Dr Jomaa
- ○ Ms Lee
- ○ Mr Lin
- ○ Ms Ziaziaris

Marker's use only

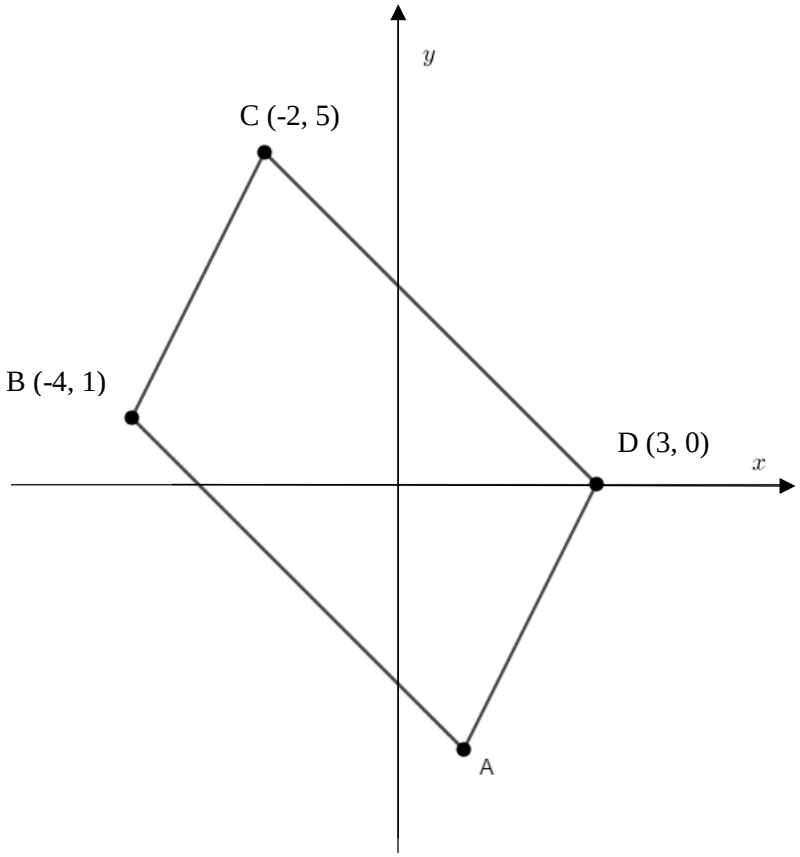
QUESTION	1	2	3	4	5	Total	%
MARKS	/10	/13	/15	/12	/8	/58	/100

58 Marks
Attempt Question 1 to 5

Write your answers in the writing booklets supplied. Additional writing booklets are available.
Your responses should include relevant mathematical reasoning and/or calculations.

Question 1 (10 marks)

In the diagram *A*, *B* (-4, 1), *C* (-2, 5) and *D* (3, 0) are the vertices of a parallelogram.



(a) Find the length of the interval *BC*. **1**

(b) Show that the equation of the line through *B* and *C* is $2x - y + 9 = 0$. **2**

(c) Find the angle of the inclination that *CD* makes with the positive direction of the x-axis. **2**

(d) Find the midpoint of *BD*. **1**

(e) Hence, or otherwise, find the co-ordinates of *A*. **1**

(f) Find the perpendicular distance from *D* to *BC*. **2**

(g) Find the area of the parallelogram *ABCD*. **1**

Question 2 (13 marks)

(a) Find the acute angle between the lines $y = 3x - 5$ and $3x + 5y - 2 = 0$ giving your answer to the nearest minute.

3

(b) The point C (5, 3) divides the interval AB externally in the ratio 3: 1. If B has coordinates (4, 1) find the coordinate of A.

3

(c) Find the value of k if the points A (-2,8), B (3, 6) and C $(6k - 4, 12 - 2k)$ are collinear. **2**

(d) Find the equation of the line passing through the point (1, 2) and through the point of intersection of the lines $3x - 2y - 1 = 0$ and $2x + 3y - 4 = 0$.

2

(e) Sketch the region on a number plane where the following hold simultaneously.

(Use at least a third of a page) **3**

$$y \leq x^2 - 3x$$

$$y > x$$

Question 3 (15 marks)

(a) Differentiate with respect to x and **<u>simplify fully</u> with common denominator where necessary.**

(i) $y = 6x\sqrt{x}$ **2**

(ii) $y = (2 - 5x^3)^7$ **2**

(iii) $y = -3x^2\sqrt{2x + 1}$ **2**

(iv) $y = \frac{5-3x}{x^2-5}$ **2**

Question 3 continues on page 4

Question 3 (continued)

(b) For the curve $y = x^3 - 4x^2 + 5$,

i. Find the equation of the tangent at $x = 2$. **3**

ii. Find the equation of the normal at $x = 2$. **1**

(c) Differentiate the following function from **first principles**. **3**

$$f(x) = 5x - x^2$$

Question 4 (12 marks)

(a) Convert $\frac{4\pi}{3}$ radians to degrees. **1**

(b) Prove that $4 \sin x \cos x \cos 2x = \sin 4x$. **2**

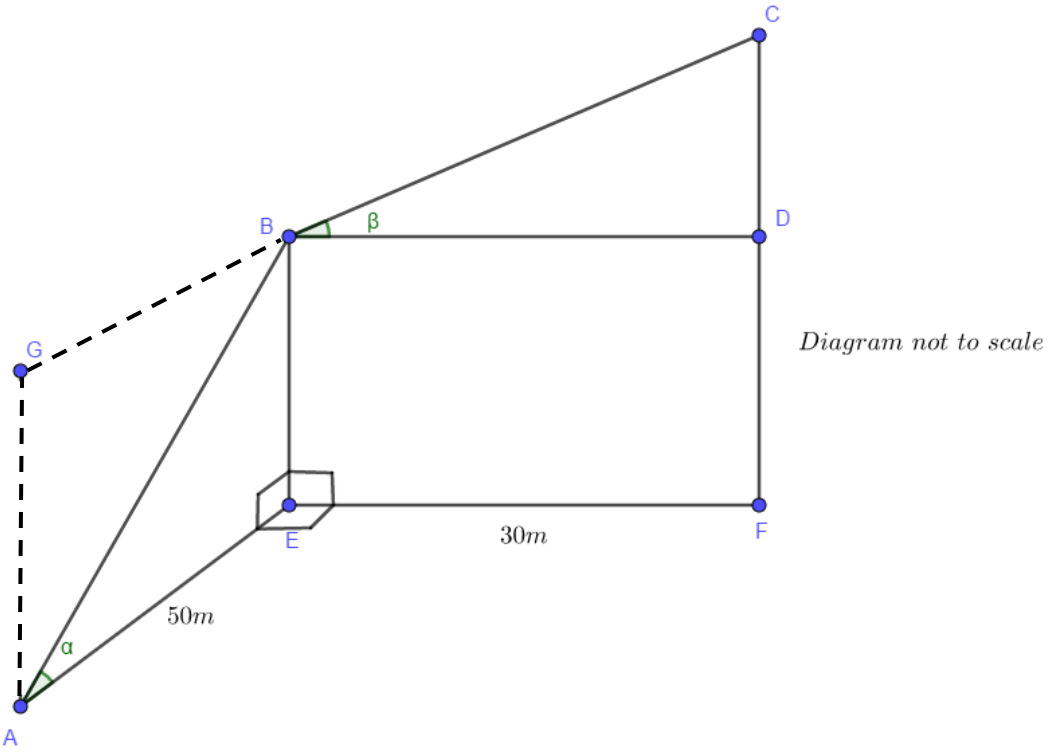
(c) **(i)** Write down the expansion of $\cos(x + y)$. **1**

(ii) Hence find the exact value of $\cos\frac{7\pi}{12}$ as a single fraction (Do NOT rationalise the denominator) **2**

(d) Given that $\tan\theta = \frac{5}{12}$, find the possible values of $\cos\frac{\theta}{2}$. **3**

(e) Solve $\sqrt{3}\tan 3x = -1$, where $-\pi \leq x \leq \pi$ **3**

Question 5 (8 marks)



(a) The diagram represents part of the inside of two of the walls (built vertically and at right angles) of a parking station. *AEF* represents the horizontal floor. *AE* is 50 metres and *EF* is 30 metres. *AB* and *BC* represent the ramp climbing up the two walls.
The ramp climbs at angles α and β, where $\tan\alpha = \frac{1}{5}$ and $\tan\beta = \frac{1}{6}$.

(i) Find the length of *FC*. **1**

(ii) Hence show that the distance of $AC = \sqrt{3625}$. **2**

(iii) Find $\angle ABC$ to the nearest minute. **2**

(b) Show that $\sin 3x - \cos 3x \tan x = 2\sin x$ **3**

End of Examination ☑

2018 Year 11 Ext 1 task 2 suggested solutions

Question 1

a) $BC = \sqrt{(-4+2)^2 + (5-1)^2}$
$= \sqrt{4+16}$
$= \sqrt{20} = 2\sqrt{5}$

b) $y - 1 = \frac{5-(1)}{-2-(-4)}(x+4)$

$y = 2x + 8 + 1$

$2x - y + 9 = 0$

$2x - y + 9 = 0 \; as \; given$

c) $m = \frac{5-0}{-2-3} = \frac{5}{-5} = -1$
$\tan^{-1}(-1) = 135°$

d) Midpoint $= \left(\frac{-4+3}{2}, \frac{0+1}{2}\right)$
$= \left(-\frac{1}{2}, \frac{1}{2}\right)$

e) $midpoint \; of \; C(-2,5) \; and \; A(x,y)$
is same as midpoint of BD

$$\left(-\frac{1}{2}, \frac{1}{2}\right) = \left(\frac{-2+x}{2}, \frac{5+y}{2}\right)$$

$$\therefore x = 1, y = -4$$

f) $\perp d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$
$= \frac{|2 \times 3 + (-1) \times 0 + 9|}{\sqrt{2^2 + (-1)^2}}$
$= \frac{|15|}{\sqrt{5}}$
$= \frac{15\sqrt{5}}{5} = 3\sqrt{5} \; or \; \frac{15}{\sqrt{5}} \; units$

g) $A = b \times h$
$2\sqrt{5} \times 3\sqrt{5} = 30 \, u^2$

Question 2

a) Angle between two lines

$$\theta = \tan^{-1} \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$m_1 = 3, \; m_2 = -\frac{3}{5}$$

$$\theta = \tan^{-1} \left| \frac{3 + \frac{3}{5}}{1 + 3 \times -\frac{3}{5}} \right| = \tan^{-1} \left| -\frac{9}{2} \right|$$

$$= 77° \, 28'$$

b) $m:n = -3:1$
$A(x,y), \quad B(4,1)$
$C(5,3)$

$$5 = \frac{nx_1 + mx_2}{m+n}, \quad 3 = \frac{ny_1 + my_2}{m+n}$$

$$5 = \frac{1x - 3(4)}{-3+1}, \quad 3 = \frac{1y_1 - 3(1)}{-3+1}$$

$$-10 = x - 12, \quad -6 = y - 3$$

$$x = 2, y = -3$$

$$A = (2, -3)$$

c) $m_{AB} = m_{CB}$

$$\frac{6-8}{3+2} = \frac{12-2k-6}{6k-4-3}$$

$$-\frac{2}{5} = \frac{6-2k}{6k-7}$$

$$-12k + 14 = 30 - 10k$$

$$-16 = 2k$$

$$\therefore k = -8$$

d) $3x-2y-1+k(2x+3y-4)=0$

$$3(1)-2(2)-1+k(2(1)+3(2)-4)=0$$

$$-2+4k=0$$

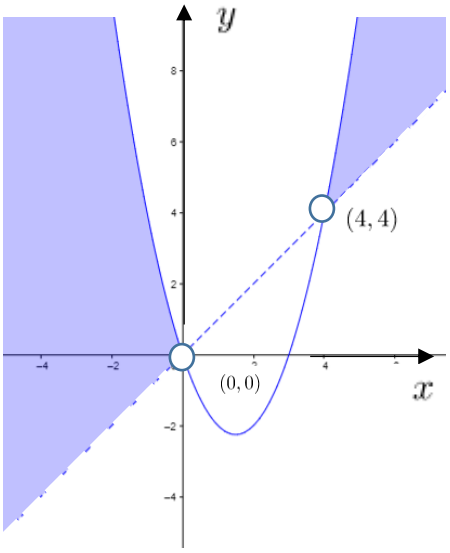
$$k=\frac{1}{2}$$

$$3x-2y-1+\frac{1}{2}(2x+3y-4)=0$$

$$6x-4y-2+2x+3y-4=0$$

$$8x-y-6=0$$

e)



1- correct shape of curves
1- correct points of intersection
1- correct lines/shading

Question 3

a) (i) $y=6x\sqrt{x}=6x^{\frac{3}{2}}$

$$=6\times\frac{3}{2}\times x^{\frac{1}{2}}$$

$$=9\sqrt{x}$$

(ii) $y=(2-5x^3)^7$

$$u=2-5x^3$$

$$u'=-15x^2$$

$$y=7\times-15x^2\times(2-5x^3)^6$$

$$=-105x^2(2-5x^3)^6$$

(iii) $y=-3x^2\sqrt{2x+1}$

$$u=-3x^2,\qquad u'=-6x$$

$$v=\sqrt{2x+1},\qquad v'=\frac{1}{\sqrt{2x+1}}$$

$$y=uv'+vu'$$

$$y=\frac{-3x^2}{\sqrt{2x+1}}-6x\sqrt{2x+1}$$

$$=\frac{-3x^2-6x(2x+1)}{\sqrt{2x+1}}$$

$$=\frac{-3x^2-12x^2-6x}{\sqrt{2x+1}}$$

$$=\frac{-15x^2-6x}{\sqrt{2x+1}}$$

$$=-\frac{3x(5x+2)}{\sqrt{2x+1}}$$

(iv) $y=\frac{5-3x}{x^2-5}$

$$u=5-3x,\qquad u'=-3$$
$$v=x^2-5,\qquad v'=2x$$

$$\frac{dy}{dx}=\frac{-3(x^2-5)-2x(5-3x)}{(x^2-5)^2}$$

$$=\frac{3x^2-10x+15}{(x^2-5)^2}$$

b)

(i) $\frac{dy}{dx} = 3x^2 - 8x$

$at\ x = 2, \quad \frac{dy}{dx} = 3(4) - 8(2) = -4$

Equation of tangent: $m = -4, (2, -3)$

$$y + 3 = -4(x - 2)$$
$$y = -4x + 5$$

(ii) $m_{normal} = \frac{1}{4}$, (2, -3)

$$y + 3 = \frac{1}{4}(x - 2)$$

$$x - 4y - 14 = 0$$

c) $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$$= \lim_{h \to 0} \frac{5(x+h) - (x+h)^2 - (5x - x^2)}{h}$$

$$= \lim_{h \to 0} \frac{5x + 5h - x^2 - 2xh - h^2 - 5x + x^2}{h}$$

$$= \lim_{h \to 0} \frac{5h - 2xh - h^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(5 - 2x - h)}{\cancel{h}}$$

$$= 5 - 2x$$

Question 4

a)

$$\frac{4\pi}{3} \times \frac{180}{\pi}$$

$$= 240°$$

b) $4 \sin x \cos x \cos 2x = \sin 4x$

$$LHS = 4 \sin x \cos x \cos 2x$$
$$= 2(2 \sin x \cos x) \cos 2x$$
$$= 2 \sin 2x \cos 2x$$
$$= \sin 4x$$
$$= RHS$$

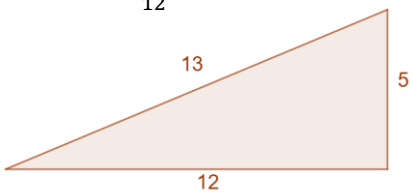
c) (i)

$\cos(x + y) = \cos x \cos y - \sin x \sin y$

(ii)

$$\cos\left(\frac{7\pi}{12}\right) = \cos(\frac{\pi}{3} + \frac{\pi}{4})$$
$$= \cos\frac{\pi}{3}\cos\frac{\pi}{4} - \sin\frac{\pi}{3}\sin\frac{\pi}{4}$$
$$= \frac{1}{2} \times \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}$$
$$= \frac{1 - \sqrt{3}}{2\sqrt{2}}$$

d) $\tan\theta = \frac{5}{12}$



$$\therefore \cos\theta = \pm\frac{12}{13}$$

$$\cos\theta = 2\cos^2\frac{\theta}{2} - 1$$

$$\cos^2\frac{\theta}{2} = \frac{\cos\theta + 1}{2}$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{\cos\theta + 1}{2}}$$

$$= \pm\sqrt{\frac{\frac{12}{13} + 1}{2}} \ or \ = \pm\sqrt{\frac{-\frac{12}{13} + 1}{2}}$$

$$= \pm\sqrt{\left(\frac{25}{26}\right)} \ or \ = \pm\sqrt{\frac{1}{26}}$$

$$= \pm\frac{5}{\sqrt{26}} \ or \ = \pm\frac{1}{\sqrt{26}}$$

e) $\sqrt{3}\tan 3x = -1, \quad -\pi \le x \le \pi$

$$\tan 3x = -\frac{1}{\sqrt{3}}, \qquad -3\pi \le 3x \le 3\pi$$

$$(3x)_b = \frac{\pi}{6}$$

$$3x = \pi - \frac{\pi}{6}, 2\pi - \frac{\pi}{6}, 2\pi + \frac{5\pi}{6}, -\frac{\pi}{6}, -\pi - \frac{\pi}{6}, -2\pi - \frac{\pi}{6}$$

$$x = \frac{5\pi}{18}, \frac{11\pi}{18}, \frac{17\pi}{18}, -\frac{\pi}{18}, -\frac{7\pi}{18}, -\frac{13\pi}{18}$$

Question 5

a)

$$\tan\alpha = \frac{1}{5} = \frac{BE}{50} \quad \therefore BE = 10$$

$$\tan\beta = \frac{1}{6} = \frac{CD}{30} \quad \therefore CD = 5$$

(i) $FC = 10 + 5$

$= 15m$

(ii) $FA^2 = 50^2 + 30^2$

$FA = \sqrt{3400}$

$CA^2 = 15^2 + 3400$

$CA = \sqrt{3625}\, m$

(iii) $AB^2 = 10^2 + 50^2$

$AB = \sqrt{2600}$

$BC^2 = 30^2 + 5^2$

$BC = \sqrt{925}$

$$\cos\angle ABC = \frac{2600 + 925 - 3625}{2 \times \sqrt{2600}\sqrt{925}}$$

$\angle ABC = 91°57'$

b) $\sin 3x - \cos 3x \tan x = 2\sin x$

$LHS = \sin 3x - \cos 3x \tan x$

$$= \sin 3x - \cos 3x \times \frac{\sin x}{\cos x}$$

$$= \frac{\cos x \sin 3x - \cos 3x \sin x}{\cos x}$$

$$= \frac{\sin(3x - x)}{\cos x}$$

$$= \frac{\sin 2x}{\cos x}$$

$$= \frac{2\sin x \cos x}{\cos x}$$

$$= 2\sin x$$

$$= RHS$$