NORTH SYDNEY GIRLS HIGH SCHOOL

2020 | **HSC Assessment Task 3**

Mathematics Extension 1

General Instructions

- Reading Time – 2 minutes
- Working Time – 55 minutes
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided
- In Questions 6 – 8, show relevant mathematical reasoning and/or calculations

Total marks – 42

Section I – 5 marks (pages 3 – 5)

- Attempt Questions 1 – 4
- Allow about 6 minutes for this section

Section II – 37 marks (pages 7 – 11)

- Attempt Questions 6 – 8
- Allow about 49 minutes for this section

NAME:____________________ **TEACHER:**________________

STUDENT NUMBER:

Question	1-4	5	6 a b c	6 d e f	7 a	7 b c d	8 a	8 b c	Total
Inverse Functions	4		5			10	5		24
Further Integration		1		7	3			7	18
									42

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Section I

4 marks
Attempt Questions 1-4
Allow about 6 minutes for this section

Use the multiple choice answer sheet for Questions 1-4.

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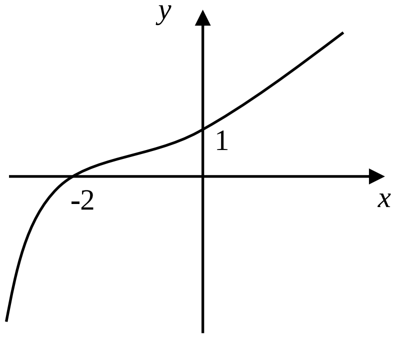
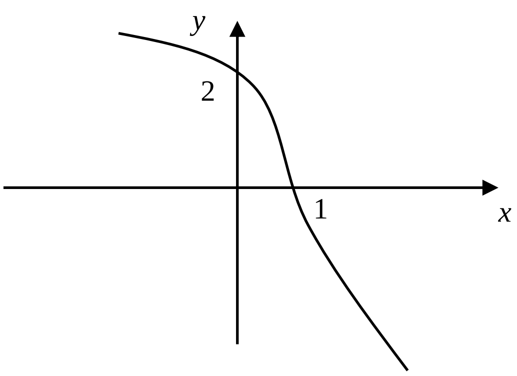
1 The graph of $y = f\left(x\right)$ is shown below.

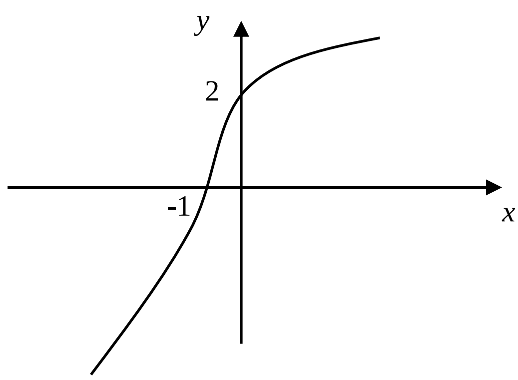


Which of the following could be the graph of $y = f^{-1}\left(x\right)$?

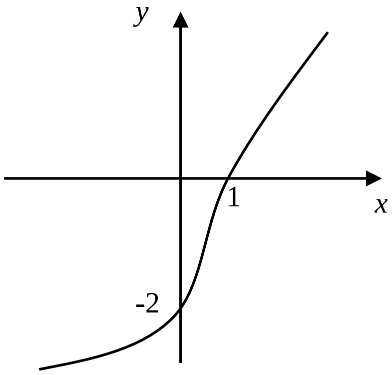
(A)



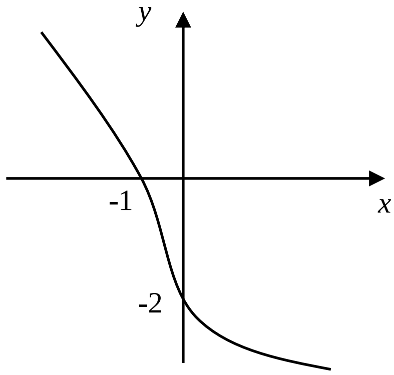
(B)



(C)



(D)



2 Which of the following types of relations is a function whose inverse is also a function?

(A) Many-to-one

(B) One-to-one

(C) Many-to-many

(D) One-to-many

3 Given that $\pi < p < \frac{3\pi}{2}$ what the value of $\tan^{-1}\left(\tan\left(p\right)\right)$?

(A) p

(B) $p+\pi$

(C) $\pi - p$

(D) $p-\pi$

4 What is the domain and range of the function $y = 4\cos^{-1}3x$?

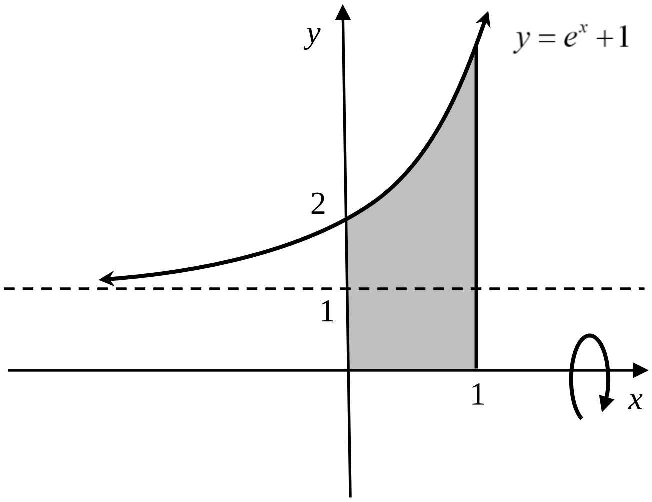
(A) Domain: $-\frac{1}{3} \leq x \leq \frac{1}{3}$ and Range: $-2\pi \leq y \leq 2\pi$

(B) Domain: $-3 \leq x \leq 3$ and Range: $-2\pi \leq y \leq 2\pi$

(C) Domain: $-\frac{1}{3} \leq x \leq \frac{1}{3}$ and Range: $0 \leq y \leq 4\pi$

(D) Domain: $-3 \leq x \leq 3$ and Range: $0 \leq y \leq 4\pi$

5 The diagram below shows the function $f\left(x\right)=e^{x}+1$. The region between the function, the *x*-axis, *y*-axis and the line $x=1$ has been shaded.



Which of the following is equivalent to the volume of the solid or revolution when the shaded region is rotated around the *x*-axis?

(A) $\pi\int_{0}^{1}\left(e^{x}+1\right)^{2}\,dx$

(B) $\pi\int_{0}^{1}\left(\left(e^{x}\right)^{2}+2^{2}\right)dx$

(C) $\pi\int_{0}^{1}\left(\left(e^{x}\right)^{2}+1\right)dx$

(D) $\pi\left[4+\int_{2}^{e+1}\left(e^{x}\right)^{2}\,dx\right]$

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Section II

37 marks
Attempt Questions 6 - 8
Allow about 49 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In questions 5-7, your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (12 marks**)** Use a SEPARATE writing booklet

(a) Differentiate $\cos^{-1}(x-3)$. **1**

(b) Find $\dfrac{d}{dx}\left(\arctan\sqrt{x}\right)$. **2**

(c) Evaluate $\displaystyle\int_0^1 \frac{dx}{\sqrt{3-4x^2}}$. **2**

(d) Find $\displaystyle\int \cos^2 2x \; dx$. **2**

(e) Evaluate $\displaystyle\int_1^{\sqrt{5}} \frac{x}{\sqrt{1+3x^2}} \; dx$ by using the substitution $u = 1+3x^2$. **3**

(f) Find $\displaystyle\int \sin 3x \cos^4 3x \; dx$. **2**

End of Question 5

Question 7 (13 marks) Use a SEPARATE writing booklet

(a) Using the substitution $u^2 = 1 - x$, find $\int x\sqrt{1-x}\ dx$. **3**

(b) Consider the function $f\left(x\right) = 3\sin^{-1}\left(2x-1\right)$.

(i) State the domain of $f\left(x\right)$. **1**

(ii) Hence sketch the graph of $y = f\left(x\right)$. Show all the important features. **2**

(c) Consider the function $f(x) = \tan^{-1}\left(\ln x\right) + \tan^{-1}\left(\dfrac{1}{\ln x}\right)$ for $x > 1$.

(i) Show that $f'(x) = 0$. **2**

(ii) Hence sketch the graph of $y = f(x)$. **2**

(d) Consider the function $f(x) = 3x - x^3$

(i) Find the largest domain containing the origin for which $y = f(x)$ has an inverse function $y = f^{-1}(x)$. **2**

(ii) State the domain of $y = f^{-1}(x)$. **1**

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End of Question 6

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Question 8 (12 marks) Use a SEPARATE writing booklet

(a) Consider a function $f(x)$, over the domain $a < x < b$, whose inverse is a function. Assume that $f'(x)$ exists for $a < x < b$ and is non-zero. Let $g(x) = f^{-1}(x)$.

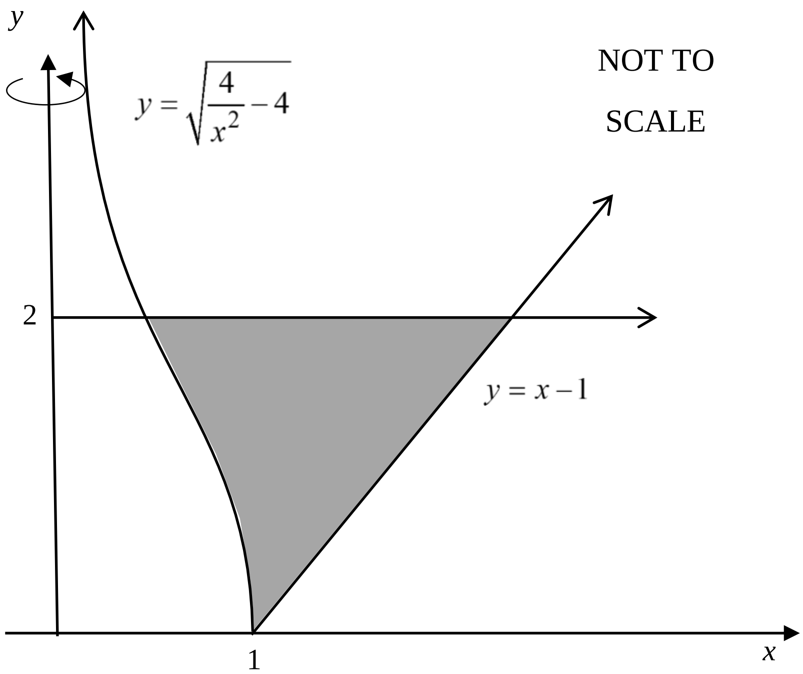
(i) Show that $g'(x) = \dfrac{1}{f'(g(x))}$. **2**

(ii) Consider $f(x) = 3 + x^2 + \sin^{-1}\left(\dfrac{x-2}{3}\right)$ for $0 < x < 3$. **1**

Explain why $g(7) = 2$.

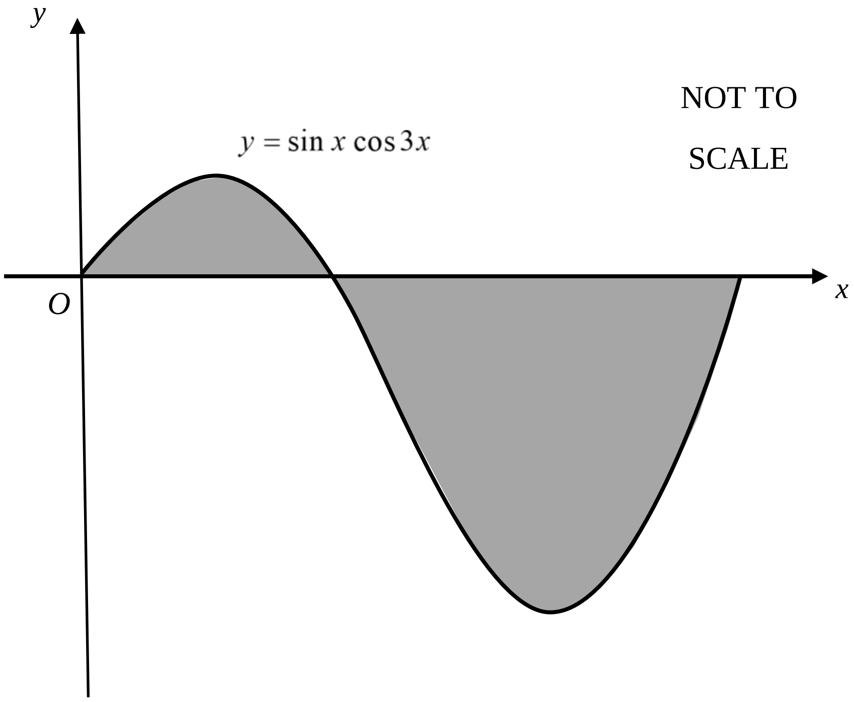
(iii) Show that the gradient of the tangent to the curve $y = g(x)$ at the point (7, 2) is $\dfrac{3}{13}$. **2**

(b) In the diagram below, the area between the graphs of $y = \sqrt{\dfrac{4}{x^2} - 4}$, $y = x - 1$ and the line $y = 2$ has been shaded. **3**



Find the volume of the solid of revolution formed when the shaded region is rotated about the y-axis.

c) In the diagram below, part of the graph of $y = \sin x \cos 3x$ is shown. **4**
The area between the graph and the x-axis has been shaded.



Find the exact area of the shaded region.

End of Paper

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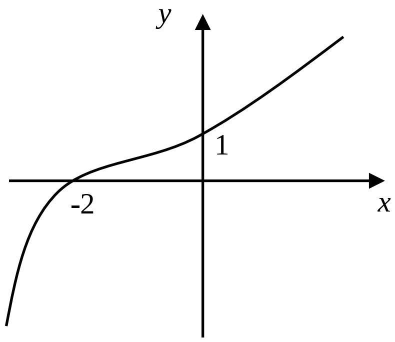

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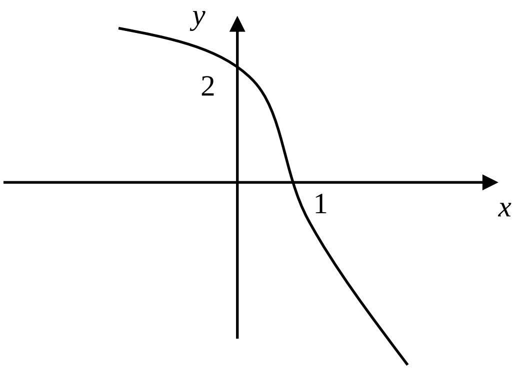

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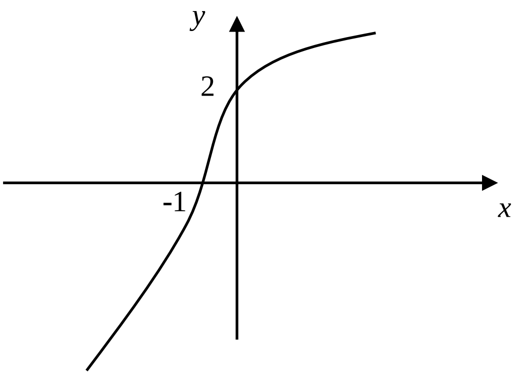

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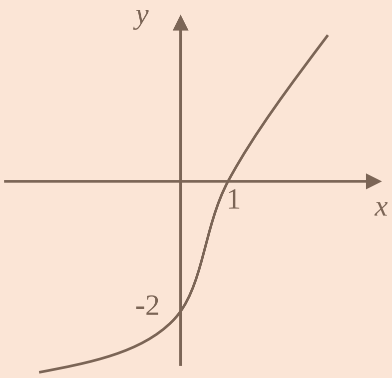
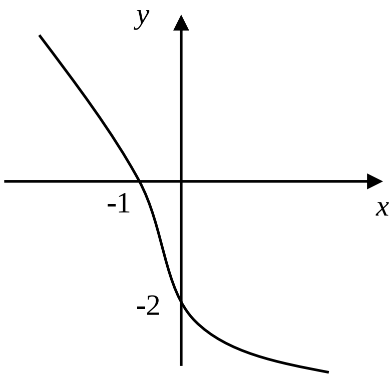


(D)



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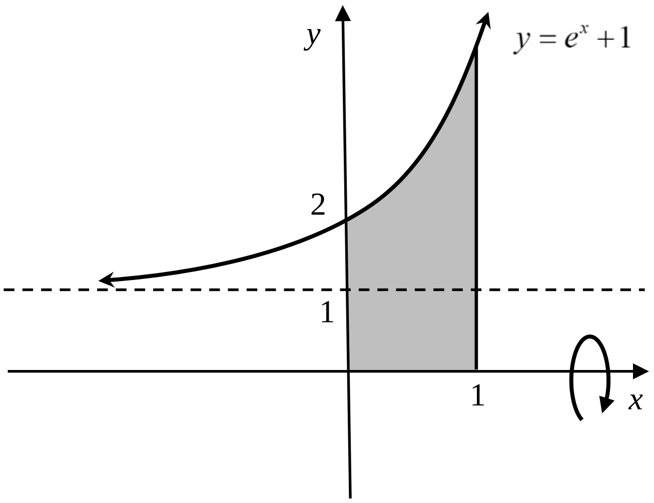

Which of the following is equivalent to the volume of the solid or revolution when the shaded region is rotated around the *x*-axis?

(A) $\pi \int_0^1 \left(e^x + 1\right)^2 dx$

(B) $\pi \int_0^1 \left(\left(e^x\right)^2 + 2^2\right) dx$

(C) $\pi \int_0^1 \left(\left(e^x\right)^2 + 1\right) dx$

(D) $\pi \left[4 + \int_2^{e+1} \left(e^x\right)^2 \, dx \right]$

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Section II

Question 6 (12 marks**)** Use a SEPARATE writing booklet

(a) Differentiate $\cos^{-1}(x-3)$. **1**

Solution

$$\frac{d}{dx}\cos^{-1}(x-3)=\frac{1}{\sqrt{1^2-(x-3)^2}}$$

(b) Find $\frac{d}{dx}\left(\arctan\sqrt{x}\right)$. **2**

Solution

$$\frac{d}{dx}\left(\arctan\sqrt{x}\right)=\frac{1}{1+\left(f(x)\right)^2}\times f'(x)$$

$$=\frac{1}{1+\left(\sqrt{x}\right)^2}\times\frac{1}{2\sqrt{x}}$$

$$=\frac{1}{2\sqrt{x}(1+x)}$$

(c) Evaluate $\int_0^1 \frac{dx}{\sqrt{3-4x^2}}$. **2**

Solution

$$\int_0^1 \frac{dx}{\sqrt{3-4x^2}}=\int_0^1 \frac{dx}{2\sqrt{\frac{3}{4}-x^2}}$$

$$=\frac{1}{2}\int_0^1 \frac{dx}{\sqrt{\left(\frac{\sqrt{3}}{2}\right)^2-x^2}}$$

$$=\frac{1}{2}\left[\sin^{-1}\left(\frac{2x}{\sqrt{3}}\right)\right]_0^1$$

No Solution, improper integral

(d) Find $\int \cos^2 2x \ dx$. **2**

Solution

$$\int \cos^2 2x \ dx = \int \frac{1}{2}(\cos 4x + 1) \ dx$$
$$= \frac{1}{2}\int (\cos 4x + 1) \ dx$$
$$= \frac{1}{2}\left(\frac{\sin 4x}{4} + x\right)$$

(e) Evaluate $\int_1^{\sqrt{5}} \frac{x}{\sqrt{1+3x^2}} \ dx$ by using the substitution $u = 1 + 3x^2$. **3**

$\int_1^{\sqrt{5}} \frac{x}{\sqrt{1+3x^2}} \ dx = \int_4^{16} \frac{1}{\sqrt{u}} \times \frac{du}{6}$ $= \frac{1}{6}\int_4^{16} u^{-\frac{1}{2}} \ du$ $= \frac{1}{6}\left[2\sqrt{u}\right]_4^{16}$ $= \frac{1}{3}\left[\sqrt{16} - \sqrt{4}\right]$ $= \frac{2}{3}$	$u = 1 + 3x^2.$ $\frac{du}{dx} = 6x$ $dx = \frac{du}{6x}$	$x = \sqrt{5}$ $u = 1 + 3\left(\sqrt{5}\right)^2$ $u = 16$ $x = 1$ $u = 1 + 3(1)^2$ $u = 4$

(f) Find $\int \sin 3x \cos^4 3x \ dx$. **2**

Solution

$$\int \sin 3x \cos^4 3x \ dx = -\frac{1}{3}\int (-3\sin 3x)(\cos 3x)^4 \ dx$$
$$= -\frac{1}{3} \times \frac{(\cos 3x)^5}{5} + C$$
$$= -\frac{\cos^5 3x}{15} + C$$

Marker's Comments

All questions were generally well done.

- a) -
- b) Most common mistake was not finding or using the derivative of the inner function.
- c) $=\frac{1}{2}\left[\sin^{-1}\left(\frac{2x}{\sqrt{3}}\right)\right]_0^1$, got full marks. No solution when x=1.
- d) -
- e) Most students followed the process correctly.
- f) Most common mistake was not recognising that f'(x) would have a factor of 3, and therefore missed the factor of a 1/3. Second most common mistake was attempting to use an incorrect substitution: $u=\cos^4 3x$, which made things very difficult.

Question 7 (13 marks) Use a SEPARATE writing booklet

(a) Using the substitution $u^2 = 1 - x$, find $\int x\sqrt{1-x}\ dx$. **3**

$\int x\sqrt{1-x}\ dx. = \int (1-u^2)\sqrt{u^2}\,(1-2u)\ du$ $= \int (1-u^2)u(2u)du$ $= \int (2u^2 - 2u^4)du$ $= \frac{2u^3}{3} - \frac{2u^5}{5} + C$ $= \frac{2(\sqrt{1-x})^3}{3} - \frac{2(\sqrt{1-x})^5}{5} + C$	$u^2 = 1 - x$ $u = \sqrt{1-x}$ $x = 1 - u^2$ $\frac{dx}{du} = 2u$ $dx = 2u\ du$

(b) Consider the function $f(x) = 3\sin^{-1}(2x-1)$.

(i) State the domain of $f(x)$. **1**

Solution

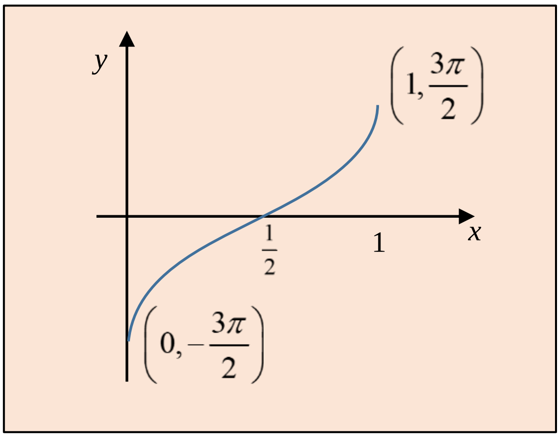
$-1 \le 2x - 1 \le 1$

$0 \le 2x \le 2$

$0 \le x \le 1$

Thus Domain of $f(x)$ is $0 \le x \le 1$

(ii) Hence sketch the graph of $y = f(x)$. Show all the important features. **2**



(c) Consider the function $f(x) = \tan^{-1}(\ln x) + \tan^{-1}\left(\frac{1}{\ln x}\right)$ for $x > 1$.

(i) Show that $f'(x) = 0$.

Solution

$$f(x) = \tan^{-1}(\ln x) + \tan^{-1}\left(\frac{1}{\ln x}\right)$$

$$\begin{aligned}
f'(x) &= \frac{1}{1+(\ln x)^2} \times \frac{1}{x} + \frac{1}{1+\left((\ln x)^{-1}\right)^2} \times \frac{-1}{x}(\ln x)^{-2} \\
&= \frac{1}{x\left[1+(\ln x)^2\right]} + \frac{1}{1+\frac{1}{(\ln x)^2}} \times \frac{-1}{x(\ln x)^2} \\
&= \frac{1}{x\left[1+(\ln x)^2\right]} + \frac{-1}{x\left[(\ln x)^2+1\right]} \\
&= 0
\end{aligned}$$

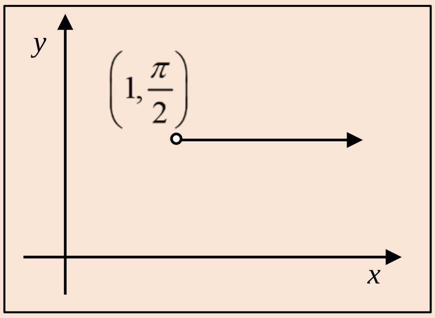
(ii) Hence sketch the graph of $y = f(x)$. **2**

Solution

Since the derivative is zero then the function will be constant for $x > 1$.

Let $x = e$

$$\begin{aligned}
f(e) &= \tan^{-1}(\ln e) + \tan^{-1}\left(\frac{1}{\ln e}\right) \\
&= \tan^{-1}(1) + \tan^{-1}(1) \\
&= \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}
\end{aligned}$$

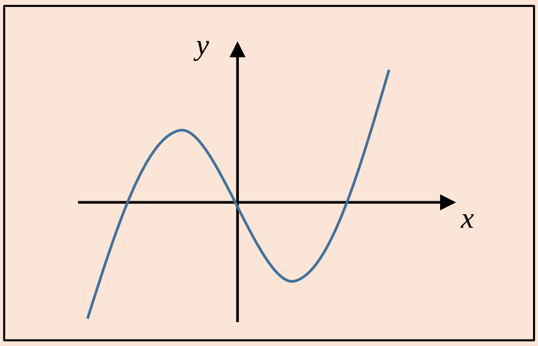


(d) Consider the function $f(x) = 3x - x^3$

(i) Find the largest domain containing the origin for which $y = f(x)$ has an inverse function $y = f^{-1}(x)$. **2**

Solution

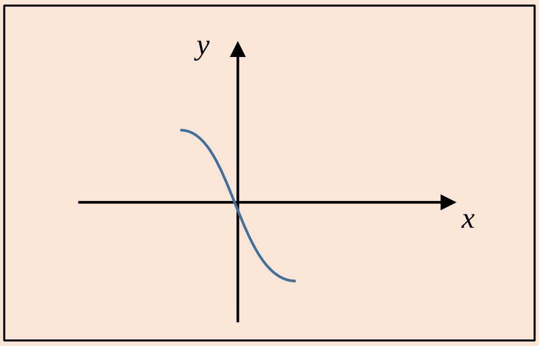
Consider the sketch of $y = 3x - x^3$



$$y = 3x - x^3$$
$$= x\left(3 - x^2\right)$$
$$= x\left(\sqrt{3} - x\right)\left(\sqrt{3} + x\right)$$

x - intercepts are at 0, $\sqrt{3}$ and $-\sqrt{3}$

The largest domain that contains the origin and the graph passes the horizontal line test is between the two x-values of the turning point.



$f'(x) = 3 - 3x^2$

Let $f'(x) = 0$

$0 = 3 - 3x^2$

$x^2 = 1$

$x \pm 1$

The largest domain containing the origin for which $y = f(x)$ has an inverse function is $-1 \le x \le 1$.

(ii) State the domain of $y = f^{-1}(x)$. **1**

The domain of an inverse function $f^{-1}(x)$ is the range of the function $f(x)$.
From part i) the range is in between the y-values of the turning points.

$f(1) = 3(1) - (1)^3 = 2$

$f(1) = 3(-1) - (-1)^3 = -2$

The range of the inverse function $f^{-1}(x)$ is $-2 \le y \le 2$

Question 8 (12 marks) Use a SEPARATE writing booklet

(a) Consider a function $f(x)$, over the domain $a < x < b$, whose inverse is a function.

Assume that $f'(x)$ exists for $a < x < b$ and is non-zero.

Let $g(x) = f^{-1}(x)$.

(i) Show that $g'(x) = \dfrac{1}{f'(g(x))}$. **2**

Aim : Show that $g'(x) = \dfrac{1}{f'(g(x))}$.

Start with finding the derivative of $f[g(x)]$. To do this use the chain rule

$\dfrac{d}{dx} f[g(x)] = g'(x) \times f'[g(x)]$ RHS is found using the Chain Rule

But since $g(x) = f^{-1}(x)$ then on the LHS

$$\frac{d}{dx} f[f^{-1}(x)] = g'(x) \times f'[g(x)]$$

$$\frac{d}{dx} x = g'(x) \times f'[g(x)]$$

$$1 = g'(x) \times f'[g(x)]$$

$$\frac{1}{f'[g(x)]} = g'(x)$$

$$\therefore g'(x) = \frac{1}{f'(g(x))}$$

(ii) Consider $f(x) = 3 + x^2 + \sin^{-1}\left(\dfrac{x-2}{3}\right)$ for $0 < x < 3$. **1**

Explain why $g(7) = 2$.

$$f(2) = 3 + (2)^2 + \sin^{-1}\left(\frac{(2)-2}{3}\right)$$

$$= 3 + 4 + \sin^{-1}(0)$$

$$= 7$$

Since $g(x) = f^{-1}(x)$ and $f(2) = 7$ therefore $g(7) = 2$

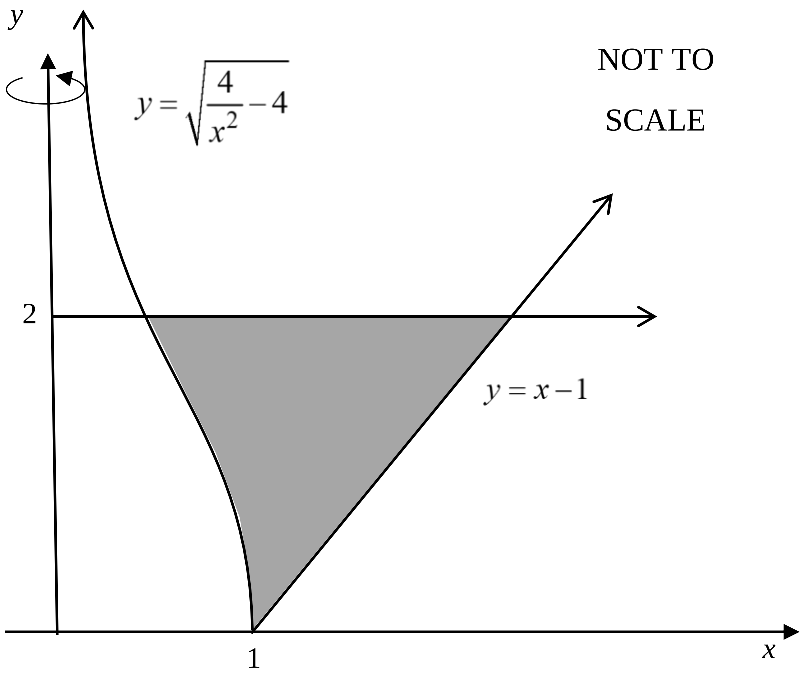
(iii) Show that the gradient of the tangent to the curve $y = g(x)$ at the point (7, 2) is $\frac{3}{13}$. **2**

Using part i) $g'(7) = \dfrac{1}{f'(g(7))}$ $= \dfrac{1}{f'(2)}$ From part ii) $g'(7) = \dfrac{1}{\frac{13}{3}}$ $= \dfrac{3}{13}$	$f(x) = 3 + x^2 + \sin^{-1}\left(\dfrac{x-2}{3}\right)$ $f'(x) = 2x + \dfrac{1}{\sqrt{1-\left(\dfrac{x-2}{3}\right)^2}} \times \dfrac{1}{3}$ $f'(2) = 2(2) + \dfrac{1}{3\sqrt{1-\left(\dfrac{(2)-2}{3}\right)^2}}$ $= 4 + \dfrac{1}{3\sqrt{1}}$ $= \dfrac{13}{3}$

Question 8 continues on Page 10

Question 8 (continued)

(b) In the diagram below, the area between the graphs of $y=\sqrt{\dfrac{4}{x^2}-4}$, $y=x-1$ and the line $y=2$ has been shaded. **3**



Find the volume of the solid of revolution formed when the shaded region is rotated about the y-axis.

Rotating around the y-axis, need to find x^2 for each function.

$$y=\sqrt{\frac{4}{x^2}-4}$$
$$y^2=\frac{4}{x^2}-4$$
$$y^2+4=\frac{4}{x^2}$$
$$x^2=\frac{4}{y^2+4}$$

$$y=x-1$$
$$y+1=x$$
$$x^2=(y+1)^2$$

Volume of solid of revolution

$$V=\pi\int_0^2\left((y+1)^2-\frac{4}{y^2+4}\right)dy$$

Volume of solid of revolution

$$V = \pi \int_0^2 \left((y+1)^2 - \frac{4}{y^2+4} \right) dy$$

$$= \pi \left[\frac{(y+1)^3}{3} - \frac{4}{2} \tan^{-1} \frac{x}{2} \right]_0^2$$

$$= \pi \left[\left(\frac{((2)+1)^3}{3} - 2\tan^{-1} \frac{(2)}{2} \right) - \left(\frac{((0)+1)^3}{3} - 2\tan^{-1} \frac{(0)}{2} \right) \right]$$

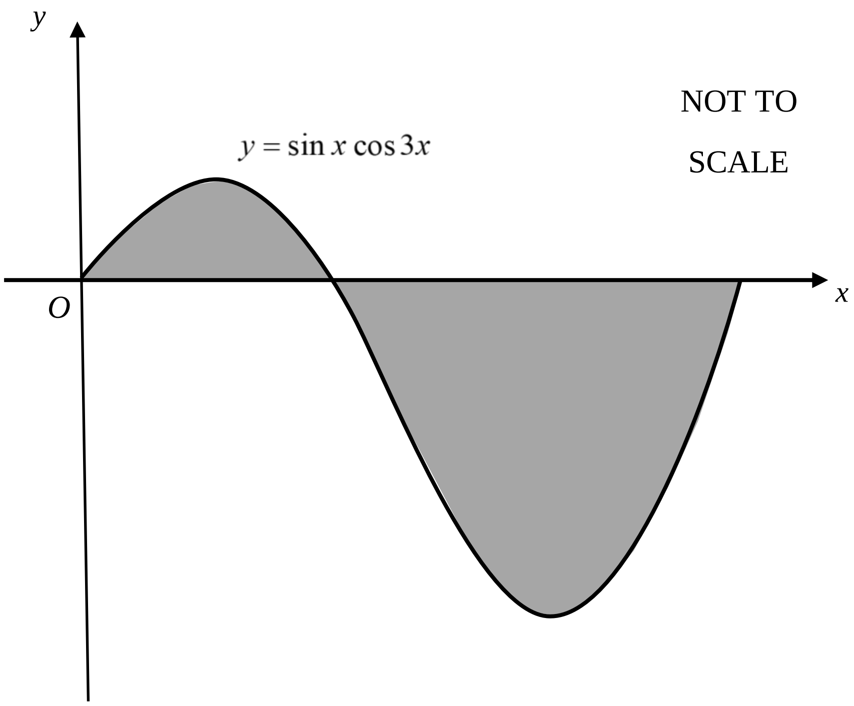
$$= \pi \left[\left(9 - \frac{2\pi}{4} \right) - \left(\frac{1}{3} \right) \right]$$

$$= \frac{\pi(56-3\pi)}{6} units^3$$

Question 8 continues on Page 11

Question 8 (continued)

(c) In the diagram below, a part of the graph of $y = \sin x \cos 3x$ is shown. **4**
The region between the graph and the x-axis has been shaded.



Find the exact area of the shaded region.

To find the first two x-intercepts solve $0 = \sin x \cos 3x$

$0 = \sin x$

$x = 0, \quad \pi, \quad 2\pi ...$

$0 = \cos 3x$

$3x = \frac{\pi}{2}, \quad \frac{3\pi}{2}...$

$x = \frac{\pi}{6}, \quad \frac{\pi}{2}...$

first two positive x-intercepts are $x = \frac{\pi}{6}$ and $x = \frac{\pi}{2}$

Shaded region A is equal to:

$$A=\int_0^{\frac{\pi}{6}} \sin x \cos 3x \, dx \;+\left|\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin x \cos 3x \, dx\right|$$

Using Products to Sums

$$\sin x \cos 3x=\frac{1}{2}\left(\sin\left(x+3x\right)+\sin\left(x-3x\right)\right)$$
$$=\frac{1}{2}\sin\left(4x\right)-\frac{1}{2}\sin\left(2x\right)$$

$$A=\frac{1}{2}\int_0^{\frac{\pi}{6}} \left(\sin 4x-\sin 2x\right)dx \;+\frac{1}{2}\left|\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left(\sin 4x-\sin 2x\right)dx\right|$$

$$=\frac{1}{2}\left[-\frac{\cos 4x}{4}+\frac{\cos 2x}{2}\right]_0^{\frac{\pi}{6}}+\frac{1}{2}\left|\left[-\frac{\cos 4x}{4}+\frac{\cos 2x}{2}\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}\right|$$

$$=\frac{1}{2}\left[\left(-\frac{\cos\frac{2\pi}{3}}{4}+\frac{\cos\frac{\pi}{3}}{2}\right)-\left(-\frac{\cos 0}{4}+\frac{\cos 0}{2}\right)\right]+\frac{1}{2}\left|\left(-\frac{\cos 2\pi}{4}+\frac{\cos \pi}{2}\right)-\left(-\frac{\cos\frac{2\pi}{3}}{4}+\frac{\cos\frac{\pi}{3}}{2}\right)\right|$$

$$=\frac{1}{2}\left[\left(\frac{1}{8}+\frac{1}{4}\right)-\left(-\frac{1}{4}+\frac{1}{2}\right)\right]+\frac{1}{2}\left|\left(-\frac{1}{4}-\frac{1}{2}\right)-\left(\frac{1}{8}+\frac{1}{4}\right)\right|$$

$$=\frac{1}{2}\left[\frac{1}{8}\right]+\frac{1}{2}\left|-\frac{9}{8}\right|$$

$$=\frac{5}{8}$$

Markers comments for Q8

Question 8

(a) Part (i) was not well done. Responses showed a very poor understanding and use of notation revealing confusion between $f^{-1}(x)$, $f'(x)$ and $\dfrac{1}{f(x)}$. Several students stated that the gradient on the inverse is the reciprocal of the gradient at the corresponding point on the original function – but this is the very result that is required to be "shown". Students should note that this is not a quotable result.
Part (ii) was better done and a majority of students were able to make the connection that if $f(2)=7$ then $g(7)=2$. Some students however, attempted to find the equation of the inverse and failed to make progress.
Part (iii) was reasonably well done. Better responses clearly identified the link with the previous two parts and explicitly showed the development of the derivative $f'(x)$.

(b) Well done. It is heartening to note that almost all students knew that they were required to square the radii before subtracting. Students are advised to take greater care when writing algebraic expressions for eg $y=\sqrt{\dfrac{4}{x^2}-4}$ became $y=\sqrt{\dfrac{4}{x^2}}-4$. Careless errors such as $y=x-1$ being rearranged as $x=y-1$ were also not infrequent. In general, the students who got the correct integral went on to successfully find and evaluate the primitive. A small number of students did not recognise the standard integral for $\dfrac{1}{4+y^2}$ resorting to integration by substitution (needless extra work) or the power rule (wrong) and are advised to become more familiar with all the standard integrals on the reference sheet. Also, as is common with volume questions, a small number of students attempted to find the volume by rotating the region about the *x*-axis but often did not do that consistently. Students must always take extra care in volumes questions to note the axis of rotation, before commencing working on the question.

(c) Reasonably well done. This was an unscaffolded question so students are advised to first plan how to proceed.
Almost all students knew that they needed to find the *x*-intercepts ie they had to solve the equation $\sin x\cos 3x=0$. This should have been straightforward but proved quite challenging for many. The temptation to expand $\cos 3x$ using compound angle results was strong or to use the products to sums results (this is useful later in the integration step but complicates the equation solving step). It is disappointing to note that a large number of students got the incorrect values for the first three intercepts.
Following this the integrals for the areas needed to be set up, taking into account that the areas under the *x*-axis would yield a negative integral. Most dealt with this aspect successfully.
Integrating $\sin x\cos 3x$ is best done by using the products to sums results to rewrite it as $\dfrac{1}{2}(\sin 4x-\sin 2x)$, however, using the compound angle results to expand $\cos 3x$ results

in $4\cos^3 x\sin x - 3\cos x\sin x$ or $\frac{1}{2}\sin 2x\cos 2x - 2\sin^3 x\cos x$ (or a couple of other variants) all of which are able to be integrated using the reverse chain rule.
Successful completion of the question then required obtaining the correct primitive (including signs), followed by correct substitution and recall of exact values and painstaking care when simplifying the numerical expression accurately.
Many students knew what to do for this question; few were careful and diligent enough to maintain accuracy throughout to obtain the correct final answer.

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