NORTH
SYDNEY
GIRLS HIGH
SCHOOL

2021	Preliminary Assessment Task 2

Preliminary Mathematics Extension

General Instructions

- Reading Time – 2 minutes
- Working Time – 55 minutes
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided
- In Questions 5 – 7, show relevant mathematical reasoning and/or calculations

Total marks – 43

Section I – 4 marks (pages 2 – 4)

- Attempt Questions 1 – 4
- Allow about 7 minutes for this section

Section II – 39 marks (pages 5 – 7)

- Attempt Questions 5 – 7
- Allow about 48 minutes for this section

NAME:______________________________ TEACHER:____________________

Question	1 – 4	5	6	7	Total
Further Algebra	/1	/5	/2	/3	**/11**
Further Functions	/1		/8	/4	**/13**
Further Trig	/2	/9	/4	/4	**/19**
	/4	**/14**	**/14**	**/11**	**/43**

Section I

4 marks
Attempt Questions 1- 4
Allow about 7 minutes for this section

Use the multiple-choice answer sheet for Questions 1-4

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1 What is the solution of the inequation $\dfrac{4}{|x+2|} < 2$?

A. $(-4,-2)\cup(-2,0)$

B. $[-\infty,-4,)\cup(0,\infty]$

C. $[-4,-2)\cup(-2,0]$

D. $(-\infty,-4,)\cup(0,\infty)$

2 If $f(x)=\dfrac{x}{x-2}$, which of the following is the range of $f^{-1}(x)$?

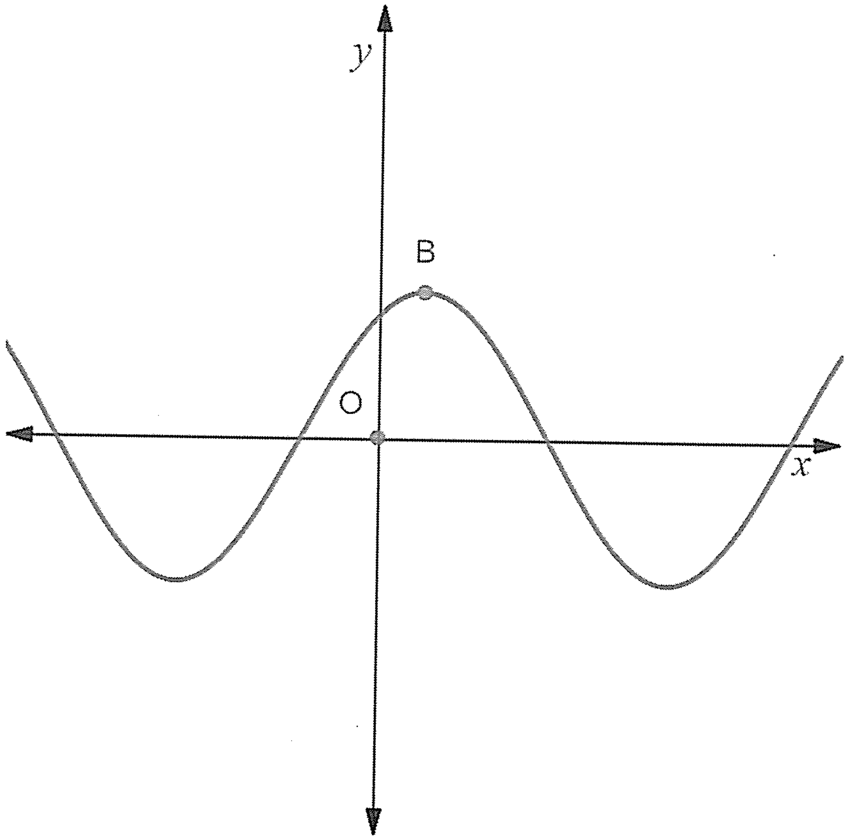
A. $y\leq 2$

B. All real y, $y\neq 0$

C. All real y, $y\neq 2$

D. $y\geq 2$

3 A part of the graph of the function $f(x) = 2\sin x + 3\cos x$ is drawn below, but only the origin and the point B, a maximum turning point, have been labelled.



What is the y value of the point B?

A. 5

B. $\sqrt{13}$

C. $\sqrt{7}$

D. 13

4 Given that $t = \tan\frac{\theta}{2}$, which of the following is an expression for $\operatorname{cosec}2\theta$?

A. $\dfrac{(1+t^2)^2}{t^4-6t^2+1}$

B. $\dfrac{t^4-6t^2+1}{(1+t^2)^2}$

C. $\dfrac{(1+t^2)^2}{4t(1-t^2)}$

D. $\dfrac{4t(1-t^2)}{(1+t^2)^2}$

Section II

39 marks
Attempt Questions 5-7
Allow about 48 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In questions 5-7, your responses should include relevant mathematical reasoning and/or calculations.

Question 5 (14 marks) Use a SEPARATE writing booklet.

(a) Solve $\dfrac{2x}{x+2} \le 4$. **3**

(b) Solve $|2x-3| < 5$. **2**

(c) By using a compound angle result, show that the exact value of $\sin 105^\circ$ is $\dfrac{\sqrt{6}+\sqrt{2}}{4}$. **2**

(d) Let $\sqrt{3}\cos x - \sin x \equiv R\cos(x+\alpha)$, where $R > 0$ and $0 \le \alpha \le \dfrac{\pi}{2}$.

(i) By expanding the right-hand side and equating coefficients of $\sin x$ and $\cos x$, find the values of R and α. **2**

(ii) Hence, solve $\sqrt{3}\cos x - \sin x = 0$ for $0 \le x \le 2\pi$. **2**

(e) Solve $\sin 2x - \cos x = 0$ for $0 \le x \le 2\pi$. **3**

End of Question 5

Question 6 (14 marks) Use a SEPARATE writing booklet.

(a) Solve $x(x-2)^2(x+1) \geq 0$. **2**

(b) Consider the function $f(x) = \sqrt{4-x}$.

(i) Find $f^{-1}(x)$ and state its domain and range. **3**

(ii) Hence solve $f(x) = f^{-1}(x)$. **2**

(c) A relation R has parametric equations $x = 3 + 5\cos\theta$ and $y = 2 + 5\sin\theta$.

(i) Find the Cartesian equation of R. **2**

(ii) Describe geometrically the relation R, mentioning any important features. **1**

(d) By using the substitution $t = \tan\frac{x}{2}$, solve $\sin x + \cos x = -1$ for $0° \leq x \leq 360°$. **4**

End of Question 6

Question 7 (11 marks) Use a SEPARATE writing booklet.

(a) (i) Square both sides of the equation $\sqrt{x}-\sqrt{y}=\sqrt{7-\sqrt{40}}$ leaving all terms involving x and y on the left hand side. **1**

(ii) Hence find the values of x and y in the equation $\sqrt{x}-\sqrt{y}=\sqrt{7-\sqrt{40}}$ given that $x > y$. **2**

(b) The acute angles α and β satisfy the relationships $7\cot^2\alpha + 6\cot\alpha = 1$ and $\tan\beta = 3$.

(i) Determine the value of $\tan\alpha$. **2**

(ii) Show that $\tan(\alpha+\beta) = -\frac{1}{2}$. **2**

(c) A bush walker is travelling along a path described by the parametric equations $x = t - 2$ and $y = -2t + 9$.
At the same time, a lion is strolling on a course described by the parametric equations $x = \frac{t}{2}, y = t + 1$, where t is the time elapsed in hours after they start.

(i) Determine where the pathways of the lion and bush walker intersect. **2**

(ii) Do the bush walker and the lion meet up with each other. Justify your answer. **2**

End of paper

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YEAR 11 Multiple Choice task 2 ext 1 2021

Q1/ $\dfrac{4}{|x+2|} < 2 \;,\; x \neq -2$

$4 < 2|x+2|$

$4 < 2x+4$ OR $4 < -2x-4$

$0 < x$ $\quad$ $8 < -2x$

$x > 0$ $\quad$ $-4 > x$

$(0, \infty)$ $\quad$ $x < -4$

$(-\infty, -4)$

(D)

Q2/ $f(x) = \dfrac{x}{x-2}$ $\quad$ D: all real x, $x \neq 2$

$\therefore f^{-1}(x)$ Range: all real y, $y \neq 2$

(C)

Q3/ $R = \sqrt{3^2 + 2^2}$

$= \sqrt{13}$

(B)

Q4/ $\operatorname{cosec} 2\theta = \dfrac{1}{\sin 2\theta}$

$= \dfrac{1}{2\sin\theta\cos\theta}$

$2\sin\theta\cos\theta = \dfrac{4t}{1+t^2} \times \dfrac{1-t^2}{1+t^2}$

(C)

$\therefore \operatorname{cosec} 2\theta = \dfrac{(1+t^2)^2}{4t(1-t^2)}$

2021 Ext 1 Yr 11 task 2

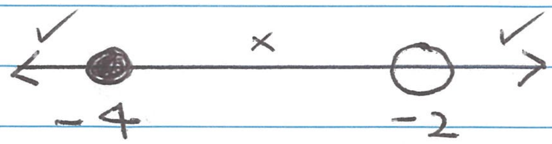
Q5) (a) $\dfrac{2x}{x+2} \leq 4$, $x \neq -2$

Check equality

$2x = 4x + 8$

$-8 = 2x$

$x = -4$



$\therefore$ $x > -2$ OR $x \leq -4$

OR $(-\infty, -4] \cup (-2, \infty)$

(b) $|2x - 3| < 5$

$2x - 3 < 5$ OR $-2x + 3 < 5$

$2x < 8$ $\quad$ $-2x < 2$

$x < 4$ OR $x > -1$

alternatively

$(-1, 4)$

(c) $\sin 105° = \sin(60 + 45)$

$= \sin 60 \cos 45 + \cos 60 \sin 45$

$= \dfrac{\sqrt{3}}{2} \times \dfrac{1}{\sqrt{2}} + \dfrac{1}{2} \times \dfrac{1}{\sqrt{2}}$

$= \dfrac{\sqrt{3} + 1}{2\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}}$

$= \dfrac{\sqrt{6} + \sqrt{2}}{4}$

(d)(i) $\sqrt{3}\cos x - \sin x \equiv R\cos x\cos\alpha - R\sin x\sin\alpha$

$\therefore \quad R\cos\alpha = \sqrt{3} \quad$ and $\quad R\sin\alpha = 1$

$\therefore \quad R^2\cos^2\alpha = 3 \quad\quad R^2\sin^2\alpha = 1$

$$R^2\sin^2\alpha + R^2\cos^2\alpha = 1 + 3$$
$$R^2(\sin^2 + \cos^2\alpha) = 4$$
$$R^2 = 4$$
$$R = 2$$

$$\frac{R\sin\alpha}{R\cos\alpha} = \frac{1}{\sqrt{3}}$$
$$\tan\alpha = \frac{1}{\sqrt{3}}$$
$$\alpha = \frac{\pi}{6}$$

(ii) $2\cos\left(x+\frac{\pi}{6}\right) = 0 \quad\quad \frac{\pi}{6} \leq x+\frac{\pi}{6} \leq 2\pi+\frac{\pi}{6}$

$$\cos\left(x+\frac{\pi}{6}\right) = 0$$
$$x+\frac{\pi}{6} = \frac{\pi}{2}, \frac{3\pi}{2}$$
$$x = \frac{\pi}{3}, \frac{4\pi}{3}$$

(e) $2\sin x\cos x - \cos x = 0 \quad , 0 \leq x \leq 2\pi$

$$\cos x(2\sin x - 1) = 0$$

$\cos x = 0 \quad$ OR $\quad \sin x = \frac{1}{2}$

$x = \frac{\pi}{2}, \frac{3\pi}{2} \quad\quad x = \frac{\pi}{6}, \frac{5\pi}{6}$

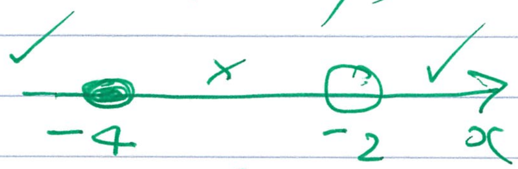
$$\therefore \quad x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$$

Q1/ $\dfrac{2x}{x+2} \leq 4 \qquad x \neq -2$

$2x = 4x + 8$ (solve equality)

$-8 = 2x$

$-4 = x$



$x > -2$ OR $x \leq -4$

(D)

Q2/ $f(x) = \dfrac{2x-1}{x+2}$

For $f^{-1}(x)$ $x = \dfrac{2y-1}{y+2}$

$xy + 2x = 2y - 1$

$2x + 1 = 2y - xy$

$2x + 1 = y(2-x)$

$\dfrac{2x+1}{2-x} = y$

$\therefore \quad f^{-1}(x) = \dfrac{2x+1}{2-x}$

(D)

Q3/ $\operatorname{cosec} 2\theta = \dfrac{1}{2\sin\theta\cos\theta}$

$= \dfrac{1}{\dfrac{2(2t)}{1+t^2} \times \dfrac{(1-t^2)}{1+t^2}}$

$= \dfrac{(1+t^2)^2}{4t(1-t^2)}$

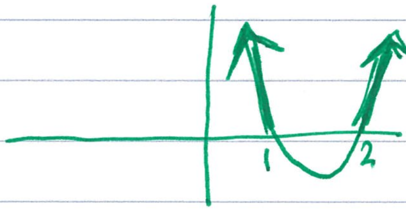
(C)

4/. Period is π

$\therefore$ one to one is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (C)

Q5/(a) $x^2 - 3x + 2 \geq 0$

$(x-2)(x-1) \geq 0$



$\therefore$ $x \geq 2$ OR $x \leq 1$

(b) $\sqrt{x+5} = 7 - x$, $x \geq -5$

$x + 5 = 49 - 14x + x^2$

$0 = x^2 - 15x - 44$

$0 = (x-4)(x-11)$

$x = 4$ OR $x = 11$

check

$\sqrt{9} = 7 - 4$

$3 = 3$

TRUE

check

$\sqrt{11+5} = 7 - 11$

$4 \neq -4$

not a solution

$\therefore$ $x = 4$

(c) $3^{2x-3} \cdot 2^{2x-3} \cdot 2^{-2x+2} = 3^{\frac{1}{2}} \cdot 2^{-1}$

$3^{2x-3} \cdot \cancel{2^{-1}} = 3^{\frac{1}{2}} \cdot \cancel{2^{-1}}$

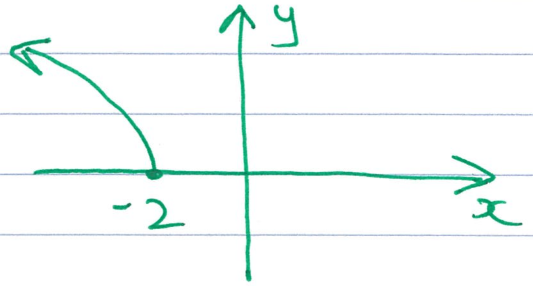
$2x - 3 = \frac{1}{2}$

$2x = \frac{7}{2}$

$x = \frac{7}{4}$

(d) $(-\infty, 2) \cup (2, \infty)$

(e)



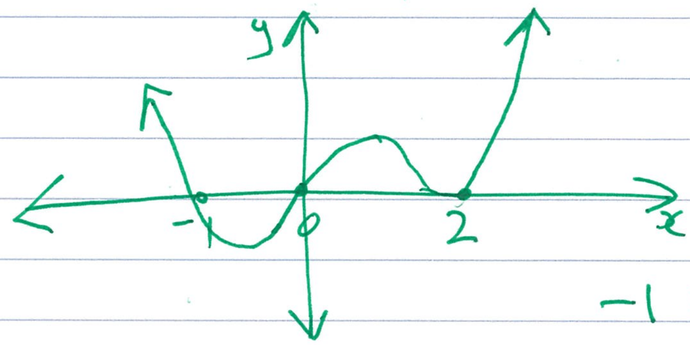
(f) $4(\cos x \sin x)^2$

$= 4\left(\frac{1}{2}\sin 2x\right)^2$

$= 4 \times \frac{1}{4}\sin 2x$

$= \sin 2x$

Q6/(a)



$-1 \leq x \leq 0$

(b) $\sqrt{x} - \sqrt{y} = \sqrt{7 - \sqrt{40}}$

$x - 2\sqrt{xy} + y = 7 - \sqrt{40}$

$= 7 - 2\sqrt{10}$

$\therefore$ $x + y = 7$ and $xy = 10$

$\therefore$ $x = 5$, $y = 2$ by inspection or solving.

(c) $\frac{x-3}{5} = \cos\theta$ $\quad\quad$ $\frac{y-2}{5} = \sin\theta$

(i) $\frac{(x-3)^2}{25} + \frac{(y-2)^2}{25} = \sin^2\theta + \cos^2\theta$

$\therefore (x-3)^2 + (y-2)^2 = 25$

(ii) Circle $\quad$ centre $(3,2)$ $\quad$ radius $= 5$

(d) $\cos 105 = \cos 60 \cos 45 - \sin 60 \sin 45$

$$= \frac{1}{2} \times \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}$$

$$= \frac{1-\sqrt{3}}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{\sqrt{2}-\sqrt{6}}{4}$$

(e) $f(x) = 2\sin\left(\frac{\pi}{2} - 3x\right)$

$$= 2\cos 3x$$

$f(-x) = 2\cos(-3x)$

$$= 2\cos 3x$$

$\therefore f(x) = f(-x)$ $\quad$ So it's an odd function

(f) $\sin\left[(a+b)-(a-b)\right]$

$= \sin\left[a+b-a+b\right]$

$= \sin 2b$

Q7 (a)(i) $x+2=t$

$y=-2(x+2)+9$
$y=-2x-4+9$
$y=-2x+5$ (Cartesian equation of bushwalker)

$t=2x$ $\quad y=2x+1$
(cartesian equation of lion)

$2x+1=-2x+5$
$4x=4$
$x=1$
$y=3$ $\quad \therefore$ paths meet at $(1,3)$

(ii) When bushwalker is at $(1,3)$ $t=3$
When lion is at $(1,3)$ $t=2$

$\therefore$ They don't meet.

(b) $\sqrt{3}\cos x-\sin x \equiv R\cos x\cos\alpha+R\sin x\sin\alpha$

$\therefore \left.\begin{array}{l} R\sin\alpha=1 \\ R\cos\alpha=\sqrt{3}\end{array}\right\}$ Divide

$\left.\begin{array}{l} R^2\sin^2\alpha=1 \\ R^2\cos^2\alpha=3\end{array}\right\}$ ADD

$\tan\alpha=\frac{1}{\sqrt{3}}$

$\alpha=\frac{\pi}{6}$

$R^2=4$
$R=2$ (as $R>0$)

$\therefore \sqrt{3}\cos x - \sin x = 2\cos\left(x+\frac{\pi}{6}\right)$

(ii) As $R = 2$ minimum value is -2

$$\therefore 2\cos\left(x+\frac{\pi}{6}\right) = -2$$

$$\cos\left(x+\frac{\pi}{6}\right) = -1$$

$$x+\frac{\pi}{6} = \pi$$

$$x = \pi - \frac{\pi}{6}$$

$$x = \frac{5\pi}{6}$$

(c)(i) $x^3 - x^2 + x - 1 = x^2(x-1) + 1(x-1)$

$$= (x-1)(x^2+1)$$

(ii) $\text{LHS} = \dfrac{\frac{1}{\sin\theta} - \sin\theta}{\frac{1}{\cos\theta} - \cos\theta}$

$$= \frac{1-\sin^2\theta}{\sin\theta} \div \frac{1-\cos^2\theta}{\cos\theta}$$

$$= \frac{\cos^2\theta}{\sin\theta} \times \frac{\cos\theta}{\sin^2\theta}$$

$$= \frac{\cos^3\theta}{\sin^3\theta}$$

$$= \cot^3\theta$$

(iii) $\cot^3\theta + \cot\theta = 1 + \cot^2\theta$

$$\cot^3\theta - \cot^2\theta + \cot\theta - 1 = 0$$

Using the part (i)

$$\cot\theta = x$$

$$(\cot^2\theta + 1)(\cot\theta - 1) = 0$$

$\cot^2\theta = -1$

no solutions

$\cot\theta = 1$

$\therefore \tan\theta = 1$

$\theta = \frac{\pi}{4}, \frac{5\pi}{4}$