2022

PRELIMINARY ASSESSMENT TASK 1

Extension 1 Mathematics

General Instructions

- Reading time – 10 minutes
- Working time – 90 minutes
- Write using blue or black pen Black pen is preferred
- For questions in Section II, show relevant mathematical reasoning and/or calculations.
- Calculators approved by NESA may be used
- DO NOT USE LIQUID PAPER OR CORRECTION TAPE

Total Marks – 60

Section I: 10 marks

- Attempt questions 1-10, using the answer sheet provided.
- Allow about 15 minutes for this section

Section II: 50 marks

- Attempt questions 11-14, answer the questions in the booklets provided.
- Allow about 75 minutes for this section

	M/C	11	12	13	14	TOTAL
Inequalities	2, 3	b (3), e (3)	b (3)	d (3)	e (4)	18
Inverse Functions	5, 10			a (4), b (2), c (2), e (2)	a (8)	20
Introduction to Functions	1, 4, 6					3
Linear, Quadratic and Cubic Functions	7	a (2), c (2)	c (3), d (3)			11
Further functions and relations	8, 9	d (3)	a (3)			8
TOTAL	10	13	12	13	12	60

STUDENT NUMBER: ______________________________ TEACHER:_______________

Section I

10 marks
Attempt Questions 1 to 10
Use the multiple-choice answer sheet for Questions 1 to 10

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1. Which of the following represents the domain of $f(x) = \sqrt{6 - 2x}$?

(A) $(-\infty, 6]$

(B) $[3, \infty)$

(C) $(3, \infty)$

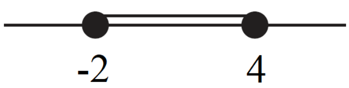
(D) $(-\infty, 3]$

2. Which diagram best represents the solution set of $x^2 - 2x - 8 \leq 0$?

(A)

(B)

(C)



(D)

3. Which set provides a solution to the inequality $|3x - 2| \leq |x + 4|$?

(A) $\left[-\frac{1}{2}, 3\right]$

(B) $\left(-\infty, -\frac{1}{2}\right] \cup [3, \infty)$

(C) $\left(-\infty, -\frac{1}{2}\right]$

(D) $(-\infty, 3]$

4. If $f(x) = 2x - 3$, then $f^{-1}(x) =$

(A) $\frac{1}{2}x + 3$

(B) $\frac{1}{2}x + \frac{3}{2}$

(C) $\frac{1}{2x - 3}$

(D) $\frac{1}{2}x - 3$

5. Which of the following functions does not have an inverse function?

(A) $f(x) = (x - 2)^2, D: [0, \infty)$

(B) $f(x) = x^3, D: (-\infty, \infty)$

(C) $f(x) = \sqrt{9 - x}, D: [-3, 3]$

(D) $f(x) = 3x + 7, D: (-\infty, \infty)$

6. The linear function $f(x) = 5 - x$ has range $[-2, 3)$. Which of the following represents its domain?

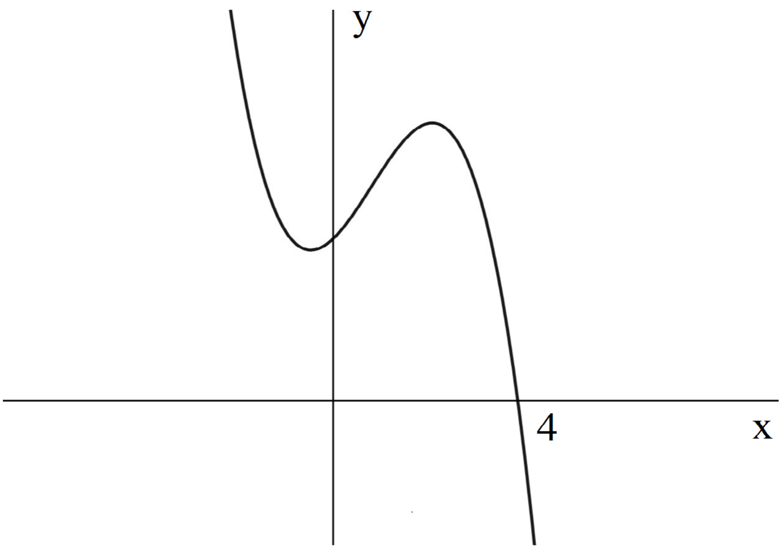
(A) $[-7, 2)$

(B) $(2, 7]$

(C) $[-2, 7)$

(D) $[2, 7)$

7. The graph of the polynomial function is shown below.



Which of the following could represent the graph above?

(A) $y = (x - 4)(x^2 + x + 1)$

(B) $y = (x + 4)(x^2 + x + 1)$

(C) $y = (4 - x)(x^2 + x + 1)$

(D) $y = (x - 4)^3$

8. Given the functions $f(x) = 4x + 10$ and $g(x) = \sqrt{x + 2}$, what is the value of a for which $f\big(g(a)\big) = 16$?

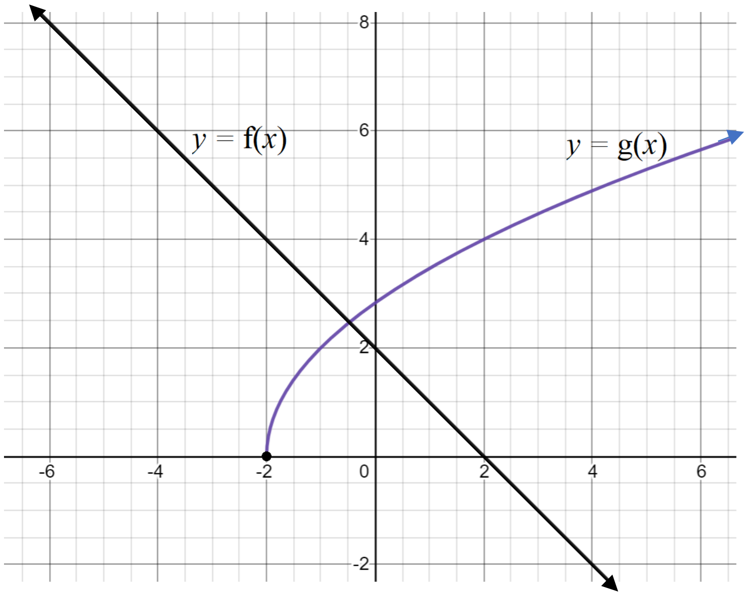
(A) $\dfrac{161}{4}$

(B) $-\dfrac{1}{2}$

(C) $\dfrac{1}{4}$

(D) $\dfrac{9}{2}$

9. The graph below shows the functions for $y = f(x)$ and $y = g(x)$. What are the domain and range of $y = f(x) - g(x)$?



(A) Domain: $[-2, \infty)$
Range: $(-\infty, \infty)$

(B) Domain: $(-\infty, \infty)$
Range: $(-\infty, -2]$

(C) Domain: $(-\infty, -2]$
Range: $(-\infty, 4]$

(D) Domain: $[-2, \infty)$
Range: $(-\infty, 4]$

10. Given $f(x) = 1 + \frac{2}{x-3}$. Which of the following gives the equations of the horizontal and vertical asymptotes of $f^{-1}(x)$?

(A) Vertical asymptote is $x = 1$ and horizontal asymptote is $y = 2$

(B) Vertical asymptote is $x = 1$ and horizontal asymptote is $y = 3$

(C) Vertical asymptote is $x = 3$ and horizontal asymptote is $y = 1$

(D) Vertical asymptote is $x = 3$ and horizontal asymptote is $y = 2$

End of Section I

Section II

50 marks
Attempt Questions 11 to 14

Answer each question in a SEPARATE writing booklet.
Extra writing booklets are available.
All necessary working should be shown in every question.

Question 11 (13 marks) Use a SEPARATE writing booklet.

(a) The coordinates of the turning point of a parabola are $(2, 6)$ and the parabola passes through the point $(3, 3)$. Find the rule for this parabola. **2**

(b) Solve for x. **3**

$$\frac{4x-3}{2} - \frac{3x-3}{3} < 3$$

(c) For what values of k does $x^2 + (k+1)x + (k-2) = 0$ have real roots? **2**

(d) Solve the following absolute value equation: **3**

$$2|x+3| = x - 2$$

(e) Sketch the graphs $y = x^2 - 4$ and $y = \frac{1}{2}|x| - 1$ on the same number plane and write down the inequality that describes the area bounded by the two curves. **3**

End of question 11

Question 12 (12 marks) Use a SEPARATE writing booklet.

(a) Describe accurately the relation below. **3**

$x^2 - 8x + y^2 - 14y + 1 = 0$

(b) Solve the inequality and draw the solutions on a number line. **3**

$\frac{6}{5x - 2} < 2$

(c) A group of students on a hiking trip decide to travel x km by bus at an average speed of 48 km/h to an unknown destination. They then plan to walk back along the same route at an average speed of 4.8 km/h and to arrive back 24 hours after setting out in the bus. If they allow for a total of 2 hours for lunch and rest, how far must the bus take them? **3**

(d) Sketch $y = 4x^3 + 3x^2 - 16x - 12$ by first factorising. **3**
Label all important features.

End of question 12

Question 13 (13 marks) Use a SEPARATE writing booklet.

(a) i. Find the inverse of the function with the rule $f(x) = \sqrt{x-2} + 4$. **2**

ii. Sketch both functions on the same number plane and hence find the point of intersection/s. **2**

(b) Consider the function $f(x) = \frac{4}{x+2}$.

Show that $f^{-1}(x) = \frac{4-2x}{x}$. **2**

(c) Given $f(x) = x^2 - 9$ is a function defined for $x \leq 0$, verify that $f^{-1}\big(f(x)\big) = x$. **2**

(d) Graph the region $x^2 + y^2 \leq 25$, for $x \leq 4$ and $y > -3$. State the domain and range of the region bounded by the inequalities. **3**

(e) What is the largest positive domain, so that $y = x^2 - 12x + 40$ has an inverse function? **2**

End of question 13

Question 14 (12 marks) Use a SEPARATE writing booklet.

(a) Consider the function $f(x) = \frac{1}{4}[(x-1)^2 + 7]$

i. Sketch the parabola $y = f(x)$. Clearly showing any intercepts with the axes and the coordinates of its vertex in the domain $[-6, 6]$ **2**

ii. Find the inverse function, $y = f^{-1}(x)$ with a range of $[1, \infty)$. **2**

iii. Find the coordinates of any points of intersection of the two curves $y = f(x)$ and $y = f^{-1}(x)$. **1**

iv. Sketch the graph of $y = f^{-1}(x)$ on the same set of axes as your graph in part i. Label the two graphs clearly. **1**

v. Let a be a real number not in the domain of $f^{-1}(x)$. Find $f^{-1}(f(a))$. **2**

(b) For what values of x is the inequality $\frac{1}{x+1} \leq \frac{x+1}{x+3}$ satisfied? **4**

End of paper

MC: Q1 D Q2C Q3A Q4B Q5 A
Q6B Q7C Q8C Q9D Q10 B

Question 11

a)

$$y = a(x-h)^2 + k$$
$$y = a(x-2)^2 + 6$$ ①

Sub (3,3) into equation

$$3 = a(3-2)^2 + 6$$
$$3 = a + 6$$
$$\therefore a = -3$$
$$y = -3(x-2)^2 + 6$$
or $y = -3x^2 + 12x - 6$ ①

Done well.
However, some students used the standard form of quadratic function, $y = ax^2 + bx + c$, and solved the equations simultaneously. This took more time than the factorised form, $y = a(x-h)^2 + k$.

b) $\frac{4x-3}{2} - \frac{3x-3}{3} < 3$

$$3(4x-3) - 2(3x-3) < 18$$
$$12x - 9 - 6x + 6 < 18$$
$$6x - 3 < 18$$
$$6x < 21$$
$$x < \frac{21}{6}$$
$$x < \frac{7}{2}$$

$\boxed{\text{or}}$

$$\frac{3(4x-3) - 2(3x-3)}{6} < 3$$
$$\frac{12x - 9 - 6x + 6}{6} < 3$$
$$6x - 3 < 18$$
$$6x < 21$$
$$x < \frac{7}{2}$$

③ if no errors
② if one error
① if two errors

* Done well
* Some students made algebraic errors but they were give CFE marks depending on the number of mistakes made.
* Large number of students did not simplify their answer.

c) $\Delta \geq 0$ for real roots.

$$(k+1)^2 - 4(1)(k-2) \geq 0$$ ①
$$(k^2 + 2k + 1) - 4k + 8 \geq 0$$
$$k^2 - 2k + 9 \geq 0$$

$k^2 - 2k + 9$ is a positive definite parabola

$\therefore \Delta \geq 0$ for all values of k. ①

$\therefore$ All real values of k.

Alternative:

$$k^2 - 2k + 9 \geq 0$$
$$k^2 - 2k + 1 \geq -9 + 1$$
$$(k-1)^2 \geq -8$$

$\therefore$ True for all real values of k since $(k-1)^2 \geq 0$.

* Poorly done.
* $\Delta > 0$ was one of the common mistakes.
* Many students managed to build the inequality for $\Delta \geq 0$ but failed to solve it for k.

11d) $2|x+3| = x-2$

* Most students worked out the two x values of -8 and $-\frac{4}{3}$, however, did not check the solution.

$2(x+3) = x-2$ for $x+3 \geq 0$ or $-2(x+3) = x-2$ for $x+3<0$

$2x+6 = x-2$ $\qquad$ $-2x-6 = x-2$

$x = -8$ for $x \geq -3$ ① $\qquad$ $-3x = 4$

$x = \frac{-4}{3}$ for $x<-3$ ①

Test solutions

$2|-8+3| = -8-2$

$2|-5| = -10$

$10 \neq -10$

$\therefore x=-8$, not a solution

$2\left|\frac{-4}{3}+3\right| = -\frac{4}{3}-2$

$2\left(\frac{5}{3}\right) = -\frac{10}{3}$

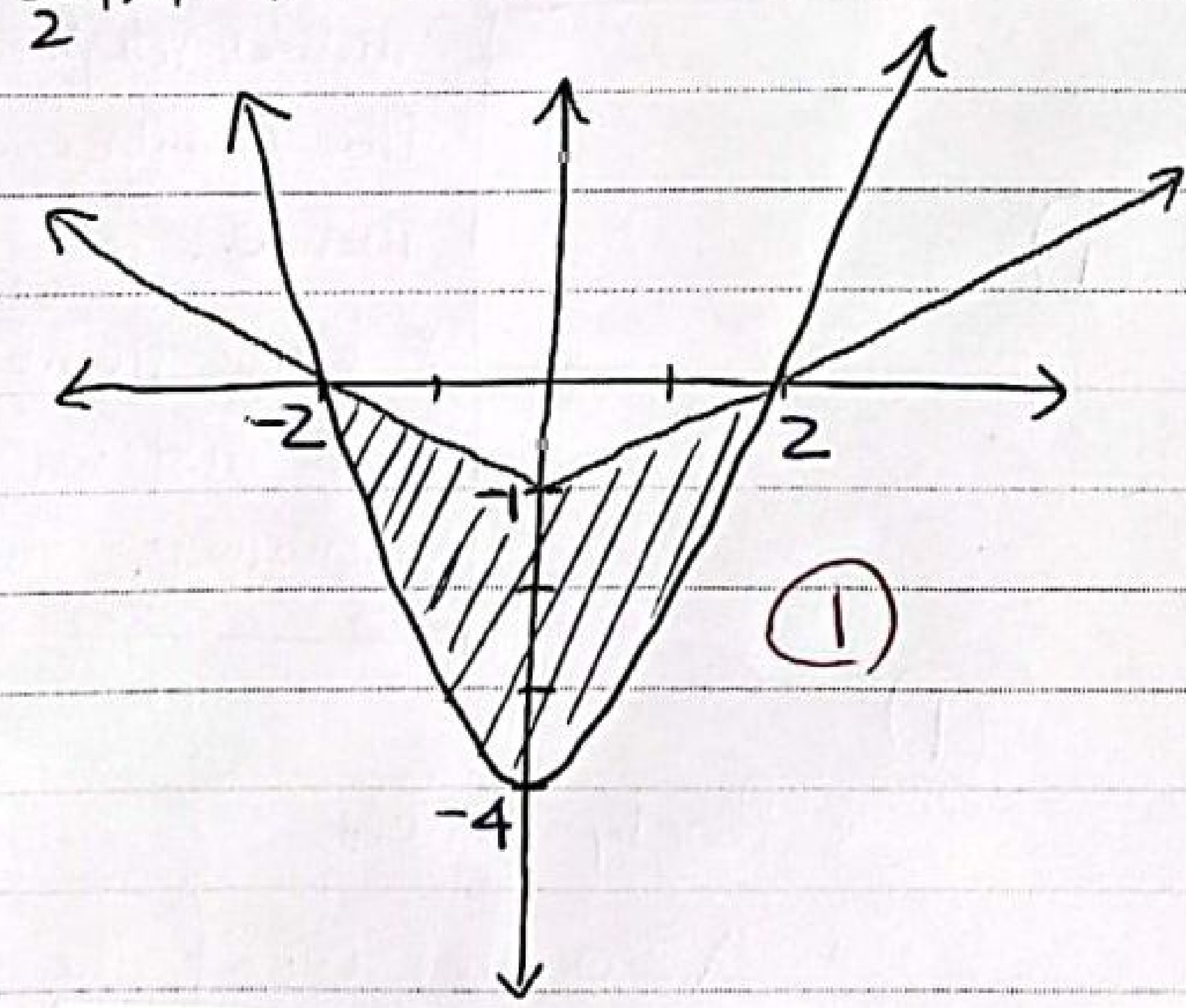
$\frac{10}{3} \neq \frac{-10}{3}$

$\therefore x = \frac{-4}{3}$, not a solution ①

$\therefore$ no solution

e) $y = x^2-4$ [1]

$y = \frac{1}{2}|x|-1$ [2]



* Poorly done.
* Most students sketched the graphs correctly but could not worked out the correct inequalities.
* Many students did not write "and" between the two inequalities. The area bounded by the curves is the intersection of the two regions
* Common mistake, $x^2-4 \leq \frac{1}{2}|x|-1$. This does not represent a region.

Substitute $(0,-2)$ into [1],

$\therefore y \geq x^2-4$

Substitute $(0,-2)$ into [2],

$\therefore y \leq \frac{1}{2}|x|-1$

$\therefore y \geq x^2-4$ and $y \leq \frac{1}{2}|x|-1$

① ①

Start here

Q12) a) $x^2 - 8x + y^2 - 14y + 1 = 0$

$$x^2 - 8x + \left(\frac{8}{2}\right)^2 + y^2 - 14y + \left(\frac{14}{2}\right)^2 = -1 + \left(\frac{8}{2}\right)^2 + \left(\frac{14}{2}\right)^2$$

$$(x-4)^2 + (y-7)^2 = 64$$

$\therefore$ The relation is a circle, centre $(4,7)$, radius 8. It is also a many-to-many relation.

NOTE:- The question is to describe, it would be a waste of exam time to sketch the graph.

- It is worth 3 marks. Simply describing it as a circle would never be enough for 3 marks. Many students completed the square but didn't use this to help in their description.

b) $\frac{6}{5x-2} < 2$ $x \neq \frac{2}{5}$

$\times(5x-2)^2$ $\times(5x-2)^2$

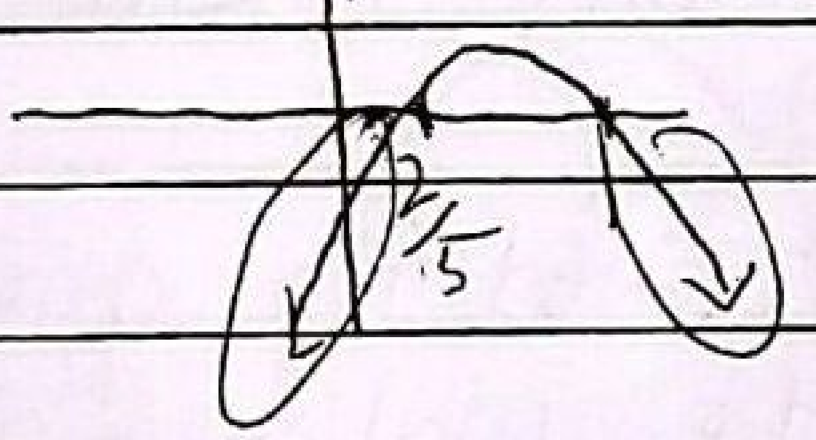
$6(5x-2) < 2(5x-2)^2$

$6(5x-2) - 2(5x-2)^2 < 0$

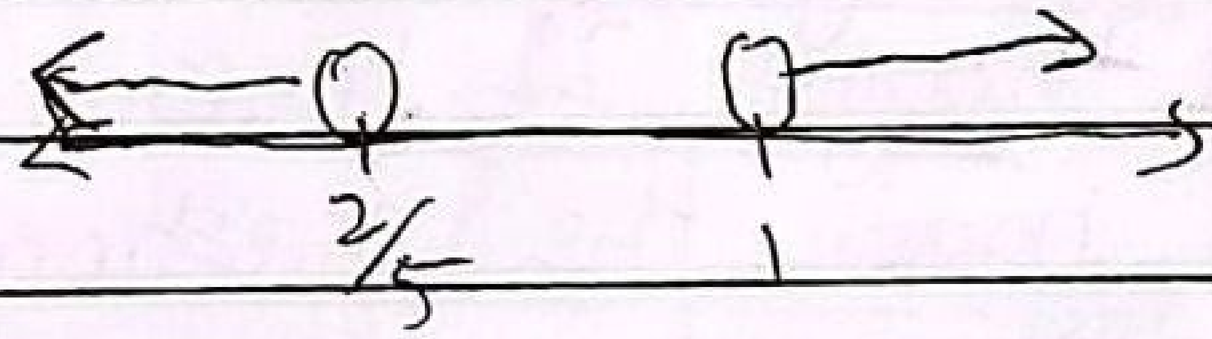
$(5x-2)(6-2(5x-2)) < 0$

$(5x-2)(-10x+10) < 0$

$-10(5x-2)(x-1) < 0$



$x < \frac{2}{5}, \ x > 1$



NOTE:- Many students forgot the write the final inequality answer and/or draw it on a number line.

- Lots of students simply multiplied by the denominator instead of its square. This made the question into a linear equation, which meant a max of 1 mark could be given. It also means you have not listened in class and haven't learnt your extension concepts.

- Lots of students changed the LHS and RHS without flipping the inequality symbol. This is incorrect.

Eg. $7x+1 > x^2$
$x^2 > 7x+1$ ✗
$x^2-7x-1 > 0$

c) 24 hours total − 2 hours rest
= 22 hours travel time.

$s = d/t \quad \therefore t = d/s$

time on bus + time walking = 22 hours

$$\frac{x}{48} + \frac{x}{4.8} = 22$$

$$x + 10x = 1056$$

$$11x = 1056$$

$$x = 96$$

∴ The bus took them 96 km.

NOTE:- Many students misinterpreted the question. Most thought 22 hours was the walking time. This made the question extremely easy ($22 \times 4.8 = 105.6$ km) So zero marks were awarded.

- This question could also have been completed using ratios. 22 hours in the ratio 1 : 10 (bus : walk)
∴ 2 hours on the bus.
∴ $2 \times 48 = 96$ km.

☐ ⇦ Tick this box if you have continued this answer in another writing booklet.

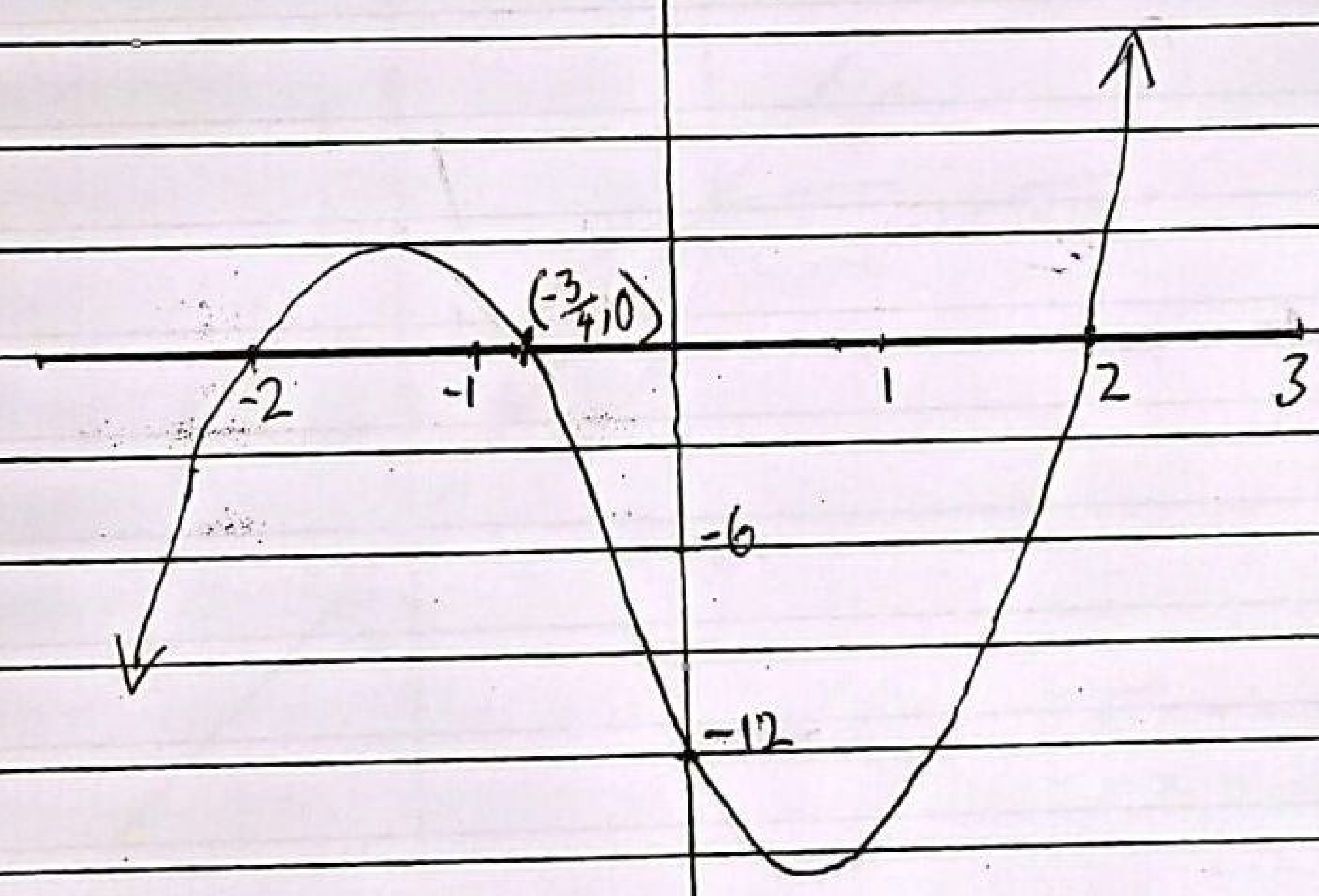
d) $y = 4x^3 + 3x^2 - 16x - 12$

$= x^2(4x+3) - 4(4x+3)$

$= (4x+3)(x^2-4)$

$= (4x+3)(x-2)(x+2)$

$\therefore$ x-ints → $x = -\frac{3}{4}, 2, -2$ y-int → $y = -12$



NOTE: The question is to sketch. Therefore is should be your focus. DO NOT draw a tiny graph that is rushed. The graph is the important bit. It should be one third of a page big, or more.

- Use a ruler to draw an accurate scale!
- $(x^2-4) \neq (x-4)^2$ a common error.

Additional writing space on back page.

- The y-int is not the vertex. The vertex is generally around the middle of the x-intercepts.

Question 13)

a) i) inverse of $f(x) = \sqrt{x-2} + 4$

$y = \sqrt{x-2} + 4$

$x = \sqrt{y-2} + 4$ (swap x with y)

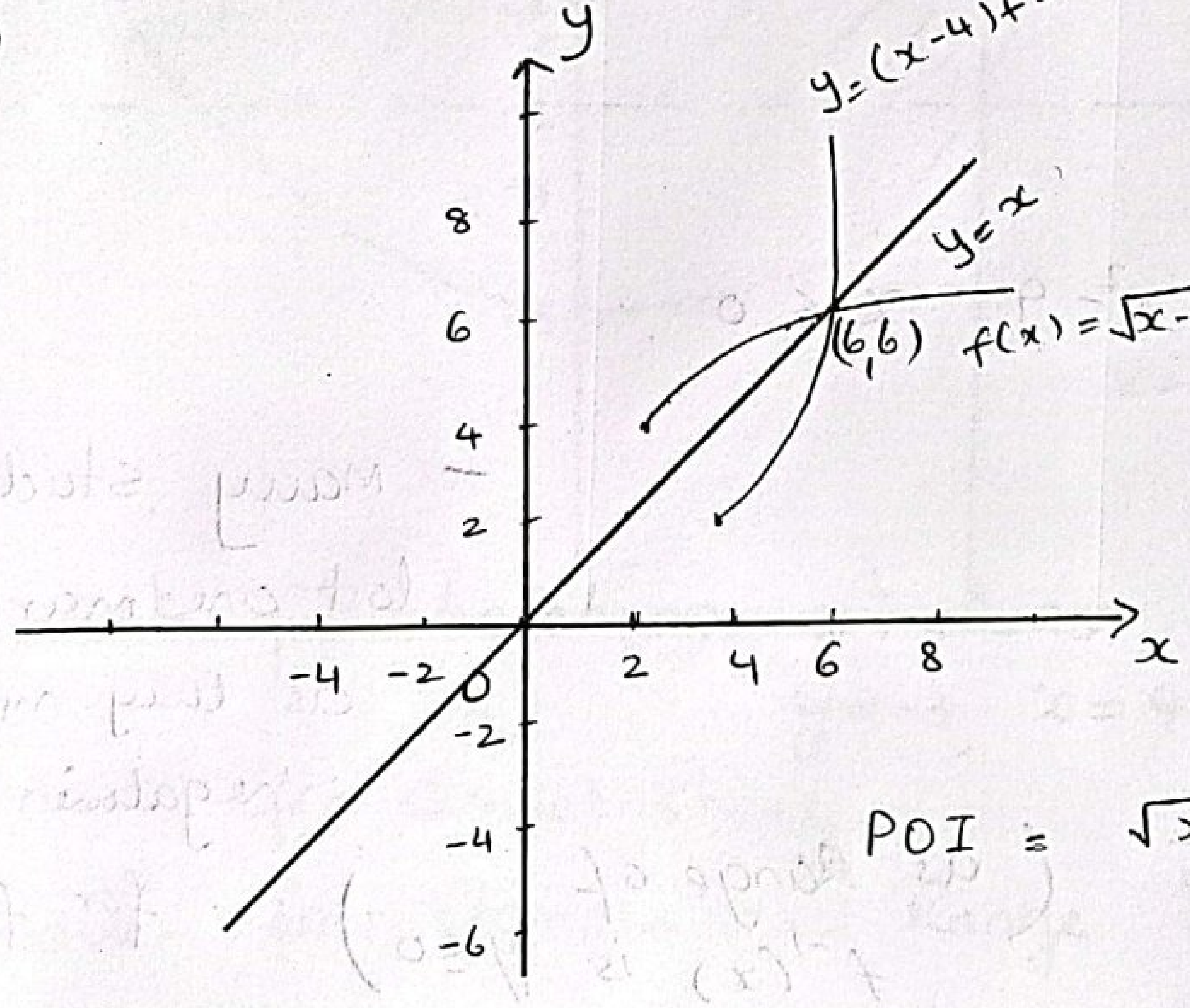
$x - 4 = \sqrt{y-2}$ ✓

$y-2 = (x-4)^2$ ✓

$y = (x-4)^2 + 2$ ✓

some algebra errors instead of adding 2 at the last step, subtracted 2.

ii)



- Many students drew both branches of $y = (x-4)^2 + 2$.

POI = $\sqrt{x-2} + 4 = x$

$x-4 = \sqrt{x-2}$

$x - 2 = x^2 - 8x + 16$

$x^2 - 9x + 18 = 0$

$(x-6)(x-3) = 0$

$(6,6)$, $(3,3)$

$(6,6)$ is the only point of intersection.

- overall poor attempt.

- students are encouraged to attempt more questions like this to improve.

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b) $f(x) = \frac{4}{x+2}$

$y = \frac{4}{x+2}$ — overall, well done.

$x = \frac{4}{y+2}$ (swap x with y)

$xy + 2x = 4$

$xy = 4 - 2x$

$y = \frac{4-2x}{x}$

c) $f(x) = x^2 - 9, \quad x \le 0$.

$y = x^2 - 9$ — Many students lost one mark as they missed negative sign for $f^{-1}(x)$.

$x = y^2 - 9$

$y^2 = x + 9$

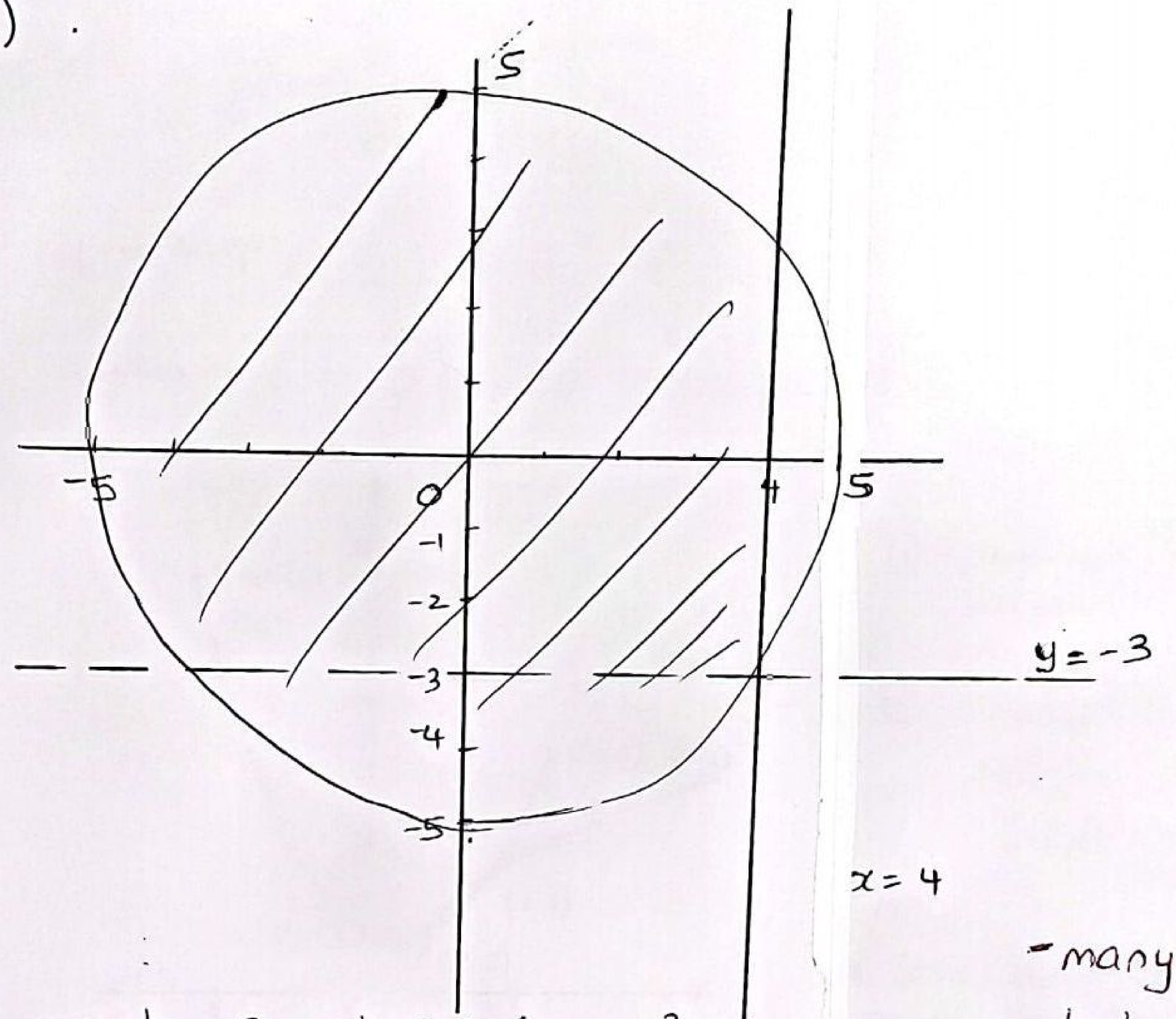
$y = -\sqrt{x+9}$ (as Range of $f^{-1}(x)$ is $y \le 0$)

$\therefore \quad f^{-1}(f(x)) = -\sqrt{x^2 \cancel{-9} \cancel{+9}}$

$= -x$

$= x$ as $x \le 0$.

d)



- many students lost mark due to incorrect shading.

1 mark for sketching $x^2+y^2=25$

$y=-3, \ x=4$

1 mark for correct shading

1 mark for domain and Range $D_f: [-5, 4]$

$R_f = (-3, 5]$

e) $y = x^2 - 12x + 40$

- many students wrote $x \geq 6$ or $x \leq 6$, lost 1 mark.

axis of symmetry $= \frac{-b}{2a}$

$= \frac{-(-12)}{2}$ ✓

$= 6$

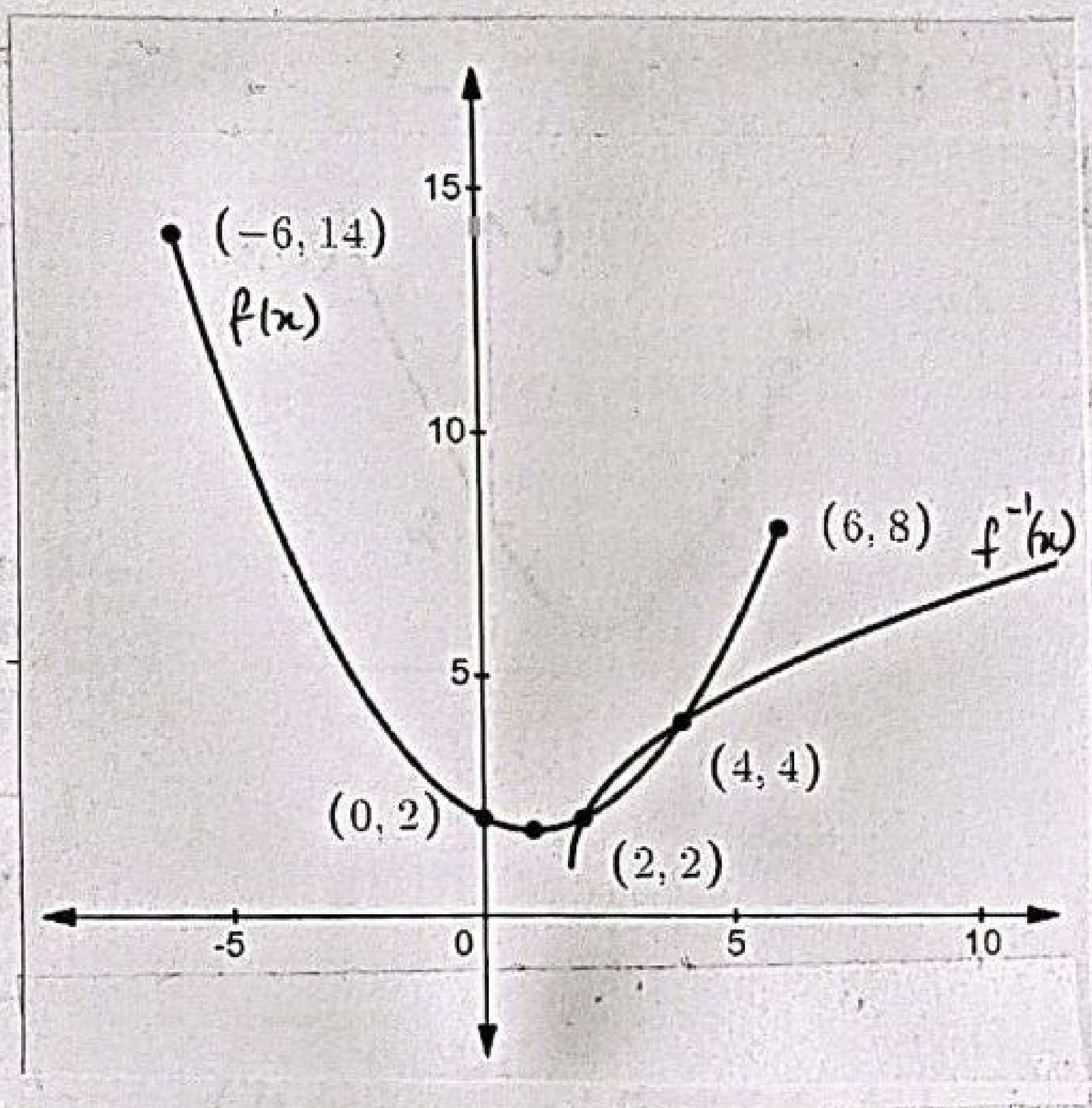
$\therefore$ $x \geq 6$ is the largest positive domain for an inverse. ✓

Question 14

a) $f(x) = \frac{1}{4}\left[(x-1)^2 + 7\right]$

i) y-intercept: $\frac{1}{4}\left[1^2+7\right] = 2$

vertex : $\left(1, \frac{7}{4}\right)$ endpoints : $(-6, -14)$, $(6, 8)$



① - Intercept + vertex labelled

① - End points labelled within domain $[-6, 6]$

NOTE :

* Some students lost marks for messy diagrams. The quality of some graphs is unacceptable.

ii) $f^{-1}(x)$: $x = \frac{1}{4}\left[(y-1)^2 + 7\right]$

$$4x = (y-1)^2 + 7$$

$$4x - 7 = (y-1)^2$$

$$\pm\sqrt{4x-7} = y - 1$$

$$y = 1 \pm \sqrt{4x-7} \quad ①$$

since range is $[1, \infty)$

$$\therefore\ y = 1 + \sqrt{4x-7} \quad ①$$

Note :

* Many students jumped to $y = 1 + \sqrt{4x-7}$ without justifying why they chose that answer. The '$\pm$' step is needed.

* Some students wrote:

$$x = \frac{1}{4}\left[(y-1)^2 + 7\right]$$

$$\frac{x}{4} = (y-1)^2 + 7.$$

iii) Points of intersection will occur at $y = x$ if they also exist the graphs $f(x)$ and $y = x$.

① $y = \frac{1}{4}[(x-1)^2 + 7]$ ② $y = x$

equate ① and ②:

$$x = \frac{1}{4}[(x-1)^2 + 7]$$

$$4x = (x-1)^2 + 7$$

$$4x = x^2 - 2x + 1 + 7$$

$$x^2 - 6x + 8 = 0$$

$$(x-4)(x-2) = 0$$

$$\therefore \quad x = 4, 2$$

sub them into ②

① $\therefore$ $(4,4)$ and $(2,2)$ are the points of intersection for $f(x)$ and $y = x$, thus by extension for $f(x)$ and $f^{-1}(x)$

NOTE:

* Many students made it difficult for themselves by trying to equate $f(x)$ and $f^{-1}(x)$. Only a few were successful in getting the answer that way.

* Some students made the claim that there are no points of intersection without any working or justification $\therefore$ no CFE for their inverse graph.

iv) on graph in i)

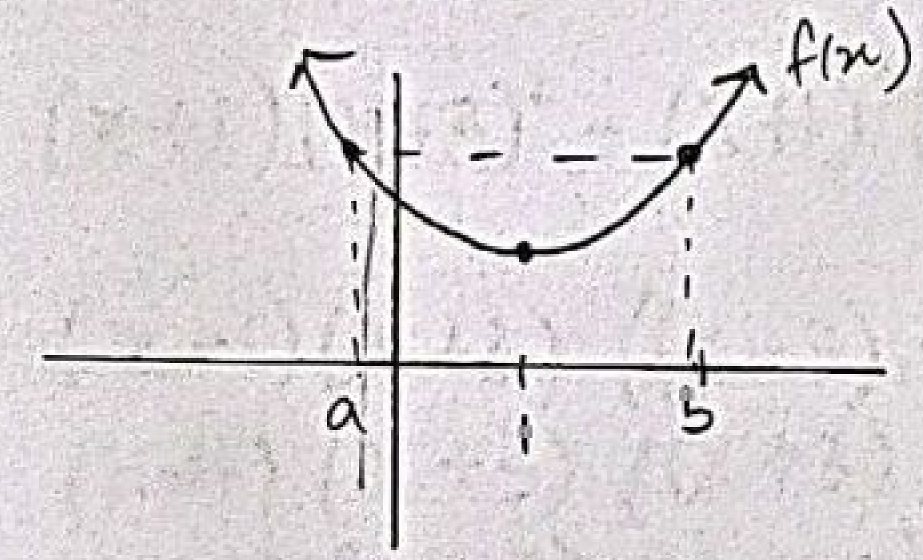
v)

Method ①:

Let $a < 1$

Now, from graph, there is a value b such that

$$f(a) = f(b)$$



NOTE:

* Most students did poorly.
* If attempting the algebraic method, only a few got 'a' as an answer (1 mark)
* A good question to revise as it's a HSC style question.

And since $b > 1$

$$f^{-1}(f(b)) = b$$

$$\therefore f^{-1}(f(a)) = b$$

By symmetry, the midpoint between a and b is 1

i.e $\frac{a+b}{2} = 1$ ①

$$b = 2 - a$$

since $f^{-1}(f(a)) = b$

$\therefore f^{-1}(f(a)) = 2 - a$ ①

Method ②:

$$f(a) = \frac{1}{4}\left[(a-1)^2 + 7\right]$$

$$f^{-1}(f(a)) = 1 + \sqrt{4\left(\frac{1}{4}(a-1)^2 + 7\right) - 7}$$

$$= 1 + \sqrt{(a-1)^2 - 7 + 7}$$

$$= 1 + \sqrt{(a-1)^2} \; *$$

$$= 1 - (a-1)$$

$= 2 - a$ // ① or if $= a$ ① only.

* Now:

① $\sqrt{(a-1)^2} = a - 1$ if $a \geq 1$

or

$= -(a-1)$ if $a < 1$

Since a is out of the domain, we take $a < 1$

)

$$\frac{1}{x+1} \le \frac{x+1}{x+3} \qquad \underline{x \neq -1, -3} \quad 1$$

$$\frac{1(x+1)^2(x+3)^2}{(x+1)} \le \frac{(x+1)(x+3)^2(x+1)^2}{(x+3)} \quad 1$$

$$(x+1)(x+3)^2 \le (x+1)^3(x+3)$$

$$(x+1)(x+3)^2 - (x+1)^3(x+3) \le 0$$

$$(x+1)(x+3)\left[(x+3) - (x+1)^2\right] \le 0$$

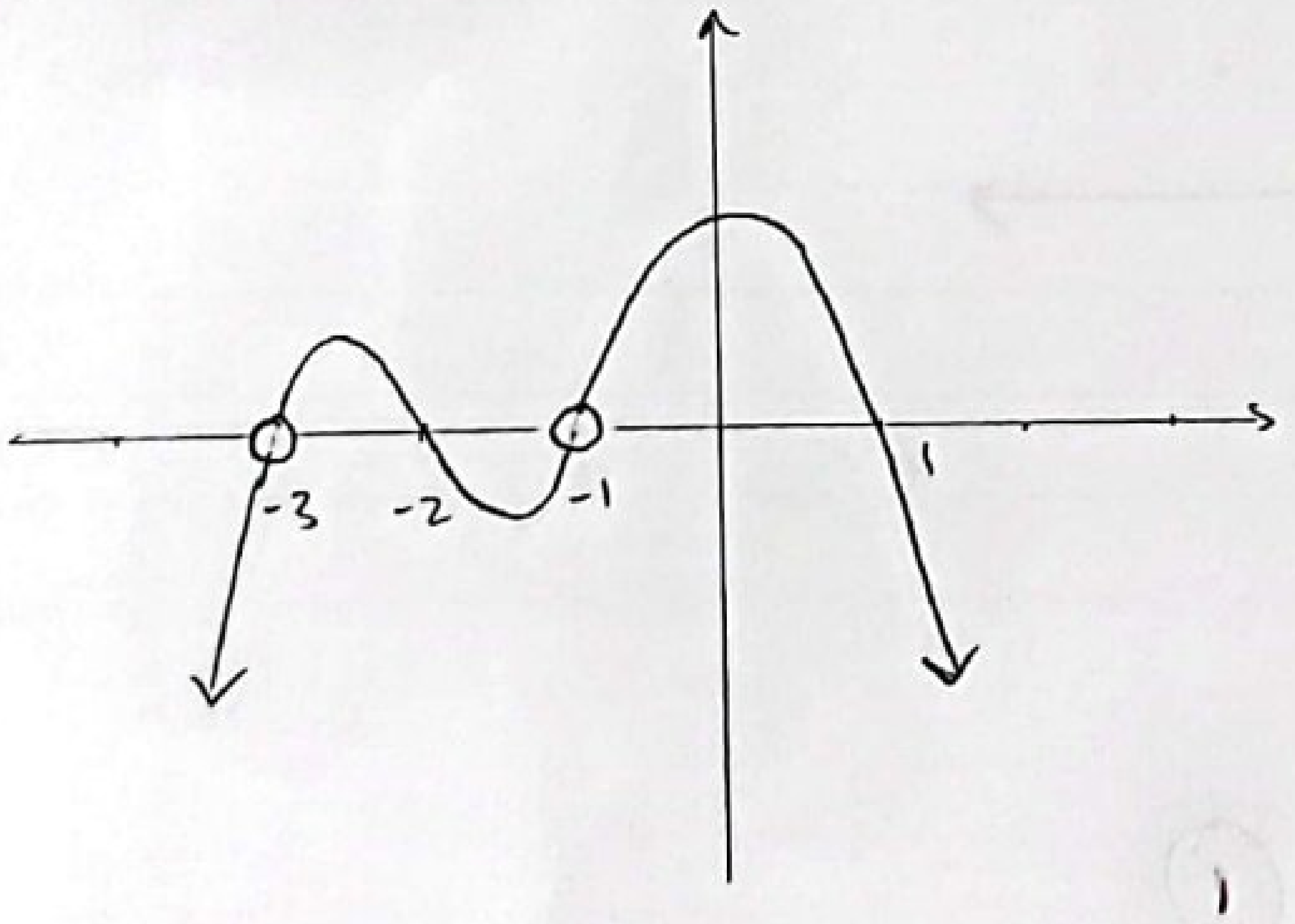
$$(x+1)(x+3)\left[(x+3) - (x^2+2x+1)\right] \le 0$$

$$(x+1)(x+3)\left[(x+3) - x^2 - 2x - 1\right] \le 0$$

$$(x+1)(x+3)(-x^2 - x + 2) \le 0$$

$$-(x+1)(x+3)(x^2 + x - 2) \le 0$$

$$-(x+1)(x+3)(x+2)(x-1) \le 0 \quad 1$$



$$1$$

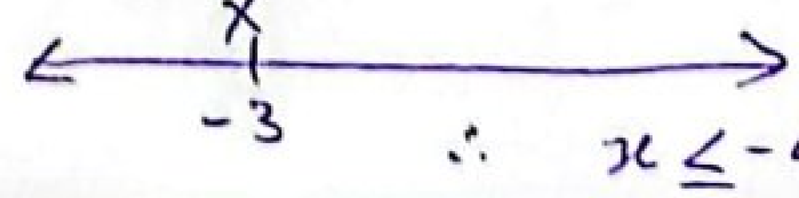
$$x < -3, \; -2 \le x < -1, \; x \geq 1$$

$$(-\infty, 3) \cup [-2, -1) \cup [1, \infty) \quad 1$$

Note:

* Done badly due to poor algebra and understanding of inequalities.
* Students that eliminated solutions and made the inequality into a quadratic lost a mark for making it easier.
* Students that did the critical points method need to be careful to not skip a range of solutions.

e.g



$\therefore \; x \le -4$

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