NORTH
SYDNEY
GIRLS HIGH
SCHOOL

2018 | Preliminary Assessment Task 2

Preliminary Mathematics Extension

General Instructions

- Reading Time – 2 minutes
- Working Time – 55 minutes
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided
- In Questions 5 – 7, show relevant mathematical reasoning and/or calculations

Total marks – 40

Section I – 4 marks (pages 2 – 3)

- Attempt Questions 1 – 4
- Allow about 6 minutes for this section

Section II – 36 marks (pages 4 – 6)

- Attempt Questions 5 – 7
- Allow about 49 minutes for this section

NAME:____________________________ TEACHER:__________________

STUDENT NUMBER:________________________

Question	1 – 2	3 – 4	5	6	7	Total
Trigonometry	/2		/6	/13	/11	/32
Geometry		/2	/6			/8
	/2	/2	/12	/13	/11	/40

Section I

4 marks
Attempt Questions 1–4
Allow about 6 minutes for this section

Use the multiple-choice answer sheet for Questions 1–4.

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1 What is the exact value of $55°$ in radians?

(A) $\dfrac{180\pi}{55}$

(B) $\dfrac{11\pi}{36}$

(C) $\dfrac{36}{11\pi}$

(D) $\dfrac{55}{180\pi}$

2 What is the exact value of $\sin 2x$ if $\cos x = \dfrac{\sqrt{2}}{5}$ and x is in the 4^{th} quadrant?

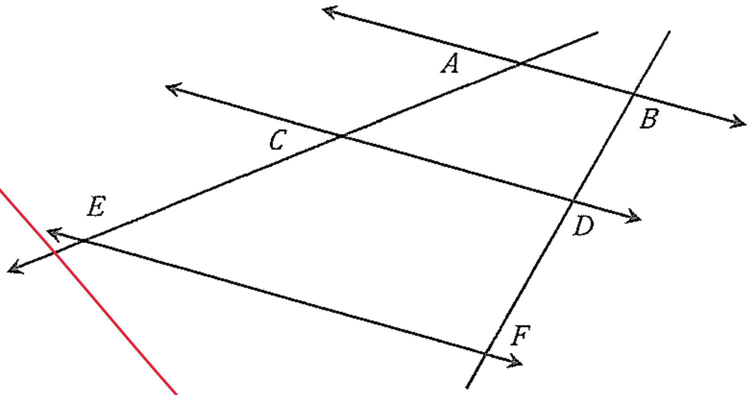
(A) $\dfrac{4\sqrt{23}}{25}$

(B) $\dfrac{2\sqrt{46}}{25}$

(C) $-\dfrac{2\sqrt{46}}{25}$

(D) $-\dfrac{4\sqrt{23}}{25}$

3 In the diagram $AB \parallel CD \parallel EF$.



Which statement is **NOT** correct?

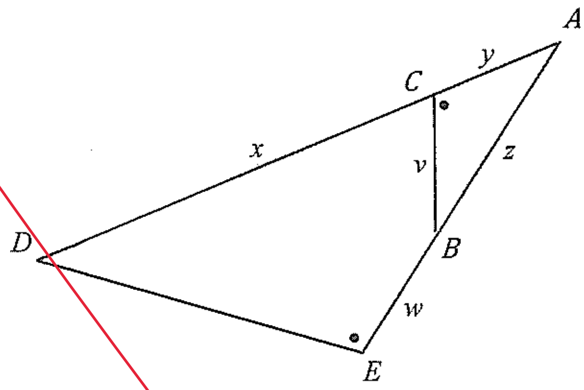
(A) $\dfrac{AC}{AE} = \dfrac{BD}{BF}$

(B) $\dfrac{AC}{CE} = \dfrac{BD}{DF}$

(C) $\dfrac{AC}{BD} = \dfrac{CE}{DF}$

(D) $\dfrac{BD}{BF} = \dfrac{CD}{EF}$

4 The two triangles in the diagram below are similar.



What is the length of DE?

(A) $\dfrac{vz + vw}{z}$

(B) $\dfrac{vx + vy}{z}$

(C) $\dfrac{vx + vy}{y}$

(D) $\dfrac{yz + yw}{v}$

Section II
36 marks
Attempt Questions 5 - 7
Allow about 49 minutes for this section
Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.
In Questions 5 – 7, your responses should include relevant mathematical reasoning and/or calculations.

Question 5 (12 marks) Use a SEPARATE writing booklet.

(a) Prove that $\cos(A+B)-\cos(A-B)=-2\sin A\sin B$. **2**

(b) Let $\sin x+\sqrt{3}\cos x\equiv R\sin(x+\alpha)$.

(i) By expanding the right-hand side and equating coefficients, find the values of R and α. **3**

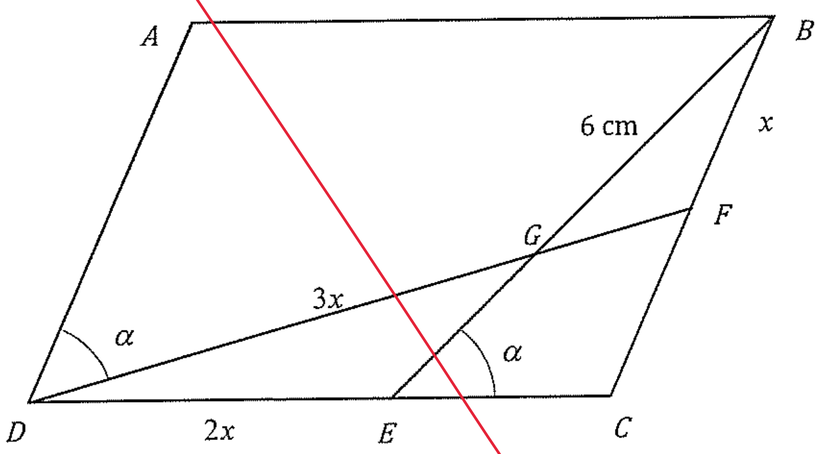
(ii) Hence, or otherwise, find the maximum value of the function **1**

$$f(x)=\sin x+\sqrt{3}\cos x-5.$$

(c) $ABCD$ is a parallelogram.

F and E are two points on BC and DC respectively such that $BF=x$, $DE=2x$ and $\angle ADF=\angle BEC=\alpha$.

FD meets BE at G such that $DG=3x$ and $BG=6\,\text{cm}$.



Copy the diagram into your writing booklet

(i) Prove that $\angle GDE=\angle GBF$. **2**

(ii) Prove that ΔGDE is similar to ΔGBF. **2**

(iii) Hence find the value of x. **2**

Question 6 (13 marks) Use a SEPARATE writing booklet.

(a) Solve the equation $\cos 2\alpha = \sin\alpha$ for $0 \le \alpha \le 2\pi$. **3**

(b) Use the substitution $t = \tan\frac{x}{2}$ to solve **3**

$$\sin x + \cos x = -1 \text{ for } 0 \le x \le 2\pi.$$

(c) (i) Find a general solution of $\cos 2x = \frac{1}{2}$. **2**

(ii) Hence or otherwise solve $\cos 2x = \frac{1}{2}$ for $-2\pi \le x \le 0$. **1**

(d) (i) Prove that $\cot\theta - \tan\theta = 2\cot 2\theta$. **2**

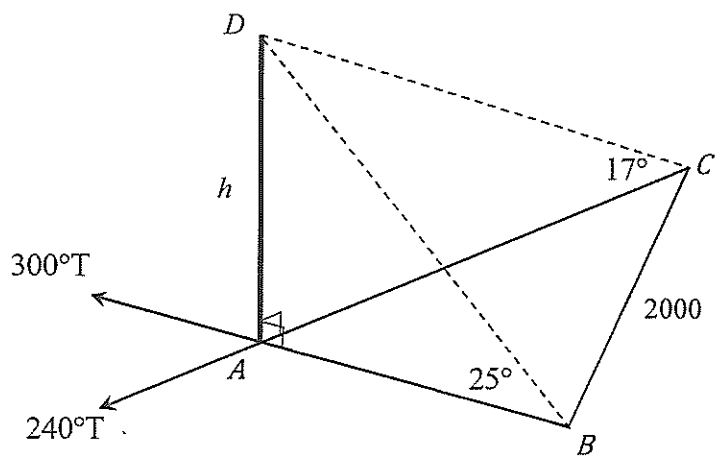
(ii) It can also be shown that:

$$\cot\theta + \tan\theta = 2\operatorname{cosec} 2\theta. \quad \text{(DO NOT PROVE THIS)}$$

Use this result and the result from part i) to find the exact value of $\cot\frac{\pi}{12}$. **2**

Question 7 (11 marks) Use a SEPARATE writing booklet.

(a) The diagram below shows Belinda at point B and Carrie at point C who are on the same horizontal level, 2000 metres apart.
An aeroplane is at point D directly above the point A at an altitude of h metres.
At the same time, Belinda observes the plane on a bearing of 300°T at an angle of elevation of 25° degrees and Carrie observes the plane on a bearing of 240°T at an angle of elevation of 17°.



(i) Show that $AB = h\cot 25°$. **1**

(ii) By first finding a similar expression for AC, show that: **3**

$$h = \frac{2000}{\sqrt{\cot^2 25° + \cot^2 17° - \cot 25° \cot 17°}}.$$

(iii) Using $h \approx 695$ m, find the bearing of C from B to the nearest degree. **2**

(b) (i) By expanding $\cos 2A$ show that $2\cos^2 A = 1 + \cos 2A$. **1**

(ii) On the same set of axes, sketch the graphs of: **2**

$$y = \sin x \text{ and } y = 1 + \cos x \quad \text{for } 0 \le x \le 2\pi.$$

(iii) By using part (i) and part (ii) solve: **2**

$$2\operatorname{cosec} x \le \sec^2\left(\frac{x}{2}\right) \quad \text{for } 0 \le x \le \pi.$$

End of Exam

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2018

Preliminary Assessment Task 2

Preliminary Mathematics Extension

SOLUTIONS

Section I

4 marks
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Allow about 6 minutes for this section

Use the multiple-choice answer sheet for Questions 1–4.

---

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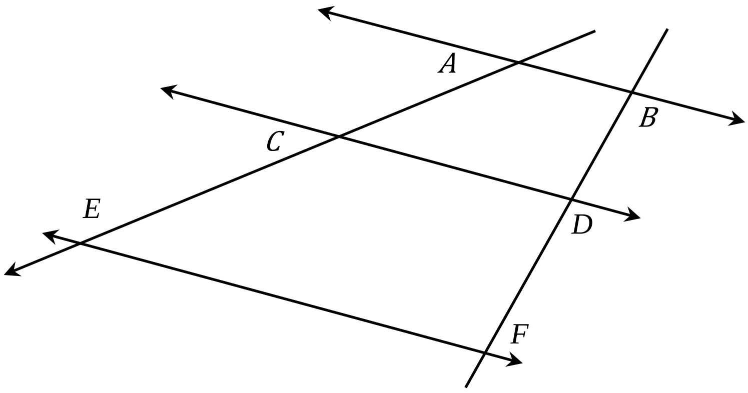

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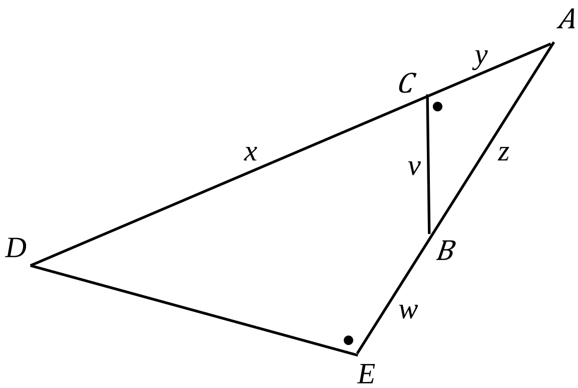


What is the length of *DE*?

(A) $\dfrac{vz + vw}{z}$

(B) $\dfrac{vx + vy}{z}$

(C) $\dfrac{vx + vy}{y}$

(D) $\dfrac{yz + yw}{v}$

Question 5 (12 marks) Use a SEPARATE writing booklet.

(a)	Prove that $\cos(A+B)-\cos(A-B)=-2\sin A\sin B$.	**2**

$$\begin{aligned}LHS &= \cos(A+B)-\cos(A-B)\\ &=(\cos A\cos B-\sin A\sin B)-(\cos A\cos B+\sin A\sin B)\\ &=\cos A\cos B-\sin A\sin B-\cos A\cos B-\sin A\sin B\\ &=-2\sin A\sin B\end{aligned}$$

(b)	Let $\sin x+\sqrt{3}\cos x\equiv R\sin(x+\alpha)$.	
	(i) By expanding the right-hand side and equating coefficients, find the values of R and α.	**3**

$$\begin{aligned}RHS &= R\sin(x+\alpha)\\ &=R(\sin x\cos\alpha+\sin\alpha\cos x)\\ &=R\cos\alpha\sin x+R\sin\alpha\cos x\end{aligned}$$

By equating co-efficients of $\cos x$ and $\sin x$

$1=R\cos\alpha \quad I \qquad$ and $\qquad \sqrt{3}=R\sin\alpha \quad II$

By squaring and adding

$1=R^2\cos^2\alpha \quad I^2 \qquad 3=R^2\sin^2\alpha \quad II^2$

$$\begin{aligned}1+3&=R^2\left(\cos^2\alpha+\sin^2\alpha\right)\\ 4&=R^2\\ R&=2\end{aligned}$$

$1=2\cos\alpha \quad I \qquad$ and $\qquad \sqrt{3}=2\sin\alpha \quad II$

$\dfrac{1}{2}=\cos\alpha \qquad$ and $\qquad \dfrac{\sqrt{3}}{2}=\sin\alpha$

Since both $\cos\alpha$ and $\sin\alpha$ are positive then α is in the 1st quadrant

$$\begin{aligned}\alpha &= \cos^{-1}\frac{1}{2}\\ &=\frac{\pi}{3}\end{aligned}$$

$$\sin x+\sqrt{3}\cos x\equiv 2\sin\left(x+\frac{\pi}{3}\right)$$

(ii) Hence, or otherwise, find the maximum value of the function **1**

$$f(x) = \sin x + \sqrt{3}\cos x - 5.$$

$f(x) = \sin x + \sqrt{3}\cos x - 5$

$f(x) = 2\sin\left(x + \frac{\pi}{6}\right) - 5$

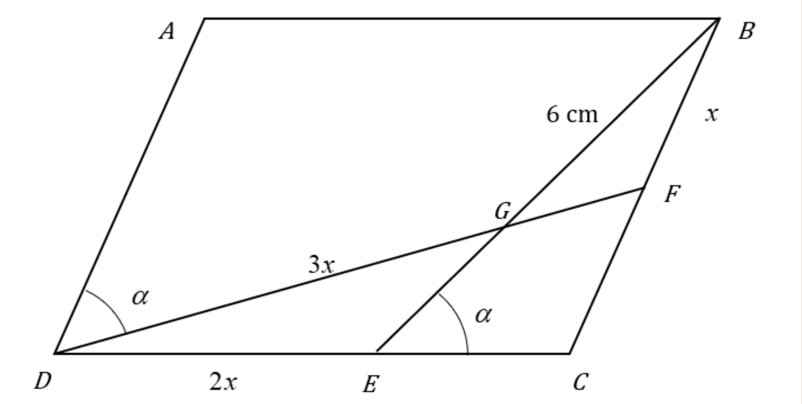
The maximum value of $2\sin\left(x + \frac{\pi}{6}\right)$ is 2.

Maximum value of $f(x)$ is $2 - 5 = -3$

(c) *ABCD* is a parallelogram.

F and *E* are two points on *BC* and *DC* respectively such that $BF = x$, $DE = 2x$ and $\angle ADF = \angle BEC = \alpha$.

FD meets *BE* at *G* such that $DG = 3x$ and $BG = 6\,\text{cm}$.



Copy the diagram into your writing booklet

(i) Prove that $\angle GDE = \angle GBF$. **2**

Let $\angle GDE = \beta$

$AB \parallel DC$ (Opposite sides of a parallelogram)

$\angle CEG = \angle ABG = \alpha$ (Alternate angles on parallel lines $AB \parallel DC$)

$\angle ADE = \angle ABF$ (Opposite angles of a parallelogram)

$\angle ADG + \angle GDE = \angle ABG + \angle GBF$

$\alpha + \angle GDE = \alpha + \angle GBF$

$\angle GDE = \angle GBF$

(ii)	Prove that ΔGDE is similar to ΔGBF .	**2**

In ΔGDE and ΔGBF

$\angle DGE = \angle AGF$ (Vertically opposite angles)

$\angle GDE = \angle GBF$ (Part i))

$\Delta GDE \parallel\!| \Delta GBF$ (Equiangular)

(iii)	Hence find the value of x.	**2**

$$\frac{GB}{GD} = \frac{BF}{DE}$$ (Matching sides in Similar Triangles)

$$\frac{6}{3x} = \frac{x}{2x} \qquad x \neq 0$$

$$\frac{2}{x} = \frac{1}{2}$$

$$x = 4$$

Question 6 (13 marks) Use a SEPARATE writing booklet.

(a)	Solve the equation $\cos 2\alpha = \sin\alpha$ for $0 \le \alpha \le 2\pi$.	**3**

$$\cos 2\alpha = \sin\alpha \qquad 0 \le \alpha \le 2\pi$$

$$1 - 2\sin^2\alpha = \sin\alpha$$

$$0 = 2\sin^2\alpha + \sin\alpha - 1$$

$$0 = (2\sin\alpha - 1)(\sin\alpha + 1)$$

$2\sin\alpha - 1 = 0$

$\sin\alpha = \dfrac{1}{2}$

x is in 1st and 2nd quadrant.

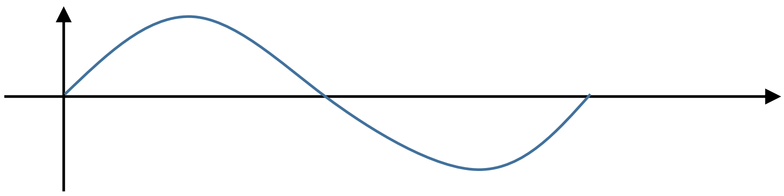
$x = \dfrac{\pi}{6}$

$x = \pi - \dfrac{\pi}{6}$

$= \dfrac{5\pi}{6}$

$\sin\alpha + 1 = 0$

$\sin\alpha = -1$



$x = \dfrac{3\pi}{2}$

(b)	Use the substitution $t = \tan\dfrac{x}{2}$ to solve $$\sin x + \cos x = -1 \text{ for } 0 \le x \le 2\pi.$$	**3**

$\sin x + \cos x = -1$ for $0 \le x \le 2\pi$.

$t = \tan\dfrac{x}{2}$ then

$\sin x = \dfrac{2t}{1+t^2}$ and $\cos x = \dfrac{1-t^2}{1+t^2}$

$$\frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2} = -1$$

$$2t + 1 - t^2 = -1 - t^2$$

$$2t + 1 = -1$$

$$2t = -2$$

$$t = -1$$

$\tan\dfrac{x}{2} = -1 \qquad 0 \le x \le 2\pi$

$\tan u = -1 \qquad 0 \le u \le \pi$

u is in the 2nd quadrant

$u = \pi - \dfrac{\pi}{4}$

$= \dfrac{3\pi}{4}$

$\dfrac{x}{2} = \dfrac{3\pi}{4}$

$x = \dfrac{3\pi}{2}$

Also test $x = \pi$ in the original equation.

$\sin\pi + \cos\pi = (0) + (-1)$

$= -1$

$\therefore x = \pi$ is also a solution

(c) (i)	Find a general solution of $\cos 2x = \frac{1}{2}$.	**2**

$$2x = 2\pi n \pm \cos^{-1}\frac{1}{2}$$

$$2x = 2\pi n \pm \frac{\pi}{3}$$

$$x = \pi n \pm \frac{\pi}{6}$$

$$x = \frac{6\pi n \pm \pi}{6}$$

$$x = \frac{(6n \pm 1)\pi}{6}$$

(ii)	Hence or otherwise solve $\cos 2x = \frac{1}{2}$ for $-2\pi \le x \le 0$.	**1**

$n = 0$	$x = \frac{(0 \pm 1)\pi}{6}$ $x = \frac{-\pi}{6}$ $\quad x = \frac{\pi}{6}$
$n = -1$	$x = \frac{(-6 \pm 1)\pi}{6}$ $x = \frac{-7\pi}{6}$ $\quad x = \frac{-5\pi}{6}$
$n = -2$	$x = \frac{(-12 \pm 1)\pi}{6}$ $x = \frac{-13\pi}{6}$ $\quad x = \frac{-11\pi}{6}$

Thus the solutions that are in the required domain are $x = \frac{-\pi}{6}$ $\quad x = \frac{-5\pi}{6}$ $\quad x = \frac{-7\pi}{6}$ and $x = \frac{-11\pi}{6}$

(d) (i)	Prove that $\cot\theta - \tan\theta = 2\cot 2\theta$.	**2**

$$LHS = \cot\theta - \tan\theta$$
$$= \frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}$$
$$= \frac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos\theta}$$
$$= \frac{\cos 2\theta}{\frac{1}{2}\sin 2\theta}$$
$$= 2\cot 2\theta$$

(ii)	It can also be shown that: $\cot\theta + \tan\theta = 2\cos ec 2\theta$. (DO NOT PROVE THIS) Use this result and the result from part i) to find the exact value of $\cot\frac{\pi}{12}$.	**2**

$$\cot\theta - \tan\theta = 2\cot 2\theta \qquad I$$
$$\cot\theta + \tan\theta = 2\operatorname{co}\sec 2\theta \qquad II$$

$I + II$

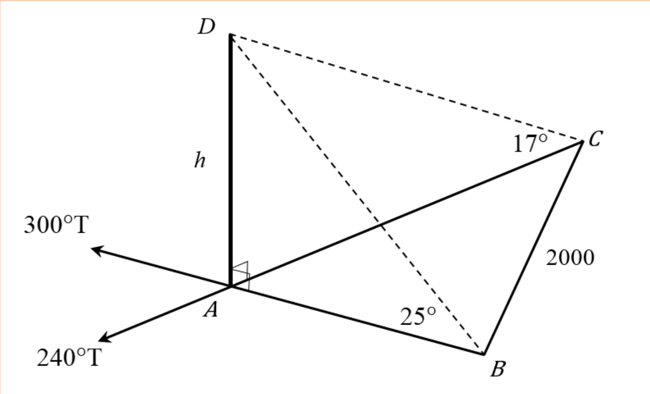
$$2\cot\theta = 2\operatorname{co}\sec 2\theta + 2\cot 2$$
$$\cot\theta = \operatorname{co}\sec 2\theta + \cot 2\theta$$

Let $\theta =$

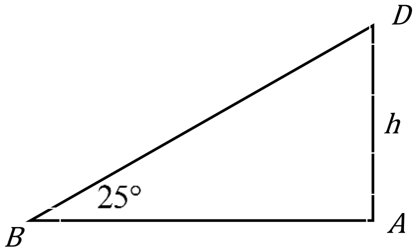
$$\cot\frac{\pi}{12} = \operatorname{co}\sec\frac{2\pi}{12} + \cot\frac{2\pi}{12}$$
$$= \operatorname{co}\sec\frac{\pi}{6} + \cot\frac{\pi}{6}$$
$$= 2 + \sqrt{3}$$

Question 7 (11 marks) Use a SEPARATE writing booklet.

(a) The diagram below shows Belinda at point B and Carrie at point C who are on the same horizontal level, 2000 metres apart.
An aeroplane is at point D directly above the point A at an altitude of h metres.
At the same time, Belinda observes the plane on a bearing of $300°\text{T}$ at an angle of elevation of $25°$ degrees and Carrie observes the plane on a bearing of $240°\text{T}$ at an angle of elevation of $17°$.



(i) Show that $AB = h\cot 25°$. **1**



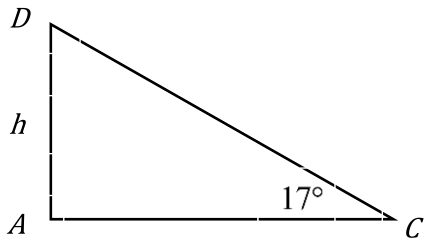
In ΔABD

$$\frac{AB}{BD} = \cot 25°$$

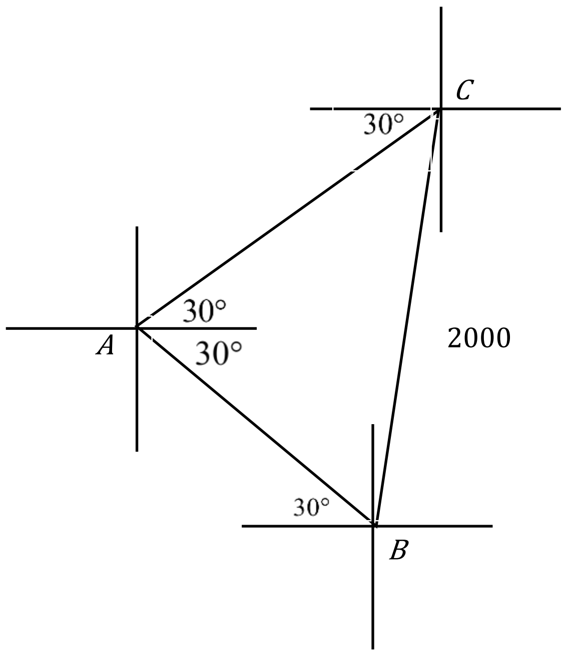
$$\frac{AB}{h} = \cot 25°$$

$$AB = h\cot 25°$$

(ii)	By first finding a similar expression for *AC*, show that: $$h=\frac{2000}{\sqrt{\cot^2 25°+\cot^2 17°-\cot 25°\cot 17°}}.$$	**3**



$AC = h\cot 17°$



In ΔABC, $\angle BAC = 60°$, $BC = 2000$, $AC = h\cot 17°$ and $AB = h\cot 25°$

Using the cosine rule in ΔABC

$$(h\cot 25°)^2+(h\cot 17°)^2-2(h\cot 17°)(h\cot 25°)\cos 60° = 2000^2$$

$$h^2\left(\cot^2 25°+\cot^2 17°-2\cot 17°\cot 25°\left(\frac{1}{2}\right)\right) = 2000^2$$

$$h^2\left(\cot^2 25°+\cot^2 17°-\cot 17°\cot 25°\right) = 2000^2$$

$$h^2 = \frac{2000^2}{\cot^2 25°+\cot^2 17°-\cot 17°\cot 25°}$$

$$h = \frac{2000}{\sqrt{\cot^2 25°+\cot^2 17°-\cot 17°\cot 25°}}$$

(iii)	Using $h \approx 695$ m, find the bearing of C from B to the nearest degree.	**2**

Method 1

Let $\angle ABC = \theta$ and $h \approx 695$

Using the cosine rule in ΔABC

$$\cos\theta = \frac{2000^2 + (695\cot 25°)^2 - (695\cot 17°)^2}{2(2000)(695\cot 25°)}$$
$$= 0.1769796411...$$
$$\theta = 79°48`$$
$$\theta \approx 80$$

Bearing of C from B is equal to $300 + 80 = 380 = 020°T$

Method 2

Let $\angle ACB = \theta$ and $h \approx 695$

Using the sine rule in ΔABC

$$\frac{\sin\theta}{h\cot 25} = \frac{\sin 60}{2000}$$
$$\sin\theta = \frac{695\cot 25°\sin 60°}{2000}$$

$\theta = \sin^{-1}\left(\frac{695\cot 25°\sin 60°}{2000}\right)$ $\qquad$ $\theta = 180 - \sin^{-1}\left(\frac{695\cot 25°\sin 60°}{2000}\right)$

$\theta = 40°$ $\qquad$ $\theta = 140°$

$$\angle ABC = 180 - 60 - 40 = 80°$$

Bearing of C from B is equal to $300 + 80 = 380 = 020°T$

Method 3

Let $\angle ABC = \theta$ and $h \approx 695$

Using the sine rule in ΔABC

$$\frac{\sin\theta}{695\cot 17} = \frac{\sin 60}{2000}$$
$$\sin\theta = \frac{695\cot 17°\sin 60°}{2000}$$

$\theta = \sin^{-1}\left(\frac{695\cot 17°\sin 60°}{2000}\right)$ $\qquad$ $\theta = 180 - \sin^{-1}\left(\frac{695\cot 25°\sin 60°}{2000}\right)$

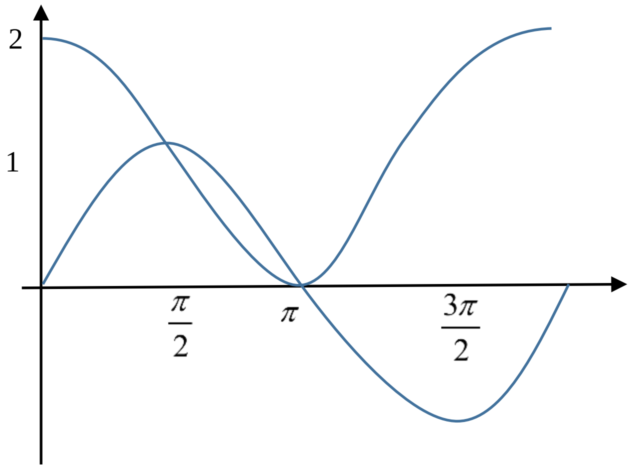
$\theta = 79°50`$ $\qquad$ $\theta = 100°10`$

Both these angles are possible in this triangle so this is an ambiguous case and eother method 1 or nethod 2 must be used.

(b) (i) By expanding $\cos 2A$ show that $2\cos^2 A = 1+\cos 2A$.		**1**

$$\begin{aligned} RHS &= 1+\cos 2A \\ &= 1+\cos^2 A - \sin^2 A \\ &= 1+\cos^2 A - \left(1-\cos^2 A\right) \\ &= 2\cos^2 A \\ &= LHS \end{aligned}$$

(ii) On the same set of axes, sketch the graphs of: $y=\sin x$ and $y=1+\cos x$ for $0 \le x \le 2\pi$.	**2**



(iii) By using part (i) and part (ii) solve: $2\operatorname{cosec} x \le \sec^2\left(\frac{x}{2}\right)$ for $0 \le x \le \pi$.	**2**

$$2\operatorname{cosec} x \le \sec^2\left(\frac{x}{2}\right)$$

$$\frac{2}{\sin x} \le \frac{1}{\cos^2\left(\frac{x}{2}\right)}$$

$$\frac{1}{\sin x} \le \frac{1}{2\cos^2\left(\frac{x}{2}\right)}$$

$\frac{1}{\sin x} \le \frac{1}{1+\cos x}$ From Part i)

$1+\cos x \le \sin x$ Since both $\sin x > 0$ and $1+\cos x > 0$ for $0 \le x \le \pi$.

Thus by considering when the graph of $y = \sin x$ is higher than $y = 1 + \cos x$ for $0 \le x \le \pi$

Solution is $\frac{\pi}{2} \le x < \pi$

NOTE: $x \ne \pi$ since $\operatorname{cosec} x$ is undefined for $x = \pi$

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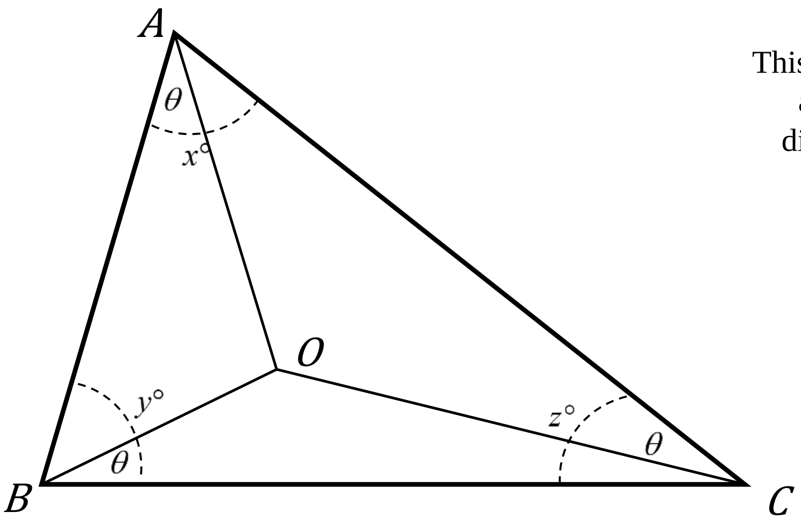
(b) Prove the identity $\cot\alpha - \cot\beta = \dfrac{\sin(\beta-\alpha)}{\sin\alpha\sin\beta}$ **2**

(c) In the ΔABC below, $\angle CAB = x$, $\angle ABC = y$ and $\angle ACB = z$.

The point O is placed inside ΔABC so that $\angle AOB = \angle BOC = \angle COA = \theta$



This is **NOT** a 3-D diagram

(i) Show that $\dfrac{OX}{OY} = \dfrac{\sin(y-\theta)}{\sin\theta}$ **2**

(ii) Henceshow that $\sin^3\theta = \sin(x-\theta)\sin(y-\theta)\sin(z-\theta)$ **2**

(iii) Hence, using parts (c) (ii) and (b) show that: **2**

$$(\cot\theta - \cot x)(\cot\theta - \cot y)(\cot\theta - \cot z) = \operatorname{cosec} x \operatorname{cosec} y \operatorname{cosec} z\ .$$

End of paper