NORTH
SYDNEY
GIRLS HIGH
SCHOOL

2020 | Preliminary Assessment Task 2

Preliminary Mathematics Extension

General Instructions

- Reading Time – 2 minutes
- Working Time – 55 minutes
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided
- In Questions 6 – 8, show relevant mathematical reasoning and/or calculations

Total marks – 41

Section I – 5 marks (pages 2 – 3)

- Attempt Questions 1 – 5
- Allow about 7 minutes for this section

Section II – 36 marks (pages 4 – 6)

- Attempt Questions 6 – 8
- Allow about 48 minutes for this section

NAME:______________________________ TEACHER:____________________

Question	1 – 5	6	7	8	Total
Further Algebra and Functions	/2	/5	/13		/20
Further Trig	/3	/7		/11	/21
	/5	/12	/13	/11	/41

Section I

5 marks
Attempt Questions 1- 5
Allow about 7 minutes for this section

Use the multiple-choice answer sheet for Questions 1- 5

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1. Which expression is equivalent to $\cos(x+80°)$?

(A) $\sin x\cos 80° + \cos x\cos 80°$

(B) $\sin x\cos 80° - \cos x\cos 80°$

(C) $\cos x\cos 80° + \sin x\sin 80°$

(D) $\cos x\cos 80° - \sin x\sin 80°$

2. If $\sin\alpha = \dfrac{5}{13}$, what are the possible values for $\sin 2\alpha$?

(A) $\dfrac{10}{13}$

(B) $\pm\dfrac{10}{13}$

(C) $\dfrac{120}{169}$

(D) $\pm\dfrac{120}{169}$

3. Which of the following describes the range of the function $f(x) = 5\sin x + 12\cos x$?

(A) $f(x) \leq 13$

(B) $f(x) \leq 17$

(C) $|f(x)| \leq 13$

(D) $|f(x)| \leq 17$

4. Which of the following inequations is NOT equivalent to the inequation $|3x-1| > 4$?

(A) $|1-3x| > 4$

(B) $3x-1 > \pm 4$

(C) $(3x-1)^2 > 16$

(D) $\dfrac{1}{|3x-1|} < \dfrac{1}{4}$

5. Consider the parametric equations:

$$x = \frac{1-t^2}{1+t^2}$$
$$y = \frac{2t}{1+t^2}$$

These equations describe:

(A) a straight line

(B) a parabola

(C) a circle

(D) a hyperbola

Section II

36 marks
Attempt Questions 6-8
Allow about 48 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In questions 6-8, your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (12 marks) Use a SEPARATE writing booklet.

(a) Solve for x: $x^3 - 4x \le 0$ **2**

(b) (i) Find, in fully expanded form, the Cartesian equation of the function with parametric equations: **2**

$$x = 2t - 1$$
$$y = 4t^2 + 1$$

(ii) Find the range of this function using interval notation, given that t is restricted to to the interval $(-2, 1)$. **1**

(c) Find a general solution to the equation $\cos 2\theta = \dfrac{\sqrt{3}}{2}$. **2**

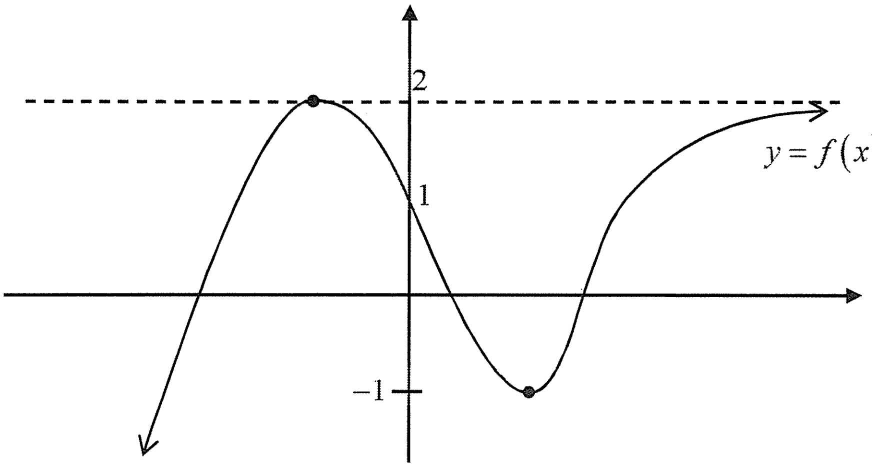
(d) (i) By writing $\cos 3x = \cos(2x + x)$, show that $\cos 3x = 4\cos^3 x - 3\cos x$. **3**

(ii) Hence fully simplify $8\cos^3\left(\dfrac{\pi}{6} - x\right) - 6\cos\left(\dfrac{\pi}{6} - x\right)$. **2**

End of Question 6

Question 7 (13 marks) Use a SEPARATE writing booklet.

(a) Consider the following graph of the function $y = f(x)$:



On the Graph Response Sheet supplied, sketch the graphs of the following relations, showing all important information:

(i) $y = \dfrac{1}{f(x)}$ **2**

(ii) $y = f(|x|)$ **2**

(b) Solve for x: $\dfrac{x}{x+2} \leq 2$ **3**

(c) (i) By using the technique of addition of ordinates, or otherwise, sketch the graph of $y = 2^x + 2^{-x}$, showing the coordinates of any important points. **2**

(ii) Write down the largest domain containing $x = -1$ for which the inverse of this function is a function. **1**

(iii) Show that the equation in part (i) can be written: $2^{2x} - y\left(2^x\right) + 1 = 0$ **1**

(iv) Hence find the equation of the inverse function specified by part (ii). **2**

End of Question 7

Question 8 (11 marks) Use a SEPARATE writing booklet.

(a) Use the product-to-sum results to show that **2**

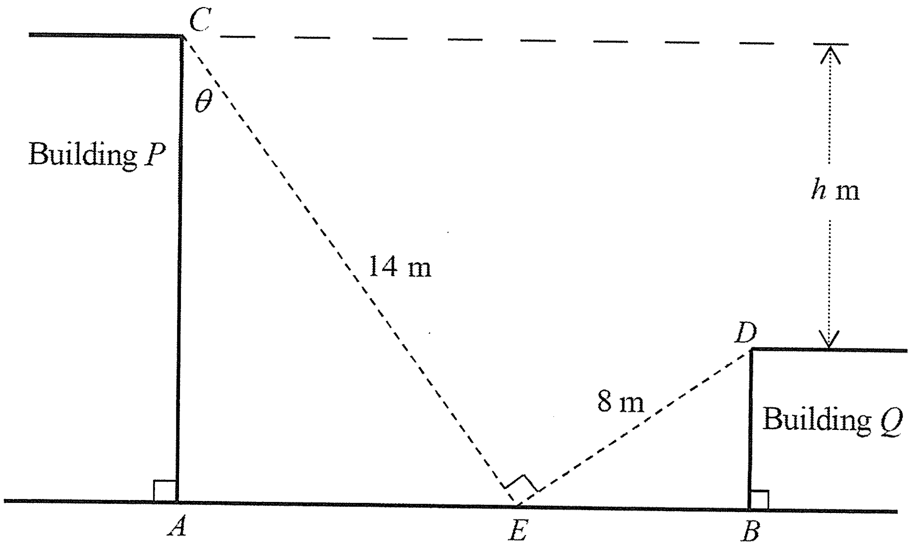
$$4\cos 4x\cos 2x\cos x = \cos 7x + \cos 5x + \cos 3x + \cos x\,.$$

(b) Let $7\cos\theta - 4\sin\theta = R\cos(\theta+\alpha)$.

(i) By expanding the right-hand side and equating coefficients, show that $R=\sqrt{65}$ and $\alpha = 29°45'$ (to the nearest minute). **3**

(ii) Hence solve $7\cos\theta = 4\sin\theta + 2$ for $0° \le \theta \le 360°$. **2**

(c) The diagram shows two buildings on opposite sides of a street, one building h metres taller than the other. A rope of length 14 metres is joined to the top C of the taller building at one end, and to a point E on the road at the other end. Another rope of length 8 metres is tied to the top D of the smaller building at one end, and to the same point E at the other. It is found that $EC \perp ED$. A, B, C, D and E all lie in the same plane. Let $\angle ACE = \theta$.



(i) Explain why $\angle BED = \theta$. **1**

(ii) Form an expression involving θ (and no other pronumerals) for h, the difference in the heights of the buildings. **1**

(iii) If building P is 4 metres taller than building Q, use your expression from part (ii) and the result of part (b) to find the height of the taller building. **2**

End of Examination

2020 Yr 11 2U Test 2 Solutions

Section I Summary: 1. D 2. D 3. C 4. B 5. C

Solutions 2. $\sin 2\alpha = 2\sin\alpha\cos\alpha$

$= 2 \cdot \frac{5}{13} \cdot \left(\pm\frac{12}{13}\right)$

$= \pm\frac{120}{169}$

$\cos^2\alpha = 1 - \sin^2\alpha$

$= 1 - \left(\frac{5}{13}\right)^2$

$= \frac{144}{169}$

$\cos\alpha = \pm\frac{12}{13}$

3. Using aux. angle method, $5\sin x + 12\cos x = R\sin(x+\alpha)$
where $R = \sqrt{5^2+12^2} = 13$
$\therefore -\sqrt{13} \le y \le \sqrt{13}$

4. (A) $|-x| = |x|$ so $|1-3x| = |3x-1|$ ✓

(C) $\sqrt{x^2} = |x|$ so $|3x-1| > 4$

$\sqrt{(3x-1)^2} > 4$

$(3x-1)^2 > 16$ ✓

(D) Provided $c>0$, if $x>c$ then $0 < \frac{1}{x} < \frac{1}{c}$ ✓

(B) ✗ correct solution: $3x-1 < -4$, $3x+1 > 4$

5. $x = \cos\theta$
$y = \sin\theta$ given $t = \tan\frac{\theta}{2}$

These are parametric equations of a circle:

$x^2 + y^2 = \cos^2\theta + \sin^2\theta$

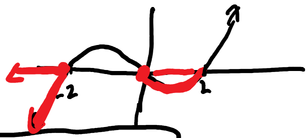
$= 1$

Question 6

(a) Solve for x: $x^3 - 4x \le 0$ **2**

$x(x^2-4) \le 0$

$x(x-2)(x+2) \le 0$



$$\boxed{\begin{array}{c} x \le -2, \quad 0 \le x \le 2 \\ \text{OR} \quad x \in (-\infty,-2] \cup [0,2] \end{array}}$$

(b) (i) Find, in fully expanded form, the Cartesian equation of the function with parametric equations: **2**

$$x = 2t - 1$$
$$y = 4t^2 + 1$$

$x = 2t - 1$

$2t = x + 1$

$t = \dfrac{x+1}{2}$

$y = 4t^2 + 1$

$y = 4\left(\dfrac{x+1}{2}\right)^2 + 1$

$y = 4 \cdot \dfrac{(x+1)^2}{4} + 1$

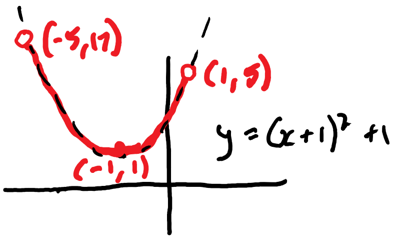
$y = (x+1)^2 + 1$

$\boxed{y = x^2 + 2x + 2}$

(ii) Find the range of this function using interval notation, given that t is restricted to to the interval $(-2, 1)$. **1**

$t = -2 \Rightarrow x = -5$
$\quad y = 17$

$t = 1 \Rightarrow x = 1$
$\quad y = 5$



Range: $1 \le y < 17$

OR $y \in [1, 17)$

(c) Find a general solution to the equation $\cos 2\theta = \dfrac{\sqrt{3}}{2}$. **2**

$2\theta = \dfrac{\pi}{6} + 2k\pi, \ \dfrac{11\pi}{6} + 2k\pi$ where k is an integer

$\theta = \dfrac{\pi}{12} + k\pi, \ \dfrac{11\pi}{12} + k\pi$

(d) (i) By writing $\cos 3x = \cos(2x + x)$, show that $\cos 3x = 4\cos^3 x - 3\cos x$. **3**

$$\begin{aligned}
\cos 3x &= \cos(2x + x) \\
&= \cos 2x \cos x - \sin 2x \sin x \\
&= (\cos^2 x - \sin^2 x)\cos x - 2\sin x \cos x \cdot \sin x \\
&= \cos^3 x - 3\sin^2 x \cos x \\
&= \cos^3 x - 3(1 - \cos^2 x)\cos x \\
&= \cos^3 x - 3\cos x + 3\cos^3 x \\
&= 4\cos^3 x - 3\cos x
\end{aligned}$$

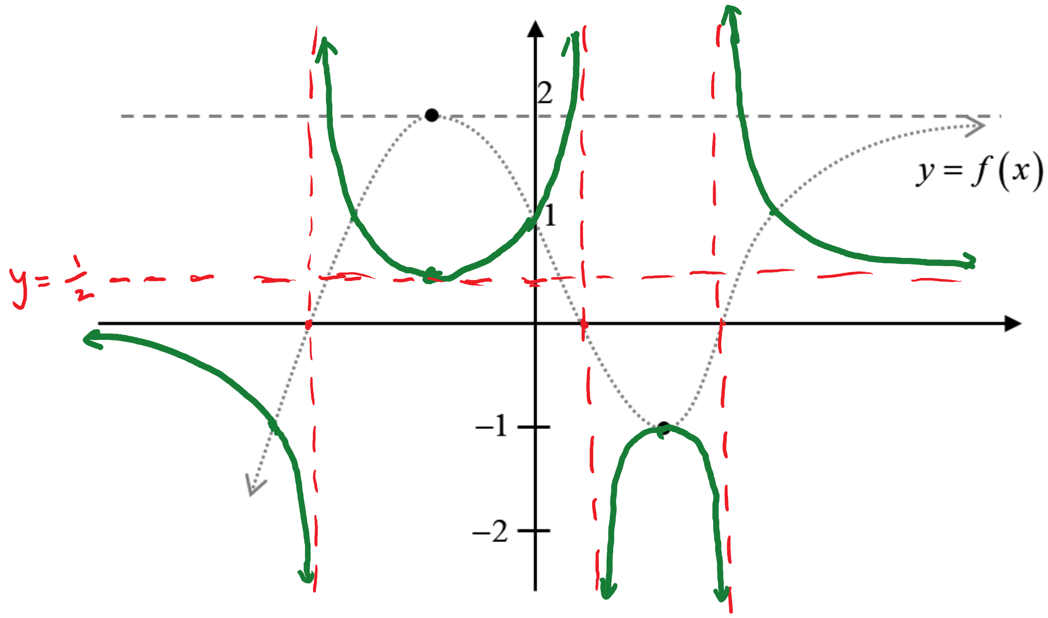
(ii) Hence fully simplify $8\cos^3\left(\frac{\pi}{6} - x\right) - 6\cos\left(\frac{\pi}{6} - x\right)$. **2**

$$\begin{aligned}
&8\cos^3\left(\tfrac{\pi}{6} - x\right) - 6\cos\left(\tfrac{\pi}{6} - x\right) \\
&= 2\left[4\cos^3\left(\tfrac{\pi}{6} - x\right) - 3\cos\left(\tfrac{\pi}{6} - x\right)\right] \\
&= 2\cos\left[3\left(\tfrac{\pi}{6} - x\right)\right] \quad \text{(from part i)} \\
&= 2\cos\left(\tfrac{\pi}{2} - 3x\right) \\
&= 2\sin 3x
\end{aligned}$$

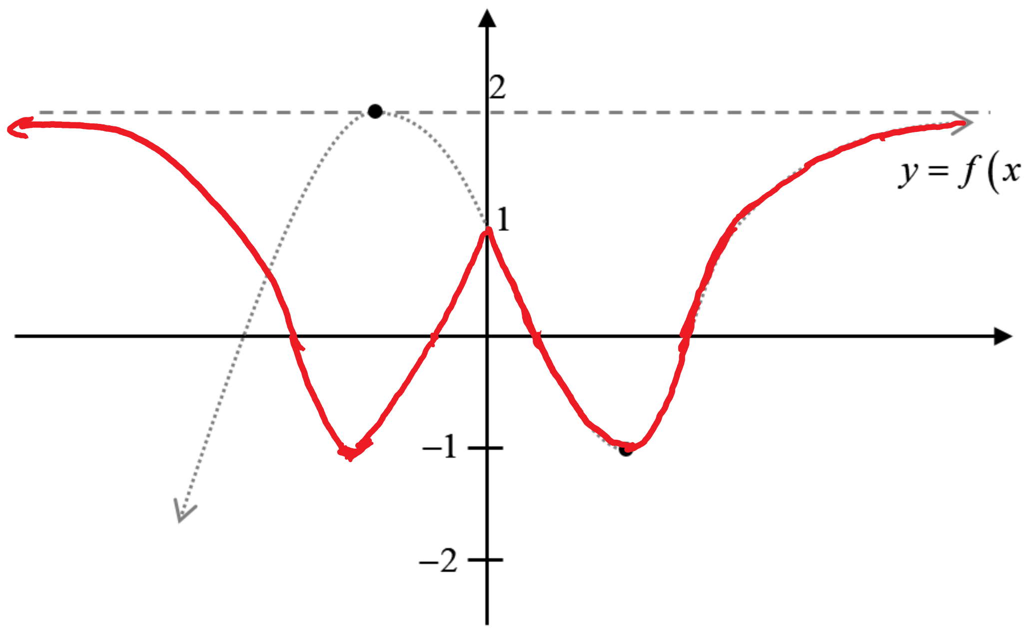
Question 7

(a)

(i) Sketch $y = \frac{1}{f(x)}$. 2



(ii) Sketch $y = f(|x|)$. 2



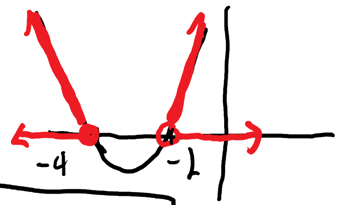
(b) Solve for x: $\dfrac{x}{x+2} \le 2$ **3**

$\times(x+2)^2$: $x(x+2) \le 2(x+2)^2$, $x \ne -2$

$2(x+2)^2 - x(x+2) \geq 0$

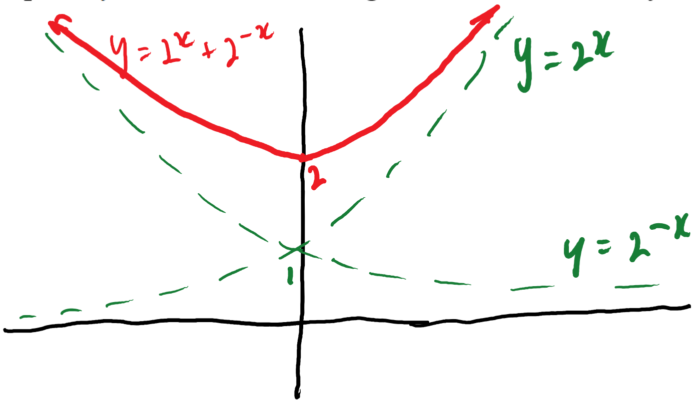
$(x+2)[2(x+2) - x] \geq 0$

$(x+2)(x+4) \geq 0$, $x \ne -2$



$\boxed{x \le -4 \cup x > -2 \quad \text{OR} \quad x \in (-\infty, -4] \cup (2, \infty)}$

(c) (i) By using the technique of addition of ordinates, or otherwise, sketch the graph of $y = 2^x + 2^{-x}$, showing the coordinates of any important points. **2**



(ii) Write down the largest domain containing $x = -1$ for which the inverse of this function is a function. **1**

$x \le 0 \quad \text{OR} \quad x \in (-\infty, 0]$

(iii) Show that the equation in part (i) can be written: $2^{2x} - y\left(2^x\right) + 1 = 0$ **1**

$y = 2^x + 2^{-x}$

$\times 2^x$: $y(2^x) = 2^{2x} + 1$

$2^{2x} - y(2^x) + 1 = 0$

(iv) Hence find the equation of the inverse function specified by part (ii). **2**

$$2^{2x} - y(2^x) + 1 = 0$$

$$2^x = \frac{y \pm \sqrt{y^2 - 4}}{2} \quad \text{(quadratic formula)}$$

for specified domain: $2^x = \dfrac{y - \sqrt{y^2 - 4}}{2}$

$$x = \log_2\left[\frac{y - \sqrt{y^2 - 4}}{2}\right]$$

$\therefore$ Inverse: $y = \log_2\left[\dfrac{y - \sqrt{y^2 - 4}}{2}\right]$

which may be written: $y = \log_2\left(y - \sqrt{y^2 - 4}\right) - 1$

(a) Use the product-to-sum results to show that **2**

$$4\cos 4x\cos 2x\cos x = \cos 7x + \cos 5x + \cos 3x + \cos x\,.$$

$$\begin{aligned}4\cos 4x\cos 2x\cos x &= 2\cos 4x\cos 2x\cdot 2\cos x\\ &= \left[\cos(4x+2x)+\cos(4x-2x)\right]\cdot 2\cos x\\ &= 2\cos 6x\cos x + 2\cos 2x\cos x\\ &= \cos(6x+x)+\cos(6x-x) + \cos(2x+x)+\cos(2x-x)\\ &= \cos 7x + \cos 5x + \cos 3x + \cos x\end{aligned}$$

(b) Let $7\cos\theta - 4\sin\theta = R\cos(\theta+\alpha)$.

(i) By expanding the right-hand side and equating coefficients, show that $R=\sqrt{65}$ and $\alpha = 29°45'$ (to the nearest minute). **3**

$$\begin{aligned}7\cos\theta - 4\sin\theta &= R\cos(\theta+\alpha)\\ &= R(\cos\theta\cos\alpha - \sin\theta\sin\alpha)\\ &= (R\cos\alpha)\cos\theta - (R\sin\alpha)\sin\theta\end{aligned}$$

Equating coefficients of $\cos\theta$ & $\sin\theta$:

$R\cos\alpha = 7$ — ①

$R\sin\alpha = 4$ — ②

①^2 + ②^2:
$$\begin{aligned}R^2\cos^2\alpha + R^2\sin^2\alpha &= 7^2+4^2\\ R^2(\cos^2\alpha+\sin^2\alpha) &= 65\\ R^2 &= 65\\ R &= \sqrt{65}\end{aligned}$$

② ÷ ①: $\tan\alpha = \frac{4}{7}$

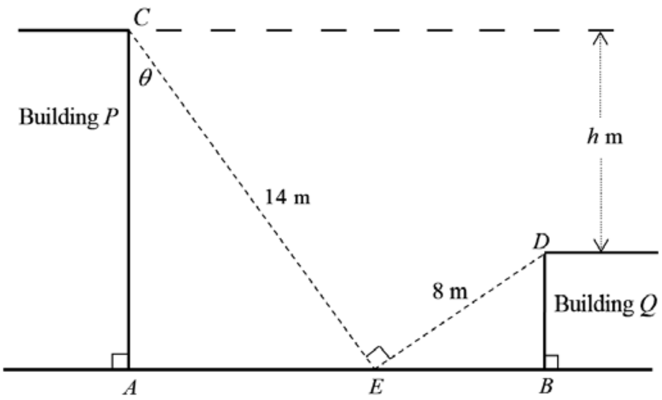
(α in 1st quadrant as $\sin\alpha>0$, $\cos\alpha>0$)

$$\begin{aligned}\alpha &= \tan^{-1}\frac{4}{7}\\ &= 29°45'\end{aligned}$$

(ii) Hence solve $7\cos\theta = 4\sin\theta + 2$ for $0° \le \theta \le 360°$. **2**

$$\begin{aligned}7\cos\theta - 4\sin\theta &= 2\\ \sqrt{65}\cos(\theta + 29°45') &= 2\\ \cos(\theta+29°45') &= \frac{2}{\sqrt{65}}\\ \theta + 29°45' &= \cos^{-1}\frac{2}{\sqrt{65}},\ 360 - \cos^{-1}\frac{2}{\sqrt{65}}\\ &= 75.64°,\ 284.36°\\ \theta &= 45°53',\ 254°37'\end{aligned}$$

(c) The diagram shows two buildings on opposite sides of a street, one building h metres taller than the other. A rope of length 14 metres is joined to the top C of the taller building at one end, and to a point E on the road at the other end. Another rope of length 8 metres is tied to the top D of the smaller building at one end, and to the same point E at the other. It is found that $EC \perp ED$. A, B, C, D and E all lie in the same plane. Let $\angle ACE = \theta$.



(i) Explain why $\angle BED = \theta$. **1**

$A\hat{E}C = 180 - 90 - A\hat{C}E$ (angle sum of $\triangle$)

$= 90 - \theta$

$B\hat{E}D + D\hat{E}C + A\hat{E}C = 180^\circ$ (angles on straight line)

$B\hat{E}D + 90 + 90 - \theta = 180$

$B\hat{E}D = \theta$

(ii) Form an expression involving θ (and no other pronumerals) for h, the difference in the heights of the buildings. **1**

$h = AC - DB$

$= 14\cos\theta - 8\sin\theta$

(iii) If building P is 4 metres taller than building Q, use your expression from part (ii) and the result of part (b) to find the height of the taller building. **2**

$h = 4$

$14\cos\theta - 8\sin\theta = 4$

$7\cos\theta - 4\sin\theta = 2$

From (b)(ii), $\theta = 45^\circ 53',\ 254^\circ 37'$

but $\theta < 90^\circ$, so $\theta = 45^\circ 53'$

height of taller building $= 14\cos 45^\circ 53'$

$= 9.7\text{ m}$