NORTH SYDNEY GIRLS HIGH SCHOOL

2017 Assessment Task 2
Term 2

Preliminary Extension 1 Mathematics

General Instructions

- Reading Time – 2 minutes
- Working Time – 55 minutes
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- In Questions 5−8, show relevant mathematical reasoning and/or calculations

Total Marks – 39 marks

Section I Pages 2−3

4 marks

- Attempt Questions 1−4
- Allow about 5 minutes for this section

Section II Pages 4−7

35 marks

- Attempt Questions 5−8
- Allow about 48 minutes for this section

Student Name:____________________________________

Student Number:______________________

Class:

- ○ 11MXR
- ○ 11MXA
- ○ 11MXI
- ○ 11MXN
- ○ 11MXB
- ○ 11MXO
- ○ 11MXW

	Q1-4	Q5	Q6	Q7	Q8	Total
Geometry	/1		/4			/5
Trigonometry	/3	/10	/5	/8	/8	/34
						/39

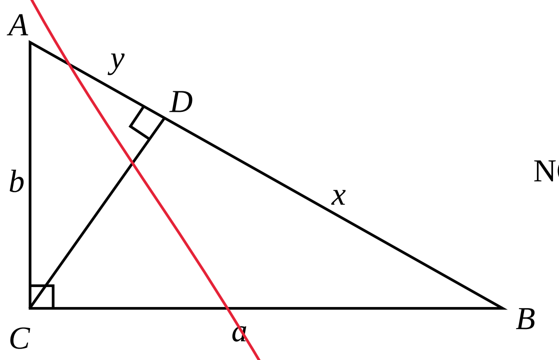
Section I

4 marks
Attempt Questions 1–4
Allow about 5 minutes for this section

Use the multiple-choice answer sheet for Questions 1–4.

---

1 In the diagram below, $\Delta ADC \parallel\!| \Delta ACB$.



NOT TO SCALE

Which of the following statements is correct?

(A) $\dfrac{b}{y} = \dfrac{a}{x}$

(B) $\dfrac{b}{y} = \dfrac{x+y}{a}$

(C) $\dfrac{b}{y} = \dfrac{x+y}{b}$

(D) $\dfrac{b}{y} = \dfrac{a}{b}$

2 If $t = \tan\dfrac{\theta}{2}$ which of the following expressions is equivalent to $1-\sin\theta$?

(A) $\dfrac{1-t^2}{1+t^2}$

(B) $\dfrac{(1-t)^2}{1+t^2}$

(C) $\dfrac{1+t^2}{1-t^2}$

(D) $\dfrac{(1+t)^2}{1+t^2}$

3 If $3\cos\theta + 2 = 0$ and $\tan\theta > 0$, what is the exact value of $\sin(180° + \theta)$?

(A) $-\dfrac{\sqrt{5}}{3}$

(B) $-\dfrac{\sqrt{5}}{2}$

(C) $\dfrac{\sqrt{5}}{2}$

(D) $\dfrac{\sqrt{5}}{3}$

4 Given $x^2 = 2\cos 2\theta$ and $y = 3\sin^2\theta$, which of the following equations is correct?

(A) $y = \dfrac{3}{2}(1 - x^2)$

(B) $y = \dfrac{3}{2}(2 - x^2)$

(C) $y = \dfrac{3}{4}(1 - x^2)$

(D) $y = \dfrac{3}{4}(2 - x^2)$

Section II

35 marks
Attempt Questions 5 - 8
Allow about 50 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.
In Questions 5 – 8, your responses should include relevant mathematical reasoning and/or calculations.

Question 5 (10 marks) Use a SEPARATE writing booklet.

(a) Find the exact value of $\cos 105°$, showing all lines of working. **2**

(b) Solve the equation $\sin 2x = \sin x$ for $-\pi \le x \le \pi$. **3**

(c) Let $\cos x - \sqrt{3}\sin x \equiv R\cos(x+\alpha)$.

(i) By expanding the right-hand side and equating coefficients, show that $R = 2$ and $\alpha = 60°$. **3**

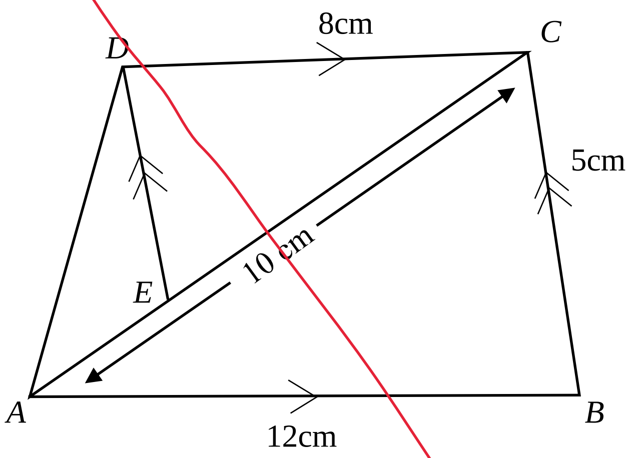
(ii) Hence solve $\cos x - \sqrt{3}\sin x = 2$ for $0° \le x \le 360°$. **2**

Question 6 (9 marks) Use a SEPARATE writing booklet.

(a) Prove that $\dfrac{\cos\alpha + \sin\alpha}{\cos\alpha - \sin\alpha} = \tan 2\alpha + \sec 2\alpha$. **3**

(b) Find a general solution of $\sin 2x = \dfrac{\sqrt{3}}{2}$. **2**

(c) In the quadrilateral $ABCD$ below, $DE \parallel CB$ and $DC \parallel AB$.



NOT TO SCALE

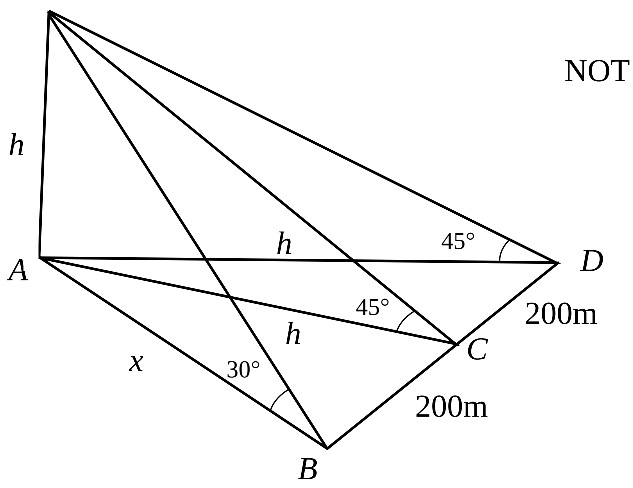
(i) Prove $\Delta CDE \,|||\, \Delta ABC$. **2**

(ii) Find the length of CE. **2**

Question 7 (8 marks) Use a SEPARATE writing booklet.

(a) By using the substitution $t = \tan\dfrac{\theta}{2}$, solve $3\sin\theta - \cos\theta = 1$ for $0 \le \theta \le 2\pi$. **3**

(b)



NOT TO SCALE

A woman travelling along a straight flat road passes three points at intervals of 200m. From these points she observes the angle of elevation of the top of a hill to the left of the road to be $30°$, $45°$ and again $45°$ as shown on the diagram above.

The lengths *AB*, *AC* and *AD* are *x*, *h* and *h* respectively, as marked.

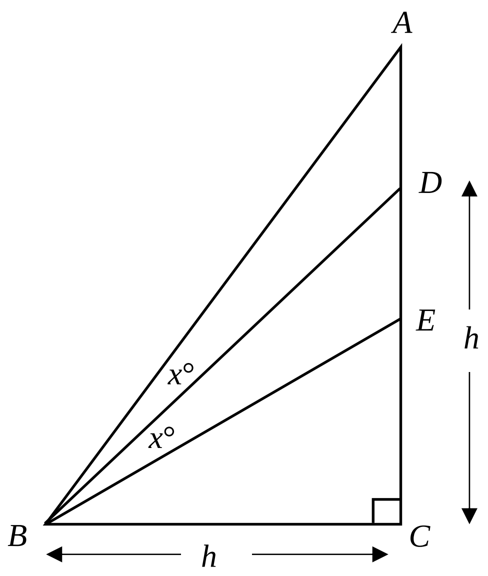
(i) Write *x* in terms of *h*. **1**

(ii) Show that $\cos\angle ACB = \dfrac{200^2 - 2h^2}{400h}$. **1**

(iii) Write a similar expression for $\cos\angle ACD$ in terms of *h*. **1**

(iv) Hence, using parts (ii) and (iii), find the height *h* of the hill. **2**

Question 8 (8 marks) Use a SEPARATE writing booklet.



In the diagram, ΔABC is a right-angled triangle, $BC = DC = h$ and $\angle EBD = \angle DBA = x°$.

(i) Write down the size of $\angle DBC$. **1**

(ii) Show that $AE = h\tan(45 + x)° - h\tan(45 - x)°$. **2**

(iii) Prove that $\tan(45 + x) - \tan(45 - x) = \dfrac{4\tan x}{1 - \tan^2 x}$. **2**

(iv) If $AE = h$, find the exact value of $\tan x$ in simplest exact form. **3**

End of paper

NORTH SYDNEY GIRLS HIGH SCHOOL

2017 Assessment Task 2
Term 2

Preliminary Extension 1 Mathematics
Suggested Solutions

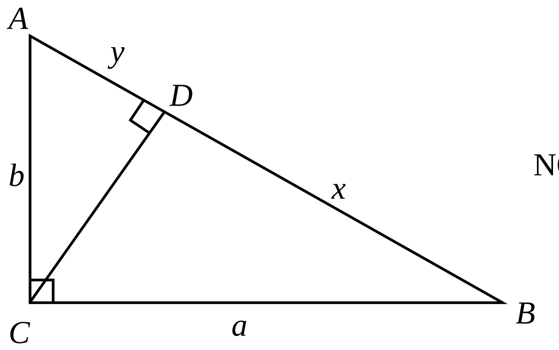
Section I

4 marks
Attempt Questions 1–4
Allow about 5 minutes for this section

Use the multiple-choice answer sheet for Questions 1–4.

---

1 In the diagram below, $\Delta ADC \parallel\!| \Delta ACB$.



NOT TO SCALE

Which of the following statements is correct?

(A) $\dfrac{b}{y} = \dfrac{a}{x}$

(B) $\dfrac{b}{y} = \dfrac{x+y}{a}$

(C) $\dfrac{b}{y} = \dfrac{x+y}{b}$

(D) $\dfrac{b}{y} = \dfrac{a}{b}$

$\Delta ADC|||\Delta ACB$

$$\frac{AD}{AC} = \frac{AC}{AB} = \frac{DC}{CB}$$

$$\frac{y}{b} = \frac{b}{x+y} = \frac{DC}{a}$$

2 If $t = \tan\dfrac{\theta}{2}$ which of the following expressions is equivalent to $1 - \sin\theta$?

(A) $\dfrac{1-t^2}{1+t^2}$

(B) $\dfrac{(1-t)^2}{1+t^2}$

(C) $\dfrac{1+t^2}{1-t^2}$

(D) $\dfrac{(1+t)^2}{1+t^2}$

$1 - \sin\theta$

$$= 1 - \frac{2t}{1+t^2}$$

$$= \frac{1+t^2-2t}{1+t^2}$$

$$= \frac{(1-t)^2}{1+t^2}$$

3 If $3\cos\theta + 2 = 0$ and $\tan\theta > 0$, what is the exact value of $\sin(180° + \theta)$?

(A) $-\frac{\sqrt{5}}{3}$

(B) $-\frac{\sqrt{5}}{2}$

(C) $\frac{\sqrt{5}}{2}$

(D) $\frac{\sqrt{5}}{3}$

$$3\cos\theta + 2 = 0$$

$$\cos\theta = -\frac{2}{3}$$

$\therefore$ θ lies in the third quadrant

$$\sin(180 + \theta) = -\sin\theta$$

$$= \frac{\sqrt{5}}{3}$$

4 Given $x^2 = 2\cos 2\theta$ and $y = 3\sin^2\theta$, which of the following equations is correct?

(A) $y = \frac{3}{2}(1 - x^2)$

(B) $y = \frac{3}{2}(2 - x^2)$

(C) $y = \frac{3}{4}(1 - x^2)$

(D) $y = \frac{3}{4}(2 - x^2)$

$x^2 = 2\cos 2\theta$

$\quad = 2(1 - 2\sin^2\theta)$

$\frac{x^2}{2} = 1 - 2\sin^2\theta$

$2\sin^2\theta = 1 - \frac{x^2}{2}$

$\sin^2\theta = \frac{2 - x^2}{4} \quad --- [1]$

$y = 3\sin^2\theta$

$\sin^2\theta = \frac{y}{3} \quad --- [2]$

Substitute [2] into [1]

$y = 3\left(\frac{2 - x^2}{4}\right) = \frac{3}{4}(2 - x^2)$

Section II

35 marks
Attempt Questions 5 - 8
Allow about 50 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.
In Questions 5 – 8, your responses should include relevant mathematical reasoning and/or calculations.

Question 5 (10 marks) Use a SEPARATE writing booklet.

(a) Find the exact value of $\cos 105°$, showing all lines of working. **2**

$$\cos 105° = \cos(180° - 75°)$$

$$= -\cos 75°$$

$$= -\cos(30° + 45°)$$

$$= -(\cos 45° \cos 30° - \sin 45° \sin 30°)$$

$$= -\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$= \frac{1-\sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{2}-\sqrt{6}}{4}$$

(b) Solve the equation $\sin 2x = \sin x$ for $-\pi \le x \le \pi$. **3**

$$\sin 2x = \sin x$$

$$2\sin x \cos x - \sin x = 0$$

$$\sin x(2\cos x - 1) = 0$$

$$\sin x = 0 \text{ or } \cos x = \frac{1}{2}$$

For $-\pi \le x \le \pi$

$$x = -\pi, 0, \pi, \text{or } x = -\frac{\pi}{3}, \frac{\pi}{3}$$

(c) Let $\cos x - \sqrt{3}\sin x \equiv R\cos(x+\alpha)$.

(i) By expanding the right-hand side and equating coefficients, show that $R = 2$ and $\alpha = 60°$. **3**

(ii) Hence solve $\cos x - \sqrt{3}\sin x = 2$ for $0° \leq x \leq 360°$. **2**

(i)

$\cos x - \sqrt{3}\sin x = Rcos\ (x+\alpha)$

$= R(\cos x\cos\alpha - \sin x\sin\alpha)$

$= (R\cos\alpha)\cos x - (R\sin\alpha)\sin x$

Equating coefficients in LHS and RHS,

$R\cos\alpha = 1$ -----[1]

$R\sin\alpha = \sqrt{3}$ -----[2]

$[1]^2 + [2]^2$: $R^2\cos^2\alpha + R^2\sin^2\alpha = 1^2 + \sqrt{3}^2$

$R^2(\cos^2\alpha + \sin^2\alpha) = 4$

$R^2 = 4$

$R = 2, R > 0$

$[2] \div [1]$: $\dfrac{R\sin\alpha}{\mathrm{R}\cos\alpha} = \sqrt{3}$

$\tan\alpha = \sqrt{3}$

$\alpha = 60°, 0 < \alpha < 90°$

$\therefore \cos x - \sqrt{3}\sin x = 2\cos(x + 60°)$

(ii)

$\cos x - \sqrt{3}\sin x = 2$

$2\cos(x+60°) = 2$

$\cos(x+60°) = 1$

$0° \leq x \leq 360°$

$60° \leq x + 60° \leq 420°$

$x + 60° = 360°$

$x = 300°$

Question 6 (9 marks) Use a SEPARATE writing booklet.

(a) Prove that $\dfrac{\cos\alpha+\sin\alpha}{\cos\alpha-\sin\alpha}=\tan 2\alpha+\sec 2\alpha$. **3**

$$\text{LHS}=\frac{\cos\alpha+\sin\alpha}{\cos\alpha-\sin\alpha}\times\frac{\cos\alpha+\sin\alpha}{\cos\alpha+\sin\alpha}$$

$$=\frac{(\cos\alpha+\sin\alpha)^2}{\cos^2\alpha-\sin^2\alpha}$$

$$=\frac{\cos^2\alpha+2\sin\alpha\cos\alpha+\sin^2\alpha}{\cos 2\alpha}$$

$$=\frac{1+2\sin\alpha\cos\alpha}{\cos 2\alpha}$$

$$=\frac{1+\sin 2\alpha}{\cos 2\alpha}$$

$$=\frac{1}{\cos 2\alpha}+\frac{\sin 2\alpha}{\cos 2\alpha}$$

$$=\sec 2\alpha+\tan 2\alpha=\text{RHS}$$

(b) Find a general solution of $\sin 2x=\dfrac{\sqrt{3}}{2}$.

$$\sin 2x=\frac{\sqrt{3}}{2}$$

$$2x=\frac{\pi}{3}+2k\pi,\frac{2\pi}{3}+2k\pi,\text{where } n\in\mathbb{Z}$$

$$x=\frac{\pi}{6}+k\pi,\frac{\pi}{3}+k\pi$$

Alternate solution to 6(b)

$$\sin 2x=\frac{\sqrt{3}}{2}$$

$$2x=k\pi+(-1)^k\cdot\frac{\pi}{3}\quad(\text{where } k \text{ is an integer})$$

$$x=\frac{k\pi}{2}+(-1)^k\cdot\frac{\pi}{6}$$

Comments:

(1) The correct logic is to apply the general solution when you undo the trig, THEN divide by 2. It is NOT to divide by 2 first then stick the result in your formula for the general solution. The entire angle must be divided by 2.

(2) You MUST state that k is an integer.

(3) $\dfrac{k\pi}{2}$ is an angle in RADIANS. You cannot add this to an angle in degrees.

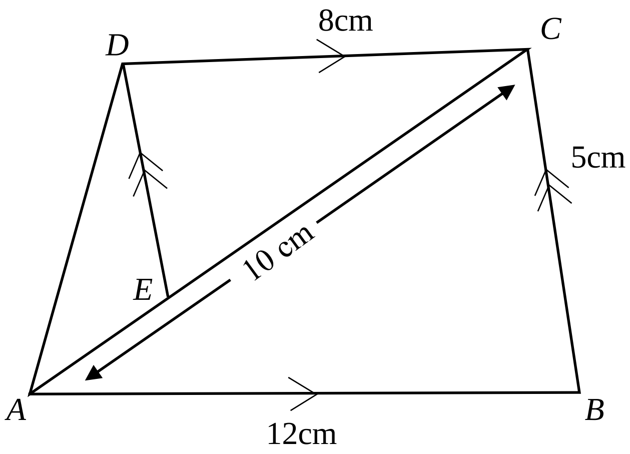
So $x=\dfrac{k\pi}{2}+(-1)^k\cdot 30$ is incorrect.

In any case, when no units are specified, the answer should always be in RADIANS. (However it wasn't penalised THIS TIME provided both parts of the solution were in the same units.)

(4) Many students stuck nonsense expressions such as $\sin^{-1}\dfrac{\pi}{3}$ into their formula.

You find the inverse sine of a RATIO, the ANSWER being an angle.

(c) In the quadrilateral $ABCD$ below, $DE \parallel CB$ and $DC \parallel AB$.



NOT TO SCALE

(i) Prove $\Delta CDE \mathbin{|||} \Delta ABC$. **2**

(ii) Find the length of CE. **2**

(i)

In ΔCDE and ΔABC

$\angle DCE = \angle CAB$ (alternate angles, $AB||DC$)

$\angle DEC = \angle ACB$ (alternate angles, $DE||BC$)

$\therefore \Delta CDE|||\Delta ABC$ (equiangular)

(ii)

$\frac{AB}{CD} = \frac{AC}{CE} = \frac{BC}{DE}$ (matching sides of similar tirangles)

$\frac{12}{8} = \frac{10}{CE} = \frac{5}{DE}$

$CE = \frac{10 \times 8}{12} = \frac{20}{3}$

Question 7 (8 marks) Use a SEPARATE writing booklet.

(a) By using the substitution $t = \tan\frac{\theta}{2}$, solve $3\sin\theta - \cos\theta = 1$ for $0 \le \theta \le 2\pi$. **3**

$3\sin\theta - \cos\theta = 1$

Let $t = \tan\frac{\theta}{2}$

$\sin\theta = \frac{2t}{1+t^2}$ and $\cos\theta = \frac{1-t^2}{1+t^2}$

$3\left(\frac{2t}{1+t^2}\right) - \left(\frac{1-t^2}{1+t^2}\right) = 1$

$6t - (1-t^2) = 1 + t^2$

$6t - 1 + t^2 = 1 + t^2$

$6t = 2$

$\therefore t = \frac{1}{3}$

$\tan\frac{\theta}{2} = \frac{1}{3}, \qquad \text{where } 0 \le \frac{\theta}{2} \le \pi$

$\frac{\theta}{2} = 0.3217\ldots$

$\therefore \theta = 0.6435\ldots$

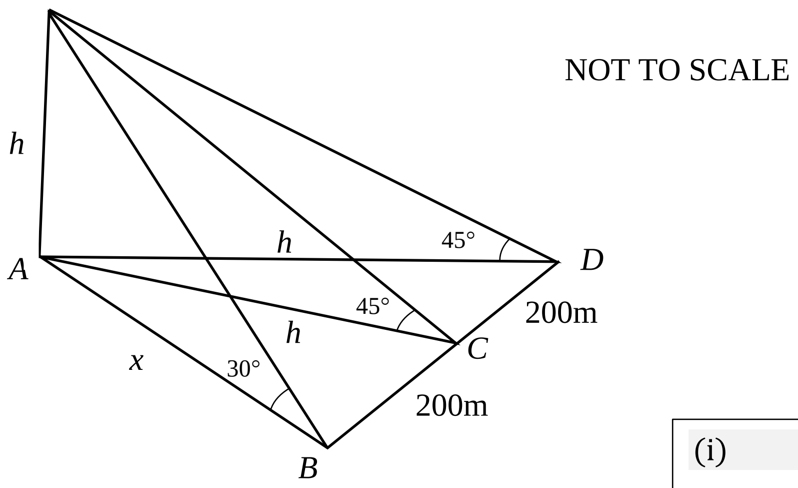
Check $\theta = \pi$

$\text{LHS} = 3\sin\pi - \cos\pi = 1 = \text{RHS}$

$\therefore \pi$ is a solutions

$\therefore \theta = 0.3217\ldots, \pi$

(b)



A woman travelling along a straight flat road passes three points at intervals of 200m.
From these points she observes the angle of elevation of the top of a hill to the left of the road to be $30°$, $45°$ and again $45°$ as shown on the diagram above.

The lengths *AB*, *AC* and *AD* are *x*, *h* and *h* respectively, as marked.

(i) Write *x* in terms of *h*. **1**

(ii) Show that $\cos\angle ACB = \dfrac{200^2 - 2h^2}{400h}$. **1**

(iii) Write a similar expression for $\cos\angle ACD$ in terms of *h*. **1**

(iv) Hence, using parts (ii) and (iii), find the height *h* of the hill. **2**

(i)

$$x = h\cot 30° = h\sqrt{3}$$

(ii)

$$\cos\angle ACB = \frac{200^2 + y^2 - x^2}{2\times 200\times y}$$

$$= \frac{200^2 + h^2 - \left(h\sqrt{3}\right)^2}{2\times 200\times h}$$

$$= \frac{200^2 + h^2 - 3h^2}{400h} = \frac{200^2 - 2h^2}{400h}$$

(iii)

$$\cos\angle ACD = \frac{200^2 + y^2 - z^2}{2\times 200\times y}$$

$$= \frac{200^2 + h^2 - h^2}{2\times 200\times h}$$

$$= \frac{200^2}{400h}$$

Now $\angle ACD = 180 - \angle ACB$

$$\cos\angle ACD = -\cos\angle ACB$$

$$\frac{200^2}{400h} = -\frac{(200^2 - 2h^2)}{400h}, h \neq 0$$

$$200^2 = -200^2 + 2h^2$$

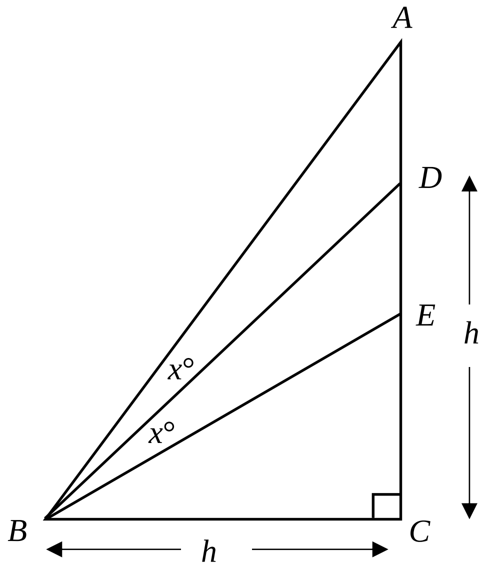
$$2h^2 = 2\times 200^2$$

$$h^2 = 200^2$$

$$h = 200, h > 0$$

The height of the hill is 200m

Question 8 (8 marks) Use a SEPARATE writing booklet.



In the diagram, ΔABC is a right-angled triangle, $BC = DC = h$ and $\angle EBD = \angle DBA = x°$.

(i) Write down the size of $\angle DBC$. **1**

(ii) Show that $AE = h\tan(45 + x)° - h\tan(45 - x)°$. **2**

(iii) Prove that $\tan(45 + x) - \tan(45 - x) = \dfrac{4\tan x}{1 - \tan^2 x}$. **2**

(iv) If $AE = h$, find the exact value of $\tan x$ in simplest exact form. **3**

(i)

$\angle DBC = 45°$ as ΔBCD is a right isosceles triangle

(ii)

$\angle EBC = (45 - x)°$ and $ABC = (45 + x)°$

$\frac{AC}{h} = \tan(45 + x)\,°$

$\therefore AC = h\tan(45 + x)°$

$\frac{EC}{h} = \tan(45 - x)\,°$

$\therefore EC = h\tan(45 - x)°$

$AE = AC - EC$

$= h\tan(45 + x)° - h\tan(45 - x)°$

(iii)

$\tan(45 + x) - \tan(45 - x)$

$= \frac{\tan 45 + \tan x}{1 - \tan 45 \tan x} - \frac{\tan 45 - \tan x}{1 + \tan 45 \tan x}$

$= \frac{1 + \tan x}{1 - \tan x} - \frac{1 - \tan x}{1 + \tan x}$

$= \frac{(1 + \tan x)^2 - (1 - \tan x)^2}{1 - \tan^2 x}$

$= \frac{1 + \tan^2 x + 2\tan x - 1 - \tan^2 x + 2\tan x}{1 - \tan^2 x}$

$= \frac{4\tan x}{1 - \tan^2 x}$

(iv)

$AE = h,$

$h = h\tan(45 + x)° - h\tan(45 - x)°$

$h = h(\tan(45 + x)° - \tan(45 - x)°)$

$\tan(45 + x)° - \tan(45 - x)° = 1$

From (i),

$\frac{4\tan x}{1 - \tan^2 x} = 1$

$4\tan x = 1 - \tan^2 x$

$\tan^2 x + 4\tan x - 1 = 0$

$\tan x = \frac{-4 \pm \sqrt{16 - 4(1)(-1)}}{2}$

$= \frac{-4 \pm \sqrt{20}}{2}$

$= -2 \pm \sqrt{5}$

As x is an acute angle, $\tan x > 0$

$\tan x = -2 + \sqrt{5}$