NORTH
SYDNEY
GIRLS HIGH
SCHOOL

2019 | Preliminary Assessment Task 2

Preliminary
Mathematics Extension

General Instructions

- Reading Time – 2 minutes
- Working Time – 55 minutes
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided
- In Questions 5 – 7, show relevant mathematical reasoning and/or calculations

Total marks – 40

Section I – 4 marks (pages 2 – 4)

- Attempt Questions 1 – 4
- Allow about 7 minutes for this section

Section II – 36 marks (pages 5 – 7)

- Attempt Questions 5 – 7
- Allow about 48 minutes for this section

NAME:______________________________ TEACHER:___________________

Question	1 – 4	5	6	7	Total
Further Trig		/8	/12	/11	**/31**
Further Algebra and Functions	/4	/5			**/9**
	/4	**/13**	**/12**	**/11**	**/40**

Section I

4 marks
Attempt Questions 1- 4
Allow about 7 minutes for this section

Use the multiple-choice answer sheet for Questions 1- 4

---

1. What is the solution of the inequation $|2x-1|<7$?

(A) $\frac{1}{2}<x<4$

(B) $-4<x<3$

(C) $-\frac{1}{2}<x<4$

(D) $-3<x<4$

2. What is the solution of the inequation $x(x-3)^2(x+3)^3 \le 0$?

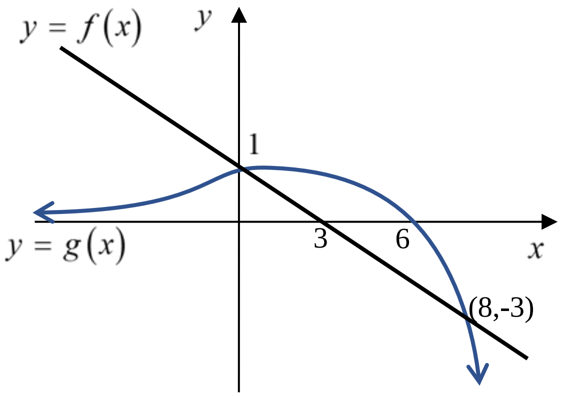
(A) $-3 \le x \le 0, \quad x \ge 3$

(B) $-3 \le x \le 0, \quad x = 3$

(C) $0 \le x \le 3, \quad x \le -3$

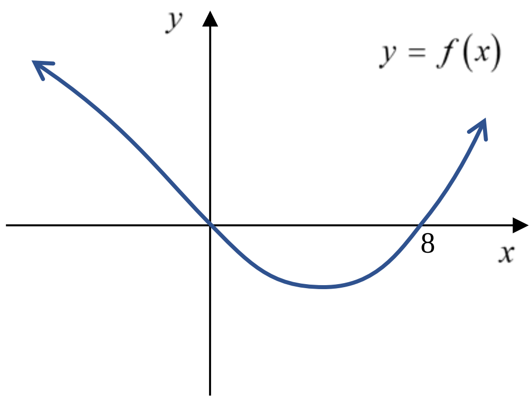
(D) $0 \le x \le 3, \quad x = -3$

3. The graphs of the functions $y = f(x)$ and $y = g(x)$ are shown on the same axes.

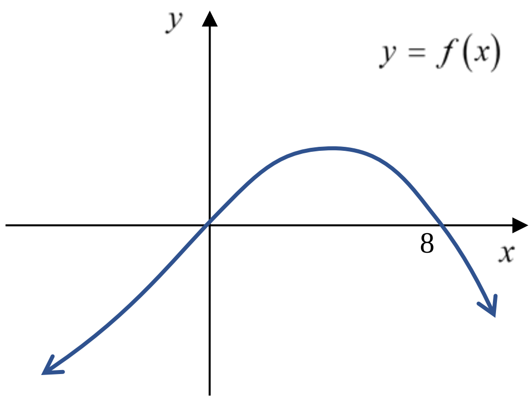


Which of the following is a possible graph of $y = f(x) - g(x)$

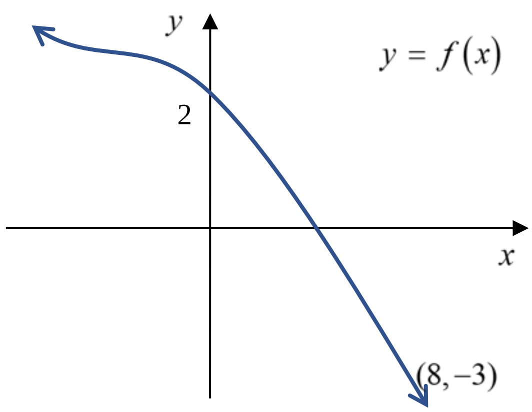
(A)



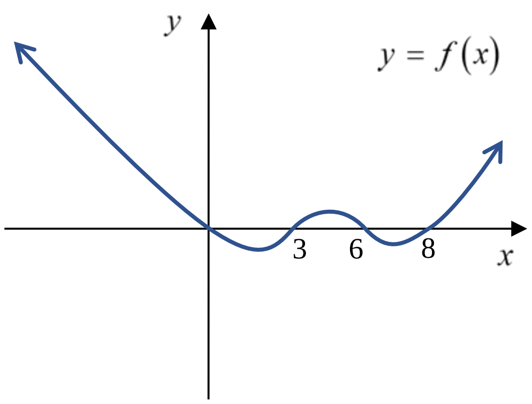
(B)



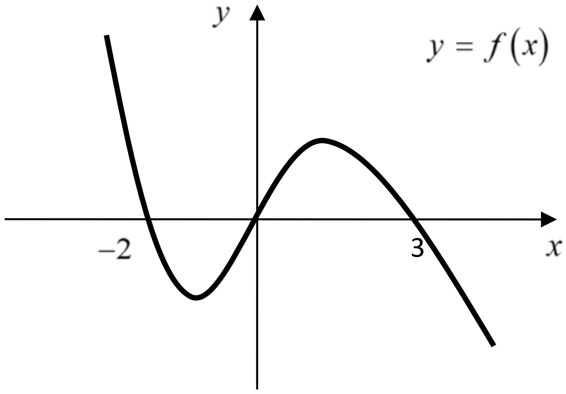
(C)



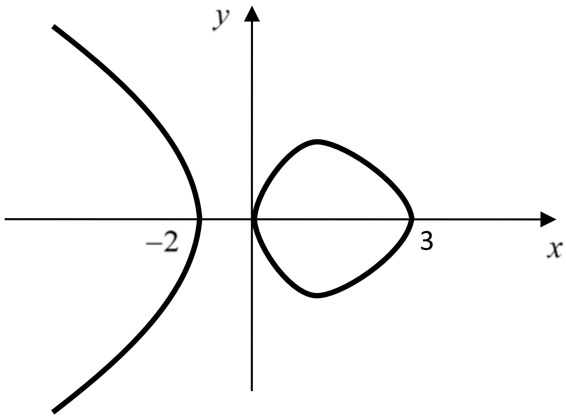
(D)



4. The graph of the function $y = f(x)$ is shown.



A second graph is obtained from the function $y = f(x)$



Which equation best represents the second graph?

(A) $y = \sqrt{f(x)}$

(B) $y = f\left(\sqrt{x}\right)$

(C) $y^2 = f(x)$

(D) $|y| = f(|x|)$

Section II

36 marks
Attempt Questions 5-7
Allow about 48 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In questions 5-7, your responses should include relevant mathematical reasoning and/or calculations.

Question 5 (13 marks) Use a SEPARATE writing booklet.

(a) If $f(x)=\dfrac{x}{x-3}$, find $g(x)$ such that $g(x)=f^{-1}(x)$ **2**

(b) Solve $\dfrac{2x}{x-3}\le x$. **3**

(c) By using a compound angle formula, determine the exact value of $\tan 75°$. **2**

(d) Let $5\cos x-\sqrt{11}\sin x\equiv R\cos\left(x+\alpha\right)$, where $R>0$ and $0°\le\alpha\le 360°$.

(i) By expanding the right-hand side and equating coefficients, show that $R=6$ and $\alpha=33°33'$ to the nearest minute. **3**

(ii) Hence, find the range of $y=6+5\cos x-\sqrt{11}\sin x$. **1**

(e) Show that $\dfrac{\sin 2\theta}{1+\cos 2\theta}=\tan\theta$. **2**

End of Question 5

Question 6 (12 marks) Use a SEPARATE writing booklet.

(a) Solve $\sin 2x - \sin x = 0$ for $-\pi \le x \le \pi$. **3**

(b) By letting $t = \tan\dfrac{\theta}{2}$, solve: **3**

$$\frac{\cos\theta + \sin\theta - 1}{\cos\theta - \sin\theta + 1} = 3 \quad \text{for} \quad 0 \le \theta \le 2\pi$$

(c) Consider the parametric equations $y = \cos 2t - 1$ and $x = \cos t$.

(i) Determine the Cartesian equation in x and y. **2**

(ii) What are the restrictions on x and y? **2**

(iii) Sketch the curve defined by $y = \cos 2t - 1$ and $x = \cos t$. **2**

End of Question 6

Question 7 (11 marks) Use a SEPARATE writing booklet.

(a) (i) Show that $\cos(\alpha-\beta)+\cos(\alpha+\beta)=2\cos\alpha\cos\beta$. **1**

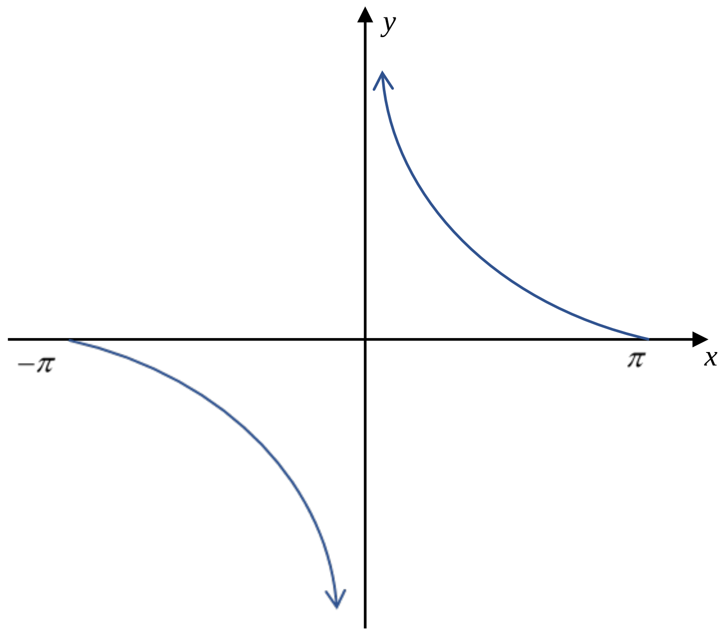
(ii) Write $2\cos 2\theta\cos\theta$ as a sum of cosine ratios. **1**

(iii) Hence solve $\cos\left(\theta-\frac{\pi}{3}\right)+\cos\theta+\cos\left(\theta+\frac{\pi}{3}\right)+\cos 3\theta=0$ for $0\le\theta\le\pi$. **3**

(b) Show that: **2**

$$\cot x+\frac{1}{2}\tan\frac{x}{2}=\frac{1}{2}\cot\frac{x}{2}.$$

(c) On the number plane below, the graph of $y=\cot\dfrac{x}{2}$ where $-\pi\le x\le\pi$ has been sketched.



Copy this sketch onto your answer sheet.

(i) On the same set of axes, sketch the graph of $y=\sin x$ where $-\pi\le x\le\pi$. **1**

(ii) Use the graphs and the result in part (b) to solve the inequality: **3**

$$2\cot x+\tan\frac{x}{2}-\sin x\le 0 \quad \text{for } -\pi\le x\le\pi.$$

End of Examination

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(D) $-3<x<4$

$|2x-1|<7$
$-7<2x-1<7$
$-6<2x<8$
$-3<x<4$

ANSWER is D

2. What is the solution of the inequation $x(x-3)^2(x+3)^3 \le 0$?

(A) $-3 \le x \le 0, \quad x \ge 3$

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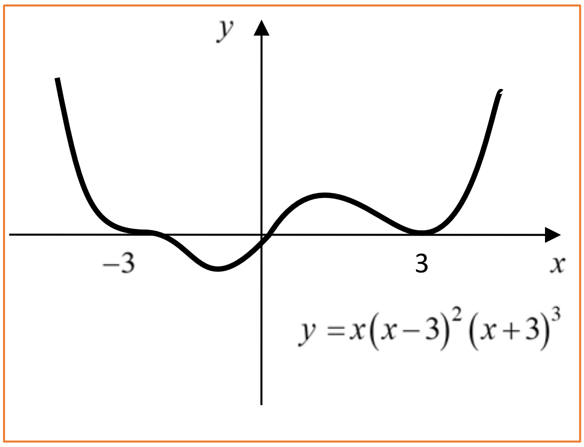
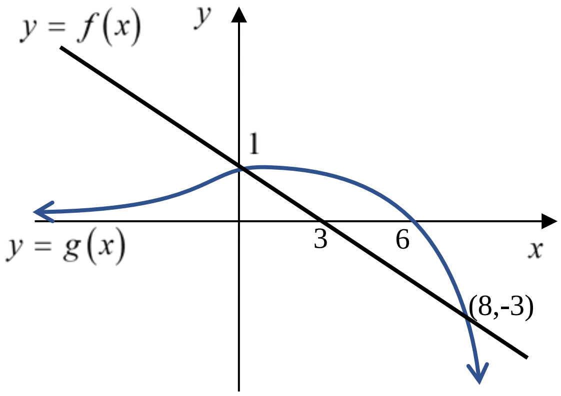


From the graph the ANSWER is B

3. The graphs of the functions $y = f(x)$ and $y = g(x)$ are shown on the same axes.



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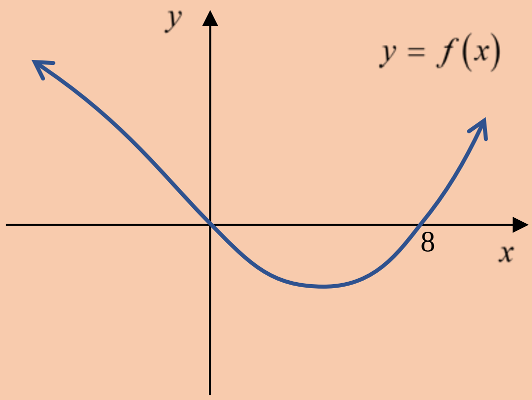
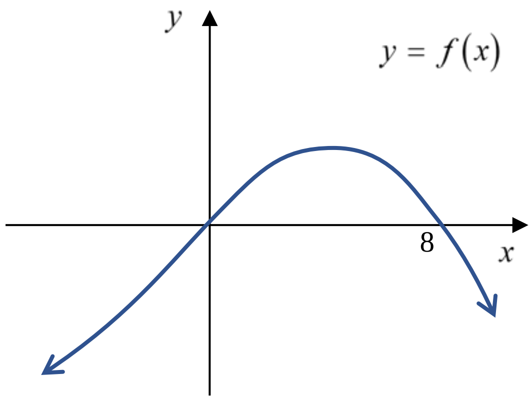


(B)



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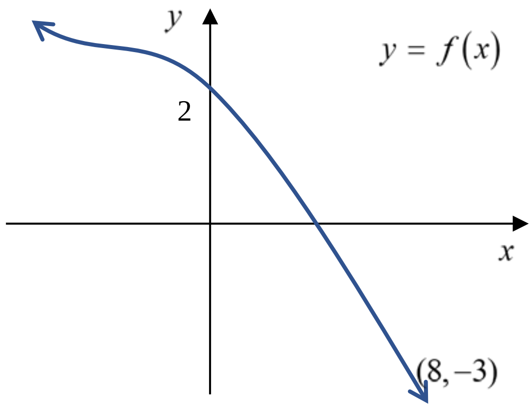
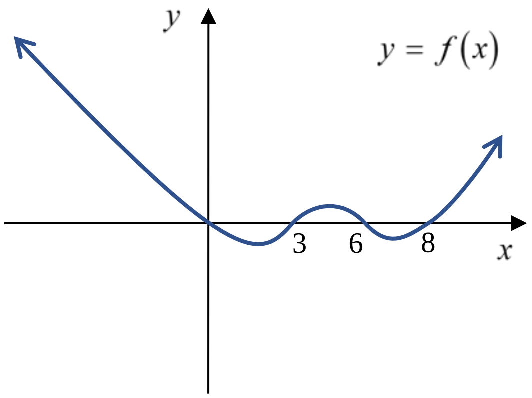


(D)



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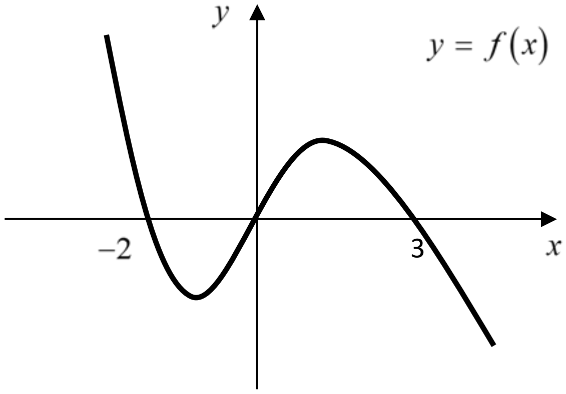

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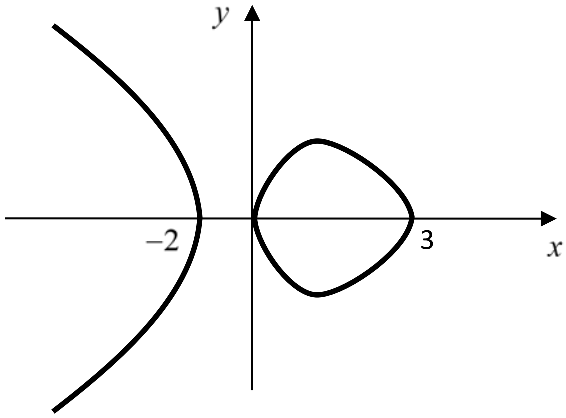

Which equation best represents the second graph?

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Question 5 (13 marks) Use a SEPARATE writing booklet.

(a) If $f(x)=\dfrac{x}{x-3}$, find $g(x)$ such that $g(x)=f^{-1}(x)$ **2**

$y=\dfrac{x}{x-3}$

Switch x and y and then make y the subject.

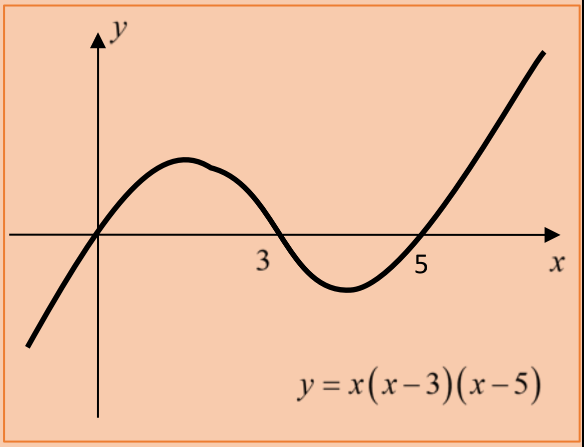
$$x=\frac{y}{y-3}$$
$$x(y-3)=y$$
$$xy-3x=y$$
$$xy-y=3x$$
$$y=\frac{3x}{x-1}$$
$$\therefore g(x)=\frac{3x}{x-1}$$

(b) Solve $\dfrac{2x}{x-3}\le x$. **3**

$\dfrac{2x}{x-3}\le x$ NOTE $x\ne 3$

Multiply by the square of the denominator

$$2x(x-3)\le x(x-3)^2$$
$$0\le x(x-3)^2-2x(x-3)$$
$$0\le x(x-3)\left[(x-3)-2\right]$$
$$0\le x(x-3)(x-5)$$



Sketch $y=x(x-3)(x-5)$ and use the graph to solve the inequality.

From the graph, $0\le x(x-3)(x-5)$ when:

$0\le x\le 3$ and $x\ge 5$ but since $x\ne 3$ the solution is $0\le x<3$ and $x\ge 5$

(c) By using a compound angle formula, determine the exact value of $\tan 75°$. **2**

$$\tan 75° = \tan\left(45° + 30°\right)$$

$$= \frac{\tan\left(45°\right) + \tan\left(30°\right)}{1 - \tan\left(45°\right)\tan\left(30°\right)}$$

$$= \frac{(1) + \left(\frac{1}{\sqrt{3}}\right)}{1 - (1)\left(\frac{1}{\sqrt{3}}\right)}$$

$$= \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$= \frac{\left(\sqrt{3}+1\right)^2}{3-1} = \frac{4+2\sqrt{3}}{2} = 2+\sqrt{3}$$

(d) Let $5\cos x - \sqrt{11}\sin x \equiv R\cos\left(x+\alpha\right)$, where $R > 0$ and $0° \le \alpha \le 360°$.

(i) By expanding the right-hand side and equating coefficients, **3**

show that $R = 6$ and $\alpha = 33°33'$ to the nearest minute.

$$5\cos x - \sqrt{11}\sin x = R\cos\left(x+\alpha\right)$$

$$= R\cos x\cos\alpha - R\sin x\sin\alpha$$

By equating co-efficients of $\sin x$ and $\cos x$

$R\cos\alpha = 5 \quad I$ $\qquad -R\sin\alpha = -\sqrt{11}$

$R\sin\alpha = \sqrt{11} \quad II$

$I^2 + II^2$

$$5^2 + \left(\sqrt{11}\right)^2 = R^2\cos^2\alpha + R^2\sin^2\alpha$$

$$36 = R^2\left(\cos^2\alpha + \sin^2\alpha\right)$$

$$6 = R^2$$

$6 = R$ since $R > 0$

Since $\cos\alpha = \frac{5}{6}$ and $\sin\alpha = \frac{\sqrt{11}}{6}$ α must be first quadrant.

$$\alpha = \cos^{-1}\frac{5}{6} = 33°33`$$

$$5\cos x - \sqrt{11}\sin x = 6\cos\left(x + 33°33`\right)$$

(ii) Hence, find the range of $y = 6 + 5\cos x - \sqrt{11}\sin x$. **1**

$-1 \le \cos\left(x + 33°33`\right) \le 1$

$-6 \le 6\cos\left(x + 33°33`\right) \le 6$

$-6 \le 5\cos x - \sqrt{11}\sin x \le 6$

$0 \le 6 + 5\cos x - \sqrt{11}\sin x \le 12$

Range is $0 \le y \le 12$

(e) Show that $\dfrac{\sin 2\theta}{1 + \cos 2\theta} = \tan\theta$. **2**

$$\begin{aligned}
\frac{\sin 2\theta}{1+\cos 2\theta} &= \frac{2\sin\theta\cos\theta}{1+2\cos^2\theta - 1} \\
&= \frac{2\sin\theta\cos\theta}{1+2\cos^2\theta - 1} \\
&= \frac{2\sin\theta\cos\theta}{2\cos^2\theta} \\
&= \frac{\sin\theta}{\cos\theta} \\
&= \tan\theta = RHS
\end{aligned}$$

End of Question 5

Question 6 (12 marks) Use a SEPARATE writing booklet.

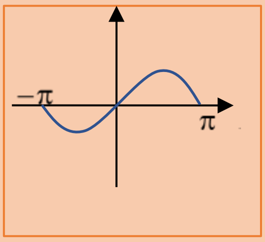
(a) Solve $\sin 2x - \sin x = 0$ for $-\pi \le x \le \pi$. **3**

$\sin 2x - \sin x = 0$ for $-\pi \le x \le \pi$.

$2\sin x\cos x - \sin x = 0$

$\sin x\left[2\cos x - 1\right] = 0$

$\sin x = 0$ $\qquad$ $2\cos x - 1 = 0$



$$\cos x = \frac{1}{2}$$

$$x = \pm\cos^{-1}\frac{1}{2}$$

$$x = \pm\frac{\pi}{3}$$

From the graph $x = -\pi,\ 0,\ \ \pi$

$$x = -\pi,\ \ -\frac{\pi}{3},\ 0,\ \ \frac{\pi}{3},\ \ \pi$$

(b) By letting $t = \tan\frac{\theta}{2}$, solve: **3**

$$\frac{\cos\theta + \sin\theta - 1}{\cos\theta - \sin\theta + 1} = 3 \quad \text{for} \quad 0 \le \theta \le 2\pi$$

$$\frac{\cos\theta + \sin\theta - 1}{\cos\theta - \sin\theta + 1} = 3$$

$$\frac{\frac{1-t^2}{1+t^2} + \frac{2t}{1+t^2} - 1}{\frac{1-t^2}{1+t^2} - \frac{2t}{1+t^2} + 1} = 3$$

$$\frac{1-t^2+2t-\left(1+t^2\right)}{1-t^2-2t+\left(1+t^2\right)} = 3$$

$$\frac{2t-2t^2}{2-2t} = 3$$

$$\frac{2t\left(1-t\right)}{2\left(1-t\right)} = 3$$

$t = 3$ $\qquad$ $\left(1-t\right) \ne 0$

$\tan\frac{\theta}{2} = 3$ $\qquad$ $0 \le \theta \le 2\pi$

$\tan\frac{\theta}{2} = 3$ $\qquad$ $0 \le \frac{\theta}{2} \le \pi$

First Quad Only

$$\frac{\theta}{2} = \tan^{-1}3$$

$$\frac{\theta}{2} = 1.249$$

$$\theta = 2.498$$

$$= 2.5$$

(c) Consider the parametric equations $y = \cos 2t - 1$ and $x = \cos t$.

(i) Determine the Cartesian equation in x and y. **2**

$y = \cos 2t - 1 \qquad x = \cos t$

$y = \left(2\cos^2 t - 1\right) - 1$

$y = 2\cos^2 t - 2$

$y = 2x^2 - 2$

(ii) What are the restrictions on x and y? **2**

$-1 \le \cos 2t \le 1 \qquad -1 \le \cos t \le 1$

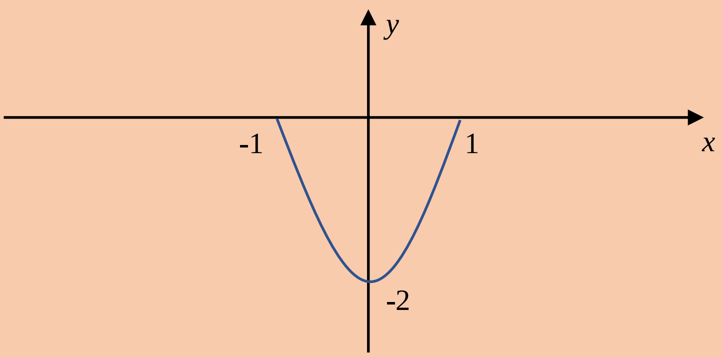
$-2 \le \cos 2t - 1 \le 0 \qquad -1 \le x \le 1$

$-2 \le y \le 0$

(iii) Sketch the curve defined by $y = \cos 2t - 1$ and $x = \cos t$. **2**

The Cartesian equation is $y = 2x^2 - 2$ with restrictions of

$-2 \le y \le 0 \qquad -1 \le x \le 1$



End of Question 6

Question 7 (11 marks) Use a SEPARATE writing booklet.

(a) (i) Show that $\cos(\alpha-\beta)+\cos(\alpha+\beta)=2\cos\alpha\cos\beta$. **1**

$$\begin{aligned} LHS &= \cos(\alpha-\beta)+\cos(\alpha+\beta) \\ &= \cos\alpha\cos\beta+\sin\alpha\sin\beta+\cos\alpha\cos\beta-\sin\alpha\sin\beta \\ &= 2\cos\alpha\cos\beta \\ &= RHS \end{aligned}$$

(ii) Write $2\cos 2\theta\cos\theta$ as a sum of cosine ratios. **1**

From part i) let $\alpha=2\theta$ and $\beta=\theta$

$$\begin{aligned} 2\cos 2\theta\cos\theta &= \cos(2\theta-\theta)+\cos(2\theta+\theta) \\ &= \cos\theta+\cos 3\theta \end{aligned}$$

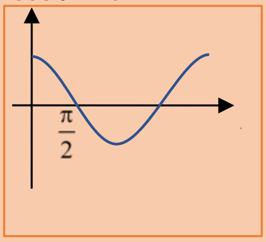
(iii) Hence solve $\cos\left(\theta-\frac{\pi}{3}\right)+\cos\theta+\cos\left(\theta+\frac{\pi}{3}\right)+\cos 3\theta=0$ for $0\le\theta\le\pi$. **3**

From part ii) $\cos\theta + \cos 3\theta = 2\cos 2\theta \cos\theta$

Also part i) $\cos\left(\theta - \frac{\pi}{3}\right) + \cos\left(\theta + \frac{\pi}{3}\right) = 2\cos\theta\cos\left(\frac{\pi}{3}\right)$

$$\cos\left(\theta - \tfrac{\pi}{3}\right) + \cos\theta + \cos\left(\theta + \tfrac{\pi}{3}\right) + \cos 3\theta = 0$$
$$2\cos 2\theta\cos\theta + 2\cos\theta\cos\left(\tfrac{\pi}{3}\right) = 0$$
$$2\cos 2\theta\cos\theta + 2(\cos\theta)\left(\frac{1}{2}\right) = 0$$
$$\cos\theta(2\cos 2\theta + 1) = 0$$

$\cos\theta = 0$



$\theta = \dfrac{\pi}{2}$

$\cos 2\theta = -\dfrac{1}{2}$

$0 \le 2\theta \le 2\pi$

$2\theta = \dfrac{2\pi}{3}, \dfrac{4\pi}{3}$

$\theta = \dfrac{\pi}{3}, \dfrac{2\pi}{3}$

$\therefore \theta = \dfrac{\pi}{2}, \quad \dfrac{\pi}{3}, \dfrac{2\pi}{3}$

(b) Show that: **2**

$$\cot x + \frac{1}{2}\tan\frac{x}{2} = \frac{1}{2}\cot\frac{x}{2}.$$

$LHS = \cot x + \dfrac{1}{2}\tan\dfrac{x}{2}$

$= \dfrac{1-t^2}{2t} + \dfrac{t}{2}$

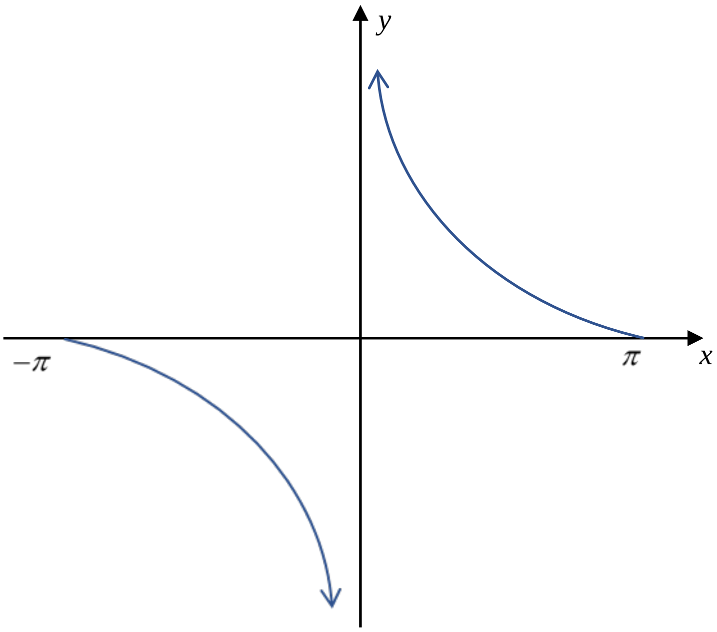
$= \dfrac{1-t^2}{2t} + \dfrac{t^2}{2t}$

$= \dfrac{1}{2t}$

$= \dfrac{1}{2\tan\dfrac{x}{2}}$

$= \dfrac{1}{2}\cot\dfrac{x}{2} = RHS$

(c) On the number plane below, the graph of $y = \cot\dfrac{x}{2}$ where $-\pi \le x \le \pi$ has been sketched.



Copy this sketch onto your answer sheet.

(i) On the same set of axes, sketch the graph of $y = \sin x$ where $-\pi \le x \le \pi$. **1**

Solving simultaneously

$$\cot\frac{x}{2} = \sin x$$

$$\frac{1}{t} = \frac{2t}{1+t^2}$$

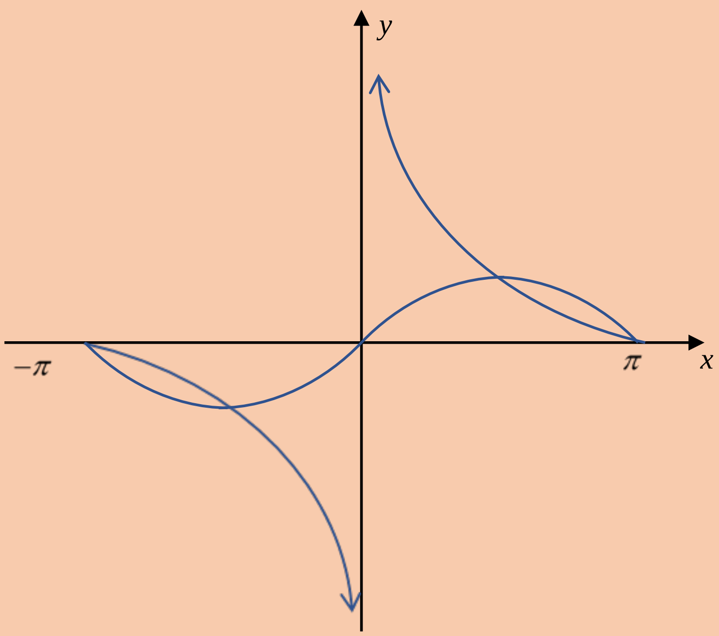
$$1+t^2 = 2t^2$$

$$t^2 = 1$$

$$\tan\frac{x}{2} = \pm 1$$

$$\frac{x}{2} = \pm\frac{\pi}{4}$$

$$x = \pm\frac{\pi}{2}$$



Graphs intersect at:

$\left(-\dfrac{\pi}{2}, -1\right)$ and $\left(\dfrac{\pi}{2}, 1\right)$

(ii) Use the graphs and the result in part (b) to solve the inequality: **3**

$$2\cot x + \tan\frac{x}{2} - \sin x \le 0 \quad \text{for } -\pi \le x \le \pi .$$

$$2\cot x + \tan\frac{x}{2} - \sin x \le 0$$

$$2\cot x + \tan\frac{x}{2} \le \sin x$$

$$2\left(\cot x + \frac{1}{2}\tan\frac{x}{2}\right) \le \sin x$$

$$2\left(\frac{1}{2}\cot\frac{x}{2}\right) \le \sin x \qquad \text{From part b)}$$

$$\cot\frac{x}{2} \le \sin x$$

Using the graph from part i) the solution is where the graph of $y = \sin x$ is above $y = \cot\frac{x}{2}$.

NOTE: $x = 0$ is not in the domain of $y = \cot x$ and

$x = \pm\pi$ is not in the domain of $y = \tan\frac{x}{2}$

So the solution to $2\cot x + \tan\frac{x}{2} - \sin x \le 0$ is:

$-\frac{\pi}{2} \le x < 0$ and $\frac{\pi}{2} \le x < \pi$

End of Examination

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