Ext 1

NORTH SYDNEY BOYS HIGH SCHOOL

2019 Year 11 Class Test

Mathematics
Extension 1

General Instructions

- Working time – 60 minutes
- Write on the lined paper in the booklet provided
- Write using blue or black pen
- Board approved calculators may be used
- All necessary working should be shown in every question
- Each new question is to be started on a **new page.**
- Attempt all questions

Class Teacher:
(Please tick or highlight)

- ○ Mr Hwang
- ○ Dr Jomaa
- ○ Ms Lee
- ○ Mr Lin
- ○ Ms Ziaziaris
- ○ Mr Zuber

Student Name:_____

(To be used by the exam markers only.)

Question No	1	2	3	4	5	Total	Total %
Mark	/14	/12	/9	/11	/8	/54	/100

Question 1 (13 marks)

(a) Factorise fully: $x^2 + 2x - y^2 - 2y$ **2**

(b) Solve $|2x - 5| = 13$ **2**

(c) Simplify fully: **3**

$$\frac{4}{x^2 - 4} - \frac{3}{8 + 2x - x^2}$$

(d) Solve the following inequation: **3**

$$\frac{4}{x - 6} \geq 3$$

(e) The fraction $\frac{\sqrt{15}+2}{\sqrt{3}+\sqrt{5}}$ can be written in the form of $a\sqrt{3} + b\sqrt{5}$. Find a and b **3**

Question 2 (12 marks)

(a) Find the **domain** and **range** of the following functions:

(i) $f(x) = \sqrt{4 - x}$ **2**

(ii) $f(x) = \frac{1}{2x+3}$ **2**

(b) Sketch $f(x) = 2(x - 2)(x + 3)^2$ **3**

(c) Given $f(x) = x^2 - 4x - 3$, evaluate $2f(x + a)$ **2**

(d) Find the values of K for which the equation **3**

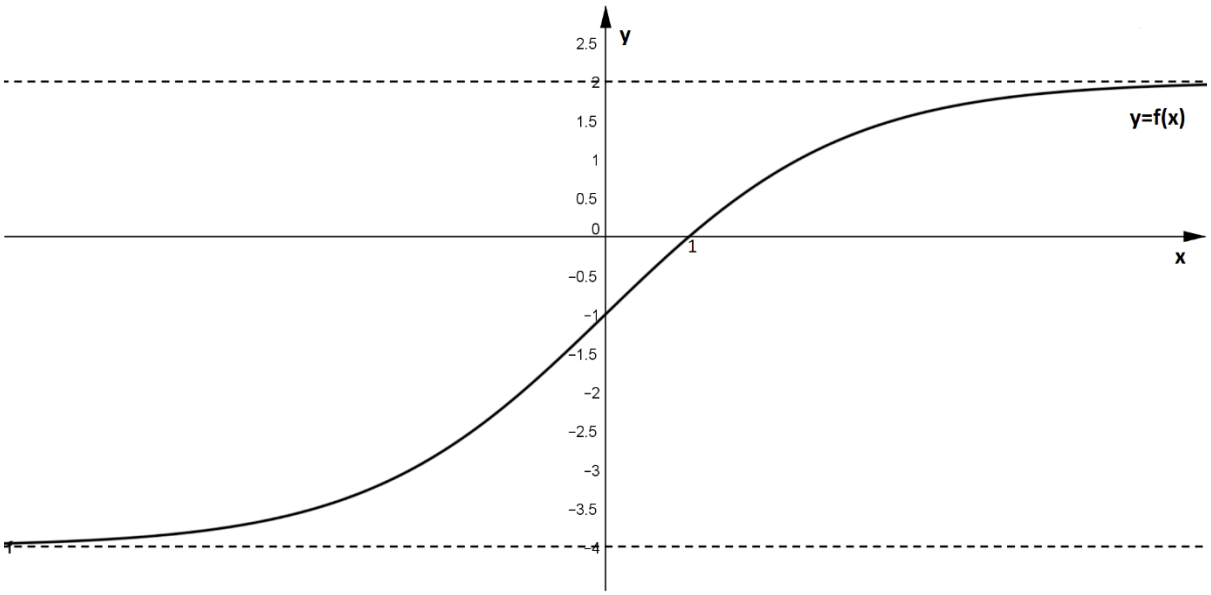
$$(2 - K)x^2 - 4(K - 2)x - 5 = 0$$

has two real distinct roots

Question 3 (9 marks)

(a) Given the function $f(x) = \frac{x}{x^2+1}$. Determine if the function is ODD, EVEN or NEITHER. Showing all working. **2**

(b) Given the following function $y = f(x)$



Sketch the following graphs on the answer sheet provided:

(a) $y = \frac{1}{f(x)}$ **2**

(b) $y^2 = f(x)$ **2**

(c) $|y| = f(-x)$ **3**

Question 4 (11 marks)

(a) Solve the trigonometric equation for $-180° \leq \theta \leq 180°$

(i) $4\sin^2 x = 3$ **2**

(ii) $\sec 2x = \frac{2\sqrt{3}}{3}$ **3**

(iii) $2\sin^2 x = 5\cos x + 4$ **3**

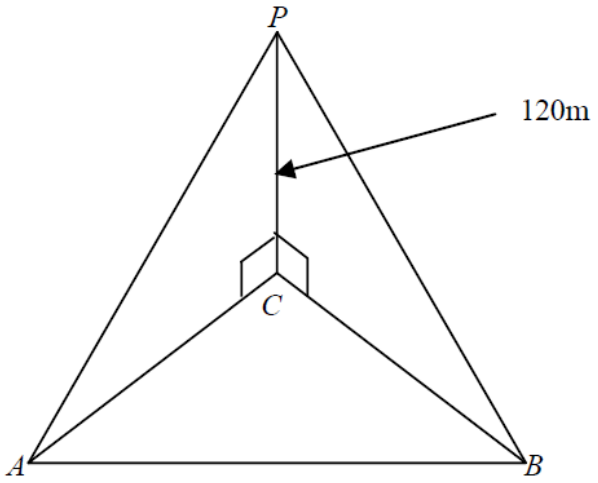
(b) Simplify fully

$$\sqrt{\frac{\text{cosec}^2 x - \cot^2 x - \cos^2 x}{\cos^2 x}}$$ **3**

Question 5 (8 marks)

(a)

Two boats at A and B are observed from the top P of a vertical cliff CP of height 120 metres. A is on a bearing of 195°T from the cliff and its angle of depression from P is 22°. B is on a bearing of 161°T from the cliff and its angle of depression from P is 27°.



(i) Find $\angle ACB$ **1**

(ii) Find the distance between the boats (to the nearest metre) **3**

(b) The function $f(n)$ is defined on the set of integers and satisfies:

$$f(n) = \begin{cases} n-3 & ,n \geq 1000 \\ f\big(f(n+5)\big) & ,n < 1000 \end{cases}$$

(i) Evaluate $f(999)$ **2**

(ii) Find the value of $f(84)$ **2**

End of Examination

Question 1

a) $|2x-5|<13$

$-13 < (2x-5) < 13$

$-8 < 2x < 18$

$-4 < x < 9$

b) x^2+2x-y^2-2y

$= x^2-y^2+2x-2y$

$= (x-y)(x+y)+2(x-y)$

$= (x-y)(x+y+2)$

c) $\dfrac{4}{(x-2)(x+2)} - \dfrac{3}{-(x-4)(x+2)}$

$= \dfrac{4}{(x-2)(x+2)} + \dfrac{3}{(x-4)(x+2)}$

$= \dfrac{4(x-4)+3(x-2)}{(x-2)(x+2)(x-4)}$

$= \dfrac{4x-16+3x-6}{(x-2)(x+2)(x-4)}$

$= \dfrac{7x-22}{(x-2)(x+2)(x-4)}$

d) $\dfrac{4}{x-6} \geq 3$, $x \neq 6$

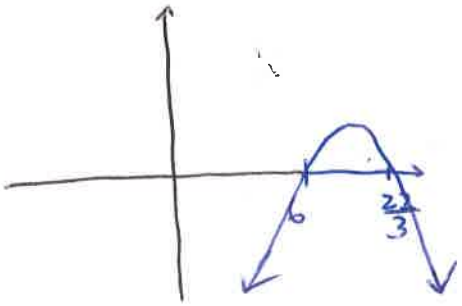
$4(x-6) \geq 3(x-6)^2$

$4(x-6) - 3(x-6)^2 \geq 0$

$(x-6)[4-3(x-6)] \geq 0$

$(x-6)(4-3x+18) \geq 0$

$(x-6)(22-3x) \geq 0$



$\therefore \quad 6 < x \leq \frac{22}{3}$,

e) $\dfrac{\sqrt{15}+2}{\sqrt{3}+\sqrt{5}} \times \dfrac{\sqrt{3}-\sqrt{5}}{\sqrt{3}-\sqrt{5}}$

$= \dfrac{\sqrt{45}-\sqrt{75}+2\sqrt{3}-2\sqrt{5}}{-2}$

$= \dfrac{3\sqrt{5}-5\sqrt{3}+2\sqrt{3}-2\sqrt{5}}{-2}$

$= \dfrac{\sqrt{5}-3\sqrt{3}}{-2}$

$= \frac{3}{2}\sqrt{3} - \frac{1}{2}\sqrt{5}$

$\therefore \quad a = \frac{3}{2} \quad b = -\frac{1}{2}$

Question 2

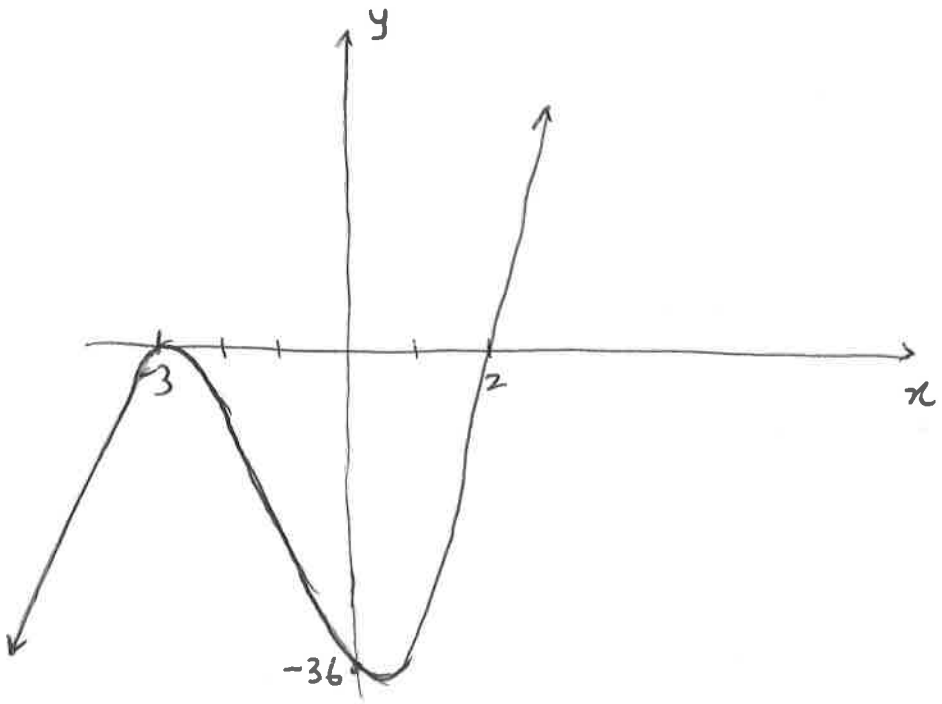
a) i) D: $x \leq 4$, $(-\infty, 4]$

R: $y \geq 0$, $[0, +\infty)$

ii) D: all real x, $x \neq -\frac{3}{2}$

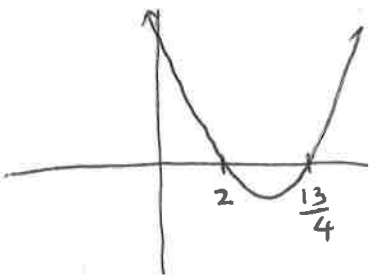
R: all real y, $y \neq 0$

b)



c) $f(x) = x^2 - 4x - 3$

$$2f(x+a) = 2\left((x+a)^2 - 4(x+a) - 3\right)$$
$$= 2\left(x^2 + 2ax + a^2 - 4x - 4a - 3\right)$$
$$= 2x^2 + 4ax + 2a^2 - 8x - 8a - 6$$



d) Two real distinct roots $\Delta > 0$

$\therefore k < 2$ or $k > \frac{13}{4}$

$$\Delta = b^2 - 4ac$$

$\Delta > 0 \Rightarrow$ $[-4(k-2)]^2 - 4(2-k)(-5)] > 0$

$$16(k-2)^2 + 20(2-k) > 0$$
$$4(k-2)[4(k-2) - 5] > 0$$
$$4(k-2)(4k-13) > 0$$

Question 3

a) $f(x) = \dfrac{x}{x^2+1}$

$$f(-x) = \frac{(-x)}{(-x)^2+1}$$

$$= -\frac{x}{x^2+1}$$

$$= -\left(\frac{x}{x^2+1}\right)$$

$$= -f(x)$$

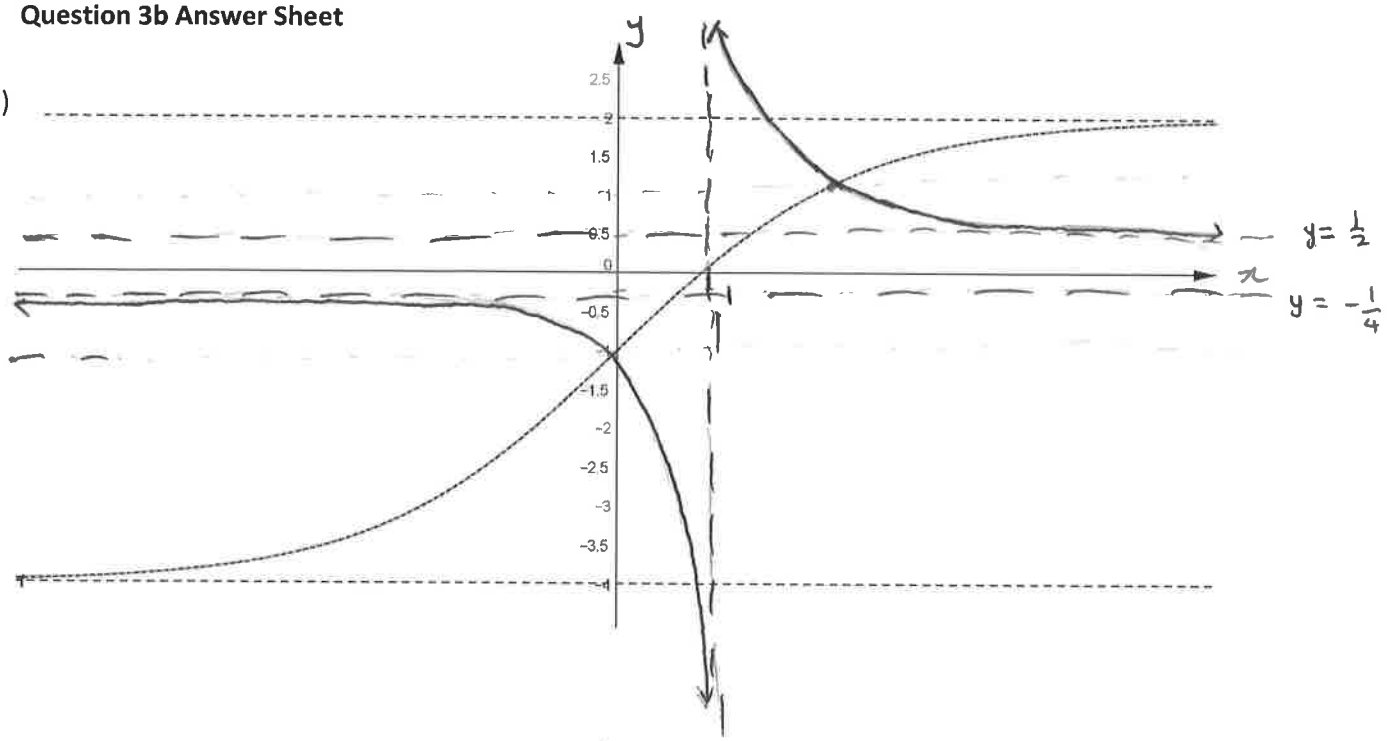
$\therefore f(x)$ is odd

b) See Answer Sheet

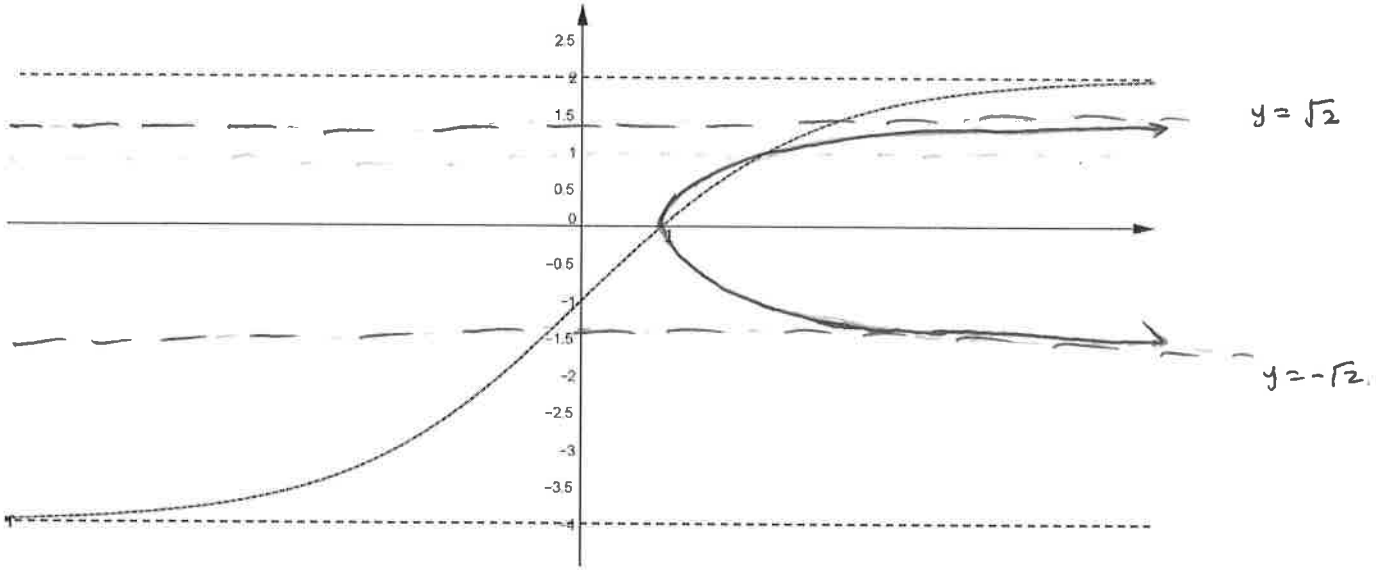
Student Number .....

Question 3b Answer Sheet

i)

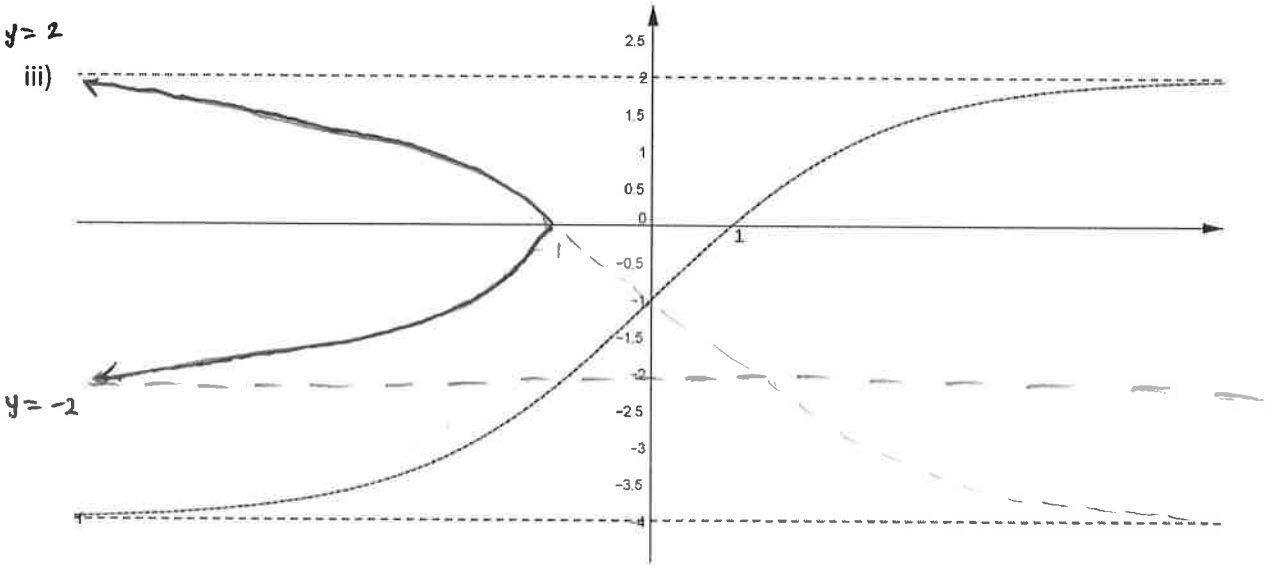


ii)



y = 2

iii)



Question 4

a) $-180^\circ \leq x \leq 180^\circ$

i) $4\sin^2 x = 3$

$\therefore \sin x = \pm \frac{\sqrt{3}}{2}$

$x = 60^\circ, 120^\circ, -120^\circ, -60^\circ$

ii) $\sec 2x = \frac{2\sqrt{3}}{3} = \frac{2}{\sqrt{3}}$ $\quad -360^\circ \leq 2x \leq 360^\circ$

$\therefore \cos 2x = \frac{\sqrt{3}}{2}$

$2x = -30^\circ, 30^\circ, 330^\circ, -330^\circ$

$x = -15^\circ, 15^\circ, 165^\circ, -165^\circ$

iii) $2\sin^2 x = 5\cos x + 4$

$\because \sin^2 x = 1 - \cos^2 x$

$2(1 - \cos^2 x) = 5\cos x + 4$

$2\cos^2 x + 5\cos x + 2 = 0$

$(2\cos x + 1)(\cos x + 2) = 0$

$\cos x = -\frac{1}{2}$ or $\cos x = -2$ (which has no solutions)

$x = -120^\circ$ or $x = 120^\circ$

b) $\sqrt{\frac{\operatorname{cosec}^2 x - \cot^2 x - \cos^2 x}{\cos^2 x}} = \sqrt{\frac{1 - \cos^2 x}{\cos^2 x}}$

as $1 + \cot^2 x = \operatorname{cosec}^2 x$

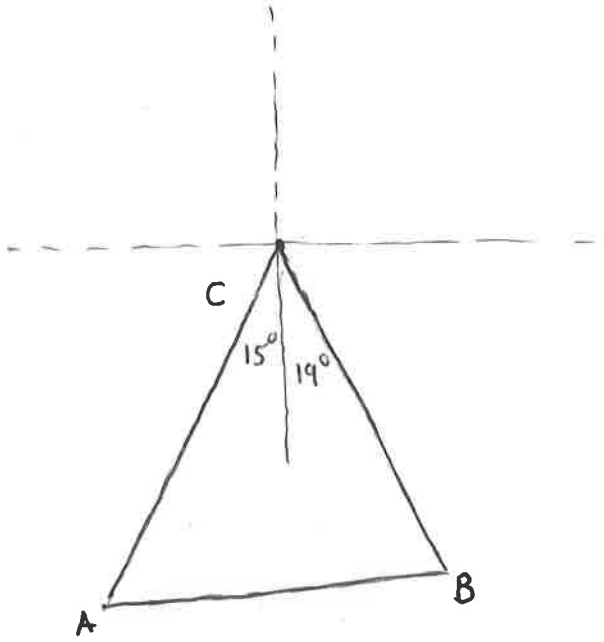
$\operatorname{cosec}^2 x - \cot^2 x = 1$

$= \sqrt{\frac{\sin^2}{\cos^2 x}}$

$= \sqrt{\tan^2 x}$

$= |\tan x|$

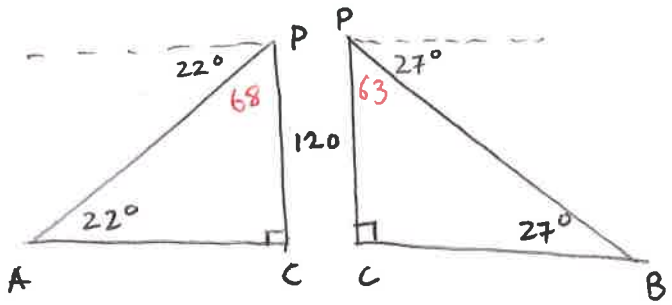
Question 5



i) $\angle ACB = 15^\circ + 19^\circ$

$= 34^\circ$

ii)



$\tan 22^\circ = \frac{120}{AC}$

$\therefore AC = 120 \cot 22^\circ \quad BC = 120 \cot 27^\circ$

$AB^2 = AC^2 + BC^2 - 2 \times AC \times BC \times \cos 34^\circ$

$= (120 \cot 22^\circ)^2 + (120 \cot 27^\circ)^2 - 2(120 \cot 27^\circ)(120 \cot 22^\circ) \cos 34^\circ$

$\doteq 27699.51$

$AB \doteq 166$ (to nearest metre)

b) i) $f(999) = f(f(1004))$

$= f(1001)$

$= 998$

ii) $f(998) = f(f(1003))$

$= f(1000)$

$= 997$

$f(997) = f(f(1002))$

$= f(999)$

$= 998$

$f(996) = f(f(1001))$

$= f(998)$

$= 997$

$f(995) = f(f(1000))$

$= f(997)$

$= 998$

function alternates.

$f(84) = f(996)$

$= 997$