YEAR 11 MATHEMATICS EXTENSION 1

Task 2

12 June 2025

General instructions

- Working time – 50 min.
- Write using a blue or black pen. Where diagrams are to be sketched, these may be done in pencil.
- NSB approved calculators may be used.
- All necessary working should be shown in every question. Marks may be deducted for illegible or incomplete working.
- Attempt **All 4** questions.

Class (please tick)

- ◯ 11 MX1.A3 – Mr Umakanthan
- ◯ 11 MX1.B7 – Mr Ireland
- ◯ 11 MX1.C7 – Ms Moss
- ◯ 11 MX1.D5 – Dr Vranešević
- ◯ 11 MX1.E6 – Mr Berry
- ◯ 11 MX1.F4– Ms Lee
- ◯ 11 MX1.G7 – Mr Lott

STUDENT NUMBER . . . .

Marker's use only.

QUESTION	MCQ	1	2	3	4	Total	%
MARKS	$\frac{\quad}{5}$	$\frac{\quad}{7}$	$\frac{\quad}{15}$	$\frac{\quad}{6}$	$\frac{\quad}{10}$	$\frac{\quad}{43}$	$\frac{\quad}{100}$

Use the multiple questions answer sheet provided to answer Q1 through to Q5

1. There are 4 letters and 4 addressed envelopes. If each letter is randomly placed in an envelope, then the number of ways in which only one letter goes in the wrong envelope is

   A. 43

   B. $\binom{4}{1} + \binom{4}{2} + \binom{4}{3}\binom{4}{4}$

   C. $\binom{4}{0} + \binom{4}{1} + \binom{4}{2} + \binom{4}{3}$

   D. 0

2. $(2x + 1)$ and $(x - 2)$ are the factors of $(2x^3 + px^2 + q)$
   What is the value of $(2p + q)$?

   A. -10

   B. $\dfrac{-22}{3}$

   C. 10

   D. $\dfrac{22}{3}$

3. The quadratic expression $x^2 - 14x + 9$ factorises as $(x - \alpha)(x - \beta)$, where α and β are positive real numbers.
   Which quadratic expression can be factorised as $(x - \sqrt{\alpha})(x - \sqrt{\beta})$?

   A. $x^2 + \sqrt{10}x + 3$

   B. $x^2 - \sqrt{20}x + 3$

   C. $x^2 - \sqrt{17}x + 81$

   D. $x^2 + \sqrt{19x + 81}$

4. The non-zero constant k is chosen so that the coefficient of x^6 in the expansion of $(1 + kx^2)^7$ and $(k + x)^{10}$ are equal.What is the value of k?.

   A. $\frac{1}{6}$

   B. 6

   C. $\frac{\sqrt{6}}{6}$

   D. $\sqrt{6}$

5. The polynomial $n^2x^{2n+3} - 25nx^{n+1} + 150x^7$ has $x^2 - 1$ as a factor

   A. for no values of n

   B. for n=10 only

   C. for n=15 only

   D. for n = 10 and n=15 only

Question 6. (7 Marks) Use the Question 1 Writing Booklet

(a) Find the coefficient of x^{10}in the expansion of $(3 + 2x)^{13}$ 1

(b) Find the independent term of x in the expansion of $\left(3x^2 - \dfrac{1}{x}\right)^{18}$ 2

(c) Find the coefficient of x^8 in the expansion of $\left(x^2 - 2x - 3\right)^5$. 2

(d) Find the values of n and r if $\dbinom{n+1}{7} = \dbinom{12}{5} + \dbinom{12}{r}$ (No working required). 2.

Question 7. (15 Marks) Use the Question 2 Writing Booklet

(a) The number of selections of r objects from an unknown number of different objects is 56 and and the total number of different arrangements of the r objects from the same unknown number of different objects is 6720. Find the value of r. 2

(b) A school library has 3 books by Dickens, 4 books by Shakespeare and 2 books by Montgomery. Find the number of ways of arranging the books on a shelf if

(i) The books written by the same author are indistinguishable. 1

(ii) The books written by the same author are distinguishable and must be together. 2

(iii) The books written by the same author are distinguishable and all the 3 books by Dickens must be separated. 2

(c) Find how many ways different arrangements can be made using all the letters of the word **CHOCOLATE**, if

(i) the vowels must be together. 2

(ii) the 2 letters **O** must be separated. 2

(d) What is the sum of all 4 digit numbers formed from $3, 5, 5, 6$, using the digits only once? 2

(e) How many 4 digit numbers can be formed with at least one digit repeated? 2

Question 8. (6 Marks) Use the Question 3 Writing Booklet

(a) Consider the expansion of $\left(\frac{2x}{3}+\frac{3}{2x}\right)^n$ in descending power of x. 3

If the ratio of the coefficient of the 4th term to that of the 5th term is $16:45$, calculate the value of n.

(b) Use the expansion of $\left(2\sin x+\frac{1}{3\cos x}\right)^{10}$ for $0\leq x<\frac{\pi}{2}$, to show that 3

$$\binom{10}{10}3^{10}+\binom{10}{9}3^{9}+\binom{10}{8}3^{8}+........+\binom{10}{1}3+\binom{10}{0}=2^{20}$$

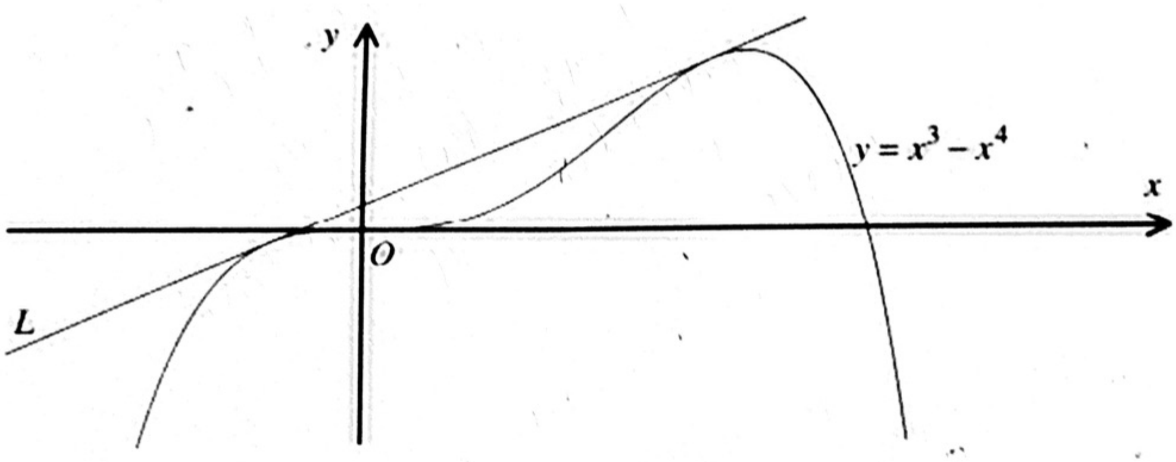
Question 9. (10 Marks) Use the Question 4 Writing Booklet

(a) The polynomial $P(x)=x^3-6x^2+ax+b$ has a root of multiplicity 3. Find the values of a and b. 3

(b) α and are the zeros of $3x^2-5x+7=0$ 3

Find the value of $3(\alpha^5+\beta^5)-5(\alpha^4+\beta^4)+7(\alpha^3+\beta^3)$

(c) The figure below shows the curve C with equation $y=x^3-x^4$. 4
The straight line L is a tangent to the C, at two distinct points. Determine the equation of L.



End of Paper

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Suggested solutions

1.) This is a derangement problem. It's impossible for exactly one letter to be in the wrong envelope

$\therefore$ Answer: 0 D

2.) Let the third factor be $(x-a)$

$\Rightarrow 2x^3 + px^2 + q = (2x+1)(x-2)(x-a)$

$= 2x^3 - (2a+3)x^2 + (3a-2)x + 2a$

Matching the coefficients,

$p = -(2a+3)$

$q = 2a, \quad 3a-2=0$

$\Rightarrow a = \frac{2}{3}, p = -\frac{13}{3}, q = \frac{4}{3}$

$\Rightarrow 2p+q = -\frac{26}{3} + \frac{4}{3} = -\frac{22}{3}$

B

Alternate method

$P(-\frac{1}{2}) = 0$

$\Rightarrow 2(-\frac{1}{2})^3 + p(-\frac{1}{2})^2 + q = 0$

$\Rightarrow p + 4q = 1$ — (1)

$P(2) = 0$

$\Rightarrow 2(2)^3 + p(2)^2 + q = 0$

$\Rightarrow 4p + q = -16$ — (2)

Solving p and q simultaneously,

$p = -\frac{13}{3}, q = \frac{4}{3}$

$\Rightarrow 2p+q = -\frac{26}{3} + \frac{4}{3}$

$= -\frac{22}{3}$

3) $x^2 - 14x + 9 = (x-\alpha)(x-\beta)$

$\Rightarrow \alpha+\beta = 14, \ \alpha\beta = 9$

$\Rightarrow (\sqrt{\alpha}+\sqrt{\beta})^2 = \alpha+\beta+2\sqrt{\alpha\beta}$

$= 14 + 6 = 20$

$\Rightarrow \sqrt{\alpha}+\sqrt{\beta} = \sqrt{20}$

$\sqrt{\alpha\beta} = \sqrt{9} = 3$

$\therefore$ The quadratic eq^n is

$x^2 - \sqrt{20}x + 3 = 0$ B

4) The coeff. of x^6 in the expansion of $(1+kx^2)^7$ is $\binom{7}{3}k^3$.

" " " " $(k+x)^{10}$ is $\binom{10}{6}k^4$

$\Rightarrow \frac{\binom{7}{3}k^3}{\binom{10}{6}k^4} = 1 \Rightarrow \frac{7!}{3!4!} \times k^3 \times \frac{6!\,4!}{10!\,k^4} = 1$

$\Rightarrow k^3 = 6k^4 \Rightarrow k^3(6k-1) = 0$

$k \neq 0 \Rightarrow k = \frac{1}{6}$ A

5) $P(x) = n^2 x^{2n+3} - 25n x^{n+1} + 150 x^7$ (x^2-1) is a factor

$\Rightarrow P(1) = 0 \Rightarrow n^2 - 25n + 150 = 0$

$\Rightarrow (n-10)(n-15) = 0$

and $\Rightarrow n = 10, n = 15$

$\Rightarrow P(-1) = 0 \Rightarrow -n^2 + 25n - 150 = 0$

$\Rightarrow n^2 - 25n + 150 = 0$

Test $n=10$ $\quad 100 - 250 + 150 = 0$ True.

Test $n=15$ $\quad 225 - 25 \times 15 + 150 \neq 0$ false

$\therefore n = 10$ only (B)

~~The coefficient of the~~

6) (a) $(r+1)$th term in the expansion of $(3+2x)^{13}$ is

$T_{r+1} = \binom{13}{r} 3^{13-r} 2^r x^r$

For the coeff. of x^{10}, $r = 10$

$\therefore$ The required coeff $= \binom{13}{10} 3^3 \cdot 2^{10}$

$\boxed{\cdot \text{ 1 mark for correct term and coefficient}}$

(b) The $(r+1)$th term in the expansion of $(3x^2 - \frac{1}{x})^{18}$ is

$T_{r+1} = \binom{18}{r} (3x^2)^{18-r} (-\frac{1}{x})^r$

$= \binom{18}{r} 3^{18-r} \cdot (-1)^r \cdot x^{36-3r}$

For independent term $36 - 3r = 0$

$\Rightarrow r = 12$

$\therefore$ The required term is $\binom{18}{12} 3^6 \cdot (-1)^{12}$

$= \binom{18}{12} 3^6$

- 1 mark for correct coeff.
- 1 mark for correct value for r

6) (c) $(x^2-2x-3)^5$

$= [x^2-(2x+3)]^5$

The $(r+1)$th term $T_{r+1} = \binom{5}{r}(x^2)^{5-r}(-1)^r(2x+3)^r$

$= \binom{5}{r}(-1)^r x^{10-2r} \cdot \binom{r}{p}(2x)^{r-p} \cdot (3)^p$

$= \binom{5}{r}\binom{r}{p}(-1)^r 2^{r-p} 3^p (x)^{10-r+p}$

<u>for x^8</u> $10-r+p=8$

$\Rightarrow r+p=2$

$\Rightarrow \left.\begin{matrix} r=2 \\ p=0 \end{matrix}\right\}$ $\left.\begin{matrix} r=1 \\ p=1 \end{matrix}\right\}$

$\Rightarrow$ required answer is

$\binom{5}{2}\binom{2}{0}(-1)^2 2^2 3^0 + \binom{5}{1}\binom{2}{1}(-1)^1 2^0 3^1$

$= 40-15$

$= \underline{25}$

(d) $n=12,\ r=6$ (note: $\binom{12}{5} = \binom{12}{7}$)

$\left[{}^{n+1}C_r = {}^nC_r + {}^nC_{r-1}\right]$

$\boxed{\begin{array}{l} \cdot\ \text{1 mark for correct values of } n \text{ and } r \\ \cdot\ \cancel{\text{1 mark for correct value of}} \end{array}}$

7 (a) (i) DISABLE has 7 distinct letters

$\therefore$ No of ways to choose 5 letters $= {}^7C_5 = 21$

- 1 marks for correct answer

(ii) No of ways to arrange 7 distinct letters

$= {}^7P_5 = 2520$

- 1 marks for correct answer

(b) (i) arrange 3 groups Dickens (3), shakespeare (4), montgomery (2)

$\therefore$ No of ways to arrange the groups

$$= \frac{9!}{3!\,4!\,2!} = \underline{\underline{1260}}$$

- 1 marks for correct answer

(ii) Total books $= 9$

First arrange the 6 non-Dickens books $= 6! = 720$

Now there are 7 spaces between/around these books to place the 3 Dickens books (so that no two are together) $= {}^7C_3 = 35$

Arrange the 3 Dicken's books $= 3! = 6$

$\therefore$ Total $= 720 \times 35 \times 6 = \underline{\underline{151,200}}$

- 1 marks for correct method (gaps + arrangements)
- 1 mark for correct calculation

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7 a (iii) Choosing 3 consonants from 4 $= \binom{4}{3}$

" 2 vowels " 3 $= \binom{3}{2}$

$\therefore$ " 3 consonants and 2 vowels $= \binom{4}{3} \times \binom{3}{2}$

$\therefore$ Arranging these 5 letters in 5 places $= \binom{4}{3}\binom{3}{2} \times 5!$

$= 960$

- 1 marks for correct expression for choosing 3 consonants + 2 vowels
- 1 marks for correct final answer)

7(c)(i) CHOCOLATE

vowels: O O A E   Consonants: C H C L T

Treat vowels as a block.

5 consonants + 1 vowel block = 6 items

Arrange these 6 items $\Rightarrow \frac{6!}{2!}$ ways (as there are 2 C's)

(1 mark: for correct grouping and arrangement,

Arrange the vowels within the block $= \frac{4!}{2!}$

1 mark. for correct calculation)

$\therefore$ Total $\frac{6!}{2!} \times \frac{4!}{2!} = 4320$

c(ii) arrangement without restriction $= \frac{9!}{2!2!}$

" with O's together $= \frac{8!}{2!}$

So arrangements with O's separated $=$

$\frac{9!}{2!2!} - \frac{8!}{2!}$

(1 mark: Correct method (total − together)

1 mark: Correct calculation)

$= 70560$

7(d) Digits 3, 5, 5, 6 (5 is repeated)

No of unique numbers $= \frac{4!}{2!} = 12$

Sum of digits $= 3+5+5+6 = 19$

Each digit appears in each place (thousands, hundreds, tens, units) $= \frac{12}{4} = 3$ times

Sum: $3 \times (1000+100+10+1) \times 19 = 63{,}327$

(1 mark: Correct method (digit sums, positions)

1 mark Correct calculation)

7(e)

digits (0 – 9)

Total 4 digit numbers = $9 \times 10 \times 10 \times 10 = 9000$

(from 1000 to 9999)

Numbers with all digits different

- first digit 9 choices
- second digit 9 (excluding first digit)
- third 8
- fourth 7

Total: $9 \times 9 \times 8 \times 7 = 4538$

So, numbers with at least one digit repeating:

$9000 - 4538$

$= \underline{4464}$

(~~or simply~~)

Q8 (a) In the descending powers of the expansion of $\left[\frac{2x}{3} + \frac{3}{2x}\right]^n$

the 4th coeff $= T_4 = \binom{n}{3}\left(\frac{2}{3}\right)^{n-3}\left(\frac{3}{2}\right)^3$

$= \binom{n}{3}\frac{2^{n-6}}{3^{n-6}}$

The 5th coeff $= T_5 = \binom{n}{4}\left(\frac{2}{3}\right)^{n-4}\left(\frac{3}{2}\right)^4$

$= \binom{n}{4}\frac{2^{n-8}}{3^{n-8}}$

$T_4 : T_5 = 16 : 45$

$$\Rightarrow \frac{\binom{n}{3}\frac{2^{n-6}}{3^{n-6}}}{\binom{n}{4}\frac{2^{n-8}}{3^{n-8}}} = \frac{16}{45}$$

$$\Rightarrow \frac{16}{9(n-3)} = \frac{16}{45}$$

$$\Rightarrow n - 3 = 5$$

$$\Rightarrow \underline{\underline{n = 8}}$$

- 1 mark for correct general term and ratio set up
- 1 mark for correct simplification
- 1 mark for correct value of n

8(b) In the expansion of $\left(2\sin x + \frac{1}{3\cos x}\right)^{10}$, the $(r+1)$th term

$$T_{r+1} = \binom{10}{r} (2\sin x)^{10-r} \cdot \left(\frac{1}{3\cos x}\right)^{r}$$

when $x = 45°$

$$T_{r+1} = \binom{10}{r} \frac{2^{10-r}}{3^r} \cdot \frac{(\sqrt{2})^r}{(\sqrt{2})^{10-r}}$$

$$= \binom{10}{r} \frac{2^{10-r}}{3^r} \cdot \frac{1}{(\sqrt{2})^{10-2r}}$$

$$= \binom{10}{r} \frac{2^{10-r}}{3^r} \cdot \frac{1}{(2)^{5-r}}$$

$$= \binom{10}{r} \frac{2^5}{3^r}$$

when $x = 45$ $\left(2\sin x + \frac{1}{3\cos x}\right)^{10} = \left(\sqrt{2} + \frac{\sqrt{2}}{3}\right)^{10} = \left(\frac{4\sqrt{2}}{3}\right)^{10} = \frac{2^{25}}{3^{10}}$

$$\Rightarrow \frac{2^{25}}{3^{10}} = \binom{10}{0} 2^5 + \binom{10}{1}\frac{2^5}{3} + \binom{10}{2}\frac{2^5}{3^2} + \cdots + \binom{10}{10}\frac{2^5}{3^{10}}$$

$$\Rightarrow 2^{20} = \binom{10}{0} 3^{10} + \binom{10}{1} \cdot 3^9 + \binom{10}{2} 3^8 \cdots + \binom{10}{10}$$

$$\Rightarrow 2^{20} = \binom{10}{10} 3^{10} + \binom{10}{1} 3^9 + \cdots \quad \cdots + \binom{10}{10}$$

- 1 marks for correct expansion
- 1 marks for correct substitution
- 1 mark for correct conclusion

Q9 (a) $p(x) = x^3 - 6x^2 + ax + b$

$p(x)$ has a root of multiplicity 3 at (say) $x = \alpha$

$\Rightarrow p(\alpha) = 0, p'(\alpha) = 0$ and $p''(\alpha) = 0$

$p(\alpha) = 0 \Rightarrow \alpha^3 - 6\alpha^2 + a\alpha + b = 0$ —— ①

$p'(\alpha) = 0 \Rightarrow 3\alpha^2 - 12\alpha + a = 0$ —— ②

$p''(\alpha) = 0 \Rightarrow 6\alpha - 12 = 0$ —— ③

from ③ $\alpha = 2$

sub $\alpha = 2$ into ② gives $\boxed{a = 12}$

sub $\alpha = 2$ and $a = 12$ into ① gives $\boxed{b = -8}$

- 1 mark for identification of $p(\alpha) = p'(\alpha) = p''(\alpha) = 0$
- 1 mark for correct value of a
- 1 mark for correct value of b

Alternate method

$p(x) = x^3 - 6x^2 + ax + b \equiv (x - \alpha)^3$

matching the coefficients,

$-3\alpha = 6 \Rightarrow \alpha = 2$

$3\alpha^2 = a \Rightarrow a = 12$

$-\alpha^3 = b \Rightarrow b = -8$

- 1 mark for correct identification of this form
- 1 mark for correct value for a
- 1 mark for correct value for b

Q9 (b)

$$3x^2 - 2(a+b+c)x + (ab+bc+ca) = 0$$

Method 1 $\Delta = 0$

Method 2 $P(x) = P'(x) = 0$

both will give

$$(a+b+c)^2 - 3(ab+bc+ca) = 0$$

$$a^2+b^2+c^2 - ab - bc - ca = 0$$

$$\Rightarrow \frac{1}{2}\left[(a-b)^2 + (b-c)^2 + (c-a)^2\right] = 0$$

as sum of 3 squares $= 0$
each must be equal to zero

$$\Rightarrow a = b = c$$

- 1 mark for set up $\Delta = 0$ or $P(x) = P'(x) = 0$
- 1 mark for simplification to sum of 3 squares
- 1 mark for correct conclusion

Q9(c) Let $L: y = mx + b$

$\Rightarrow x^4 - x^3 + mx + b = 0$ has roots $\alpha, \alpha, \beta, \beta$

L is touching the curve at 2 different points

$\therefore$ 2 coincident roots

$\Rightarrow$ sum of roots $= 1 \Rightarrow 2(\alpha+\beta) = 1 \Rightarrow \alpha + \beta = \frac{1}{2}$ — (1)

$\Rightarrow$ sum of roots taken 2 at a time $= 0$

$\Rightarrow \alpha^2 + 4\alpha\beta + \beta^2 = 0$ — (2)

$\Rightarrow$ sum of roots taken 3 at a time $= -m$

$\Rightarrow 2(\alpha^2\beta + \alpha\beta^2) = -m$ — (3)

$\Rightarrow$ product of roots $= b$

$\Rightarrow \alpha^2\beta^2 = b$ — (4)

Solving equations (1), (2), (3) and (4) simultaneously gives

$$m = \frac{1}{8} \text{ and } b = \frac{1}{64}$$

$$\therefore L: \boxed{y = \frac{1}{8}x + \frac{1}{64}}$$

- 1 mark for identifying there are 2 coincident roots
- 1 mark for writing the 4 relationships correctly between roots and coefficients
- 2 marks for correct values of m and b
- 1 mark for correct answer for L