2023

PRELIMINARY ASSESSMENT TASK 1

Extension 1 Mathematics

General Instructions

- Reading time – 10 minutes
- Working time – 90 minutes
- Write using black pen only

- For questions in Section II, show relevant mathematical reasoning and/or calculations.

- Calculators approved by NESA may be used
- DO NOT USE LIQUID PAPER OR CORRECTION TAPE

Total Marks – 61

Section I: 10 marks

- Attempt questions 1-10, using the answer sheet provided.
- Allow about 15 minutes for this section

Section II: 51 marks

- Attempt questions 11-14, answer the questions in the booklets provided.
- Allow about 75 minutes for this section

	M/C	11	12	13	14	TOTAL
Inequalities	1-5 /5	/13	/14			/32
Inverse Functions	6-10 /5			/11	/13	/29
TOTAL						/61

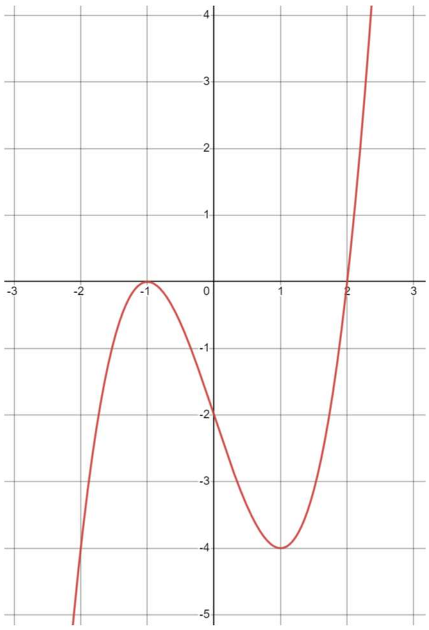
STUDENT NUMBER: ____________________________ TEACHER:______________

Section I

10 marks
Attempt Questions 1 to 10
Use the multiple-choice answer sheet for Questions 1 to 10

---

1. Given the graph of $y = (x-2)(x+1)^2$, which of the following is a solution to the inequation $(x-2)(x+1)^2 < 0$?



(A) $x < 2$

(B) $x \leq 2$

(C) $x < -1, -1 < x < 2$

(D) $x > 2$

2. Which solution below matches the diagram shown?

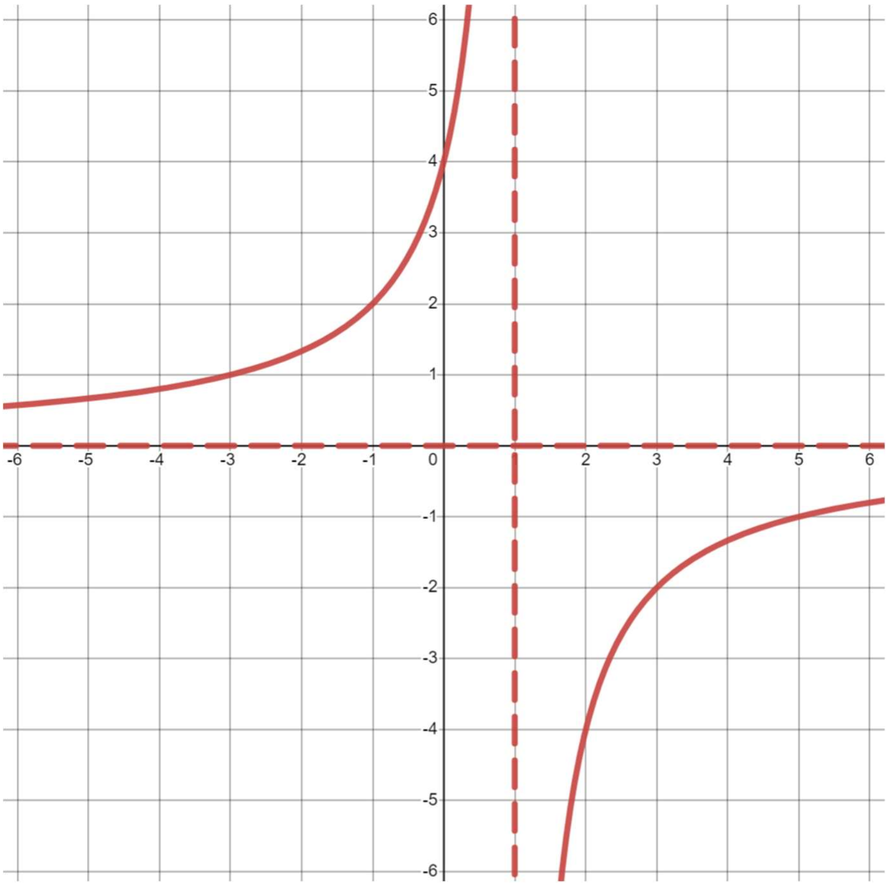
(A) $[-\infty, -2] \cup [4, \infty]$

(B) $(-\infty, -2] \cup [4, \infty)$

(C) $(-\infty, -2) \cup (4, \infty)$

(D) $[-\infty, -2) \cup (4, \infty]$

3. The graph of $y = -\frac{4}{x-1}$ is shown below.



Use this to solve the inequality $\frac{4}{x-1} < -2$

(A) $x < 1, \ \ x > 1$

(B) $1 < x < 3$

(C) $-1 < x < 1$

(D) $x < 1, \ \ x > 3$

4. Solve $3 - \frac{1+3x}{2} \geq -14$.

(A) $x \leq 11$

(B) $x \geq 11$

(C) $x \leq -11$

(D) $x \geq -11$

5. The graphs of $y = \sqrt{4 - x^2}$ and $y = 3x - 2$ intersect when:

(A) $x = 0$ and $x = \frac{5}{6}$

(B) $x = 0$ and $x = \frac{6}{5}$

(C) $x = \frac{6}{5}$

(D) $x = 0$ and $x = \sqrt{\frac{4}{5}}$

6. Given a set of coordinates or ordered pairs from a function, the method to finding its inverse is to:

(A) Reverse all the positive and negative signs.

(B) Reciprocate all the x and y values.

(C) Switching all x values with their associated y value.

(D) All of the above.

7. The function $f(x) = 2x^2 - 5x - 3$ will have an inverse function for the domain:

(A) $D: [\frac{5}{4}, \infty)$

(B) $D: \left[-\frac{1}{2}, 3\right]$

(C) $D: \left[-3, \frac{1}{2}\right]$

(D) $D: [-\frac{5}{4}, \infty)$

8. Given $h(x) = \sqrt{5-x}$, which of the following are the domain and range for $h^{-1}(x)$:

(A) $x > 0, y < 5$

(B) $x \geq 0, y \leq 5$

(C) $x < 5, y > 0$

(D) $x \leq 5, y \geq 0$

9. Given $f(x) = x^2 + 7x - 2$, for $x \leq -\frac{7}{2}$, find $f^{-1}(-2)$

(A) -7

(B) 0

(C) $-\dfrac{291}{40}$

(D) 12

10. Which of the following functions is its own inverse.

(A) $y = 2x$

(B) $y = \dfrac{2}{x}$

(C) $y = x^2$

(D) $y = x^3$

End of Section I

Section II

51 marks
Attempt Questions 11 to 14

Answer each question in a SEPARATE writing booklet.
Extra writing booklets are available.
All necessary working should be shown in every question.

Question 11 (13 marks) Use a SEPARATE writing booklet.

(a) Solve $\frac{2x+4}{3} - \frac{4x-7}{2} \geq 5$ **2**

(b) (i) Sketch the graphs of $y = x^2$ and $y = -x + 2$ on the same set of axis, using the domain restriction [-3,3] for both graphs. **3**

(ii) Explain how the graphs you have sketched in part (i) can be used to solve the inequality $x^2 + x - 2 < 0$. **2**

(iii) Sketch the graph of $y = x^2 + x - 2$, identifying all important features. Sketch this on a separate set of axis to part (i) **3**

(iv) Explain how the graph you have sketched in part (iii) can be used to solve the inequality $x^2 + x - 2 < 0$. **1**

(v) Use the algebraic method (also known as the cases method, testing points method or critical points method) to solve the inequality $x^2 + x - 2 < 0$ **2**

End of question 11

Question 12 (14 marks) Use a SEPARATE writing booklet.

(a) Solve $3x(2x-5)(x+3)^2 < 0$. Express your answer in interval notation. **2**

(b) The 2022, Year 11, Term 1, Mathematics Extension 1 examination included the following multiple choice question: **3**

> **3.** Which set provides a solution to the inequality $|3x-2| \leq |x+4|$?
>
> **(A)** $\left[-\frac{1}{2}, 3\right]$
>
> **(B)** $\left(-\infty, -\frac{1}{2}\right] \cup [3, \infty)$
>
> **(C)** $\left(-\infty, -\frac{1}{2}\right]$
>
> **(D)** $(-\infty, 3]$

Demonstrate and explain with clear detail, how the correct solution could have been found without any knowledge of solving inequalities.

(c) For what values of k does $x^2 + kx - 2 + k$ have real roots? **2**

(d) Solve $\left|\frac{x}{2} - 2\right| > 4$. Express your answer in interval notation. **2**

(e) Solve $\frac{3}{x-2} \leq 4$. **3**

(f) Solve $\frac{2}{|x+4|} \leq 1$. **2**

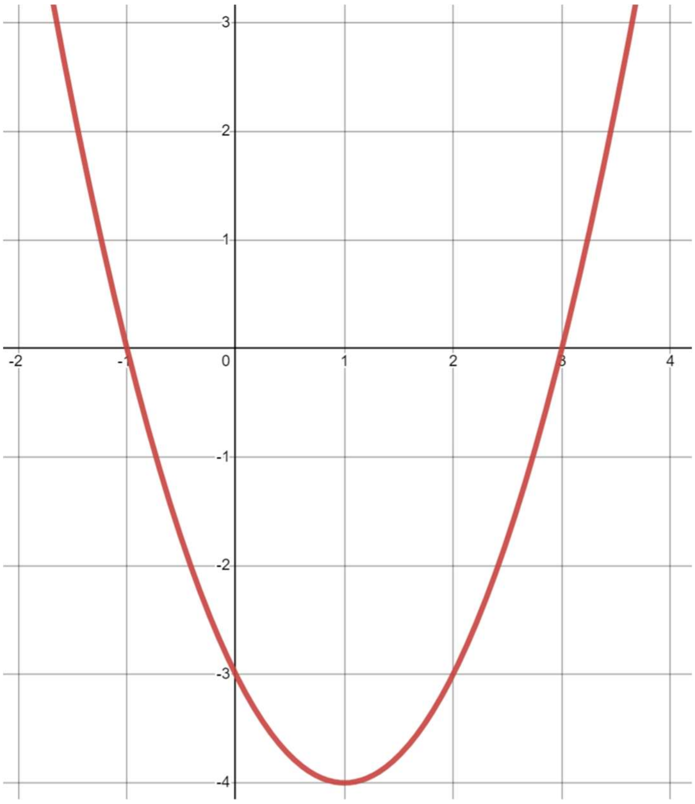
End of question 12

Question 13 (11 marks) Use a SEPARATE writing booklet.

(a) Given the function $f(x) = 2x - 5$, show that, its inverse function is $f^{-1}(x) = \frac{x+5}{2}$. **2**

(b) A **separate** piece of paper has been provided for this question. **2**
Answer this question on the separate piece of paper provided.

(c) The graph of $y = x^2 - 2x - 3$ is shown below. **4**



Anne, Benny, Carol and Dane were asked to restrict the domain of the function, so that its inverse function could be calculated. The domain restrictions they each decided on are shown below.

Anne: $x \leq 1$ Benny: $x \geq 3$ Carol: $x \geq -1$ Dane: $x \geq 1$

Discuss the effectiveness of each students' domain restriction.

(d) Given the function $g(x) = \sqrt{3x + 2}$ for $x \geq -\frac{2}{3}$, find $g^{-1}(x)$ and state its domain. **3**

End of question 13

Question 14 (13 marks) Use a SEPARATE writing booklet.

(a) The 2022, Year 11, Term 1, Mathematics Extension 1 examination included the following multiple choice question: **3**

> The function $f(x) = -2x^2 + 4x + 2$, where $x \geq 1$, has inverse $f^{-1}(x)$.
> For what value of a will $f(a) = f^{-1}(a)$?
>
> (A) $\frac{\sqrt{3}+1}{2}$ (B) 2
>
> (C) $\frac{2+\sqrt{2}}{2}$ (D) $\frac{5}{2}$

Demonstrate and explain with clear detail, how the correct solution could have been found without calculating $f^{-1}(x)$.

(b) Let $f(x) = x^2 + 4x$ for $x \geq -2$. This function has an inverse, $f^{-1}(x)$. **4**
Find an expression for $f^{-1}(x)$ and state its domain.

(c) (i) A **separate** piece of paper has been provided for this question. **2**
Answer this question on the separate piece of paper provided.

(ii) State the domain of $f^{-1}(x)$. **2**

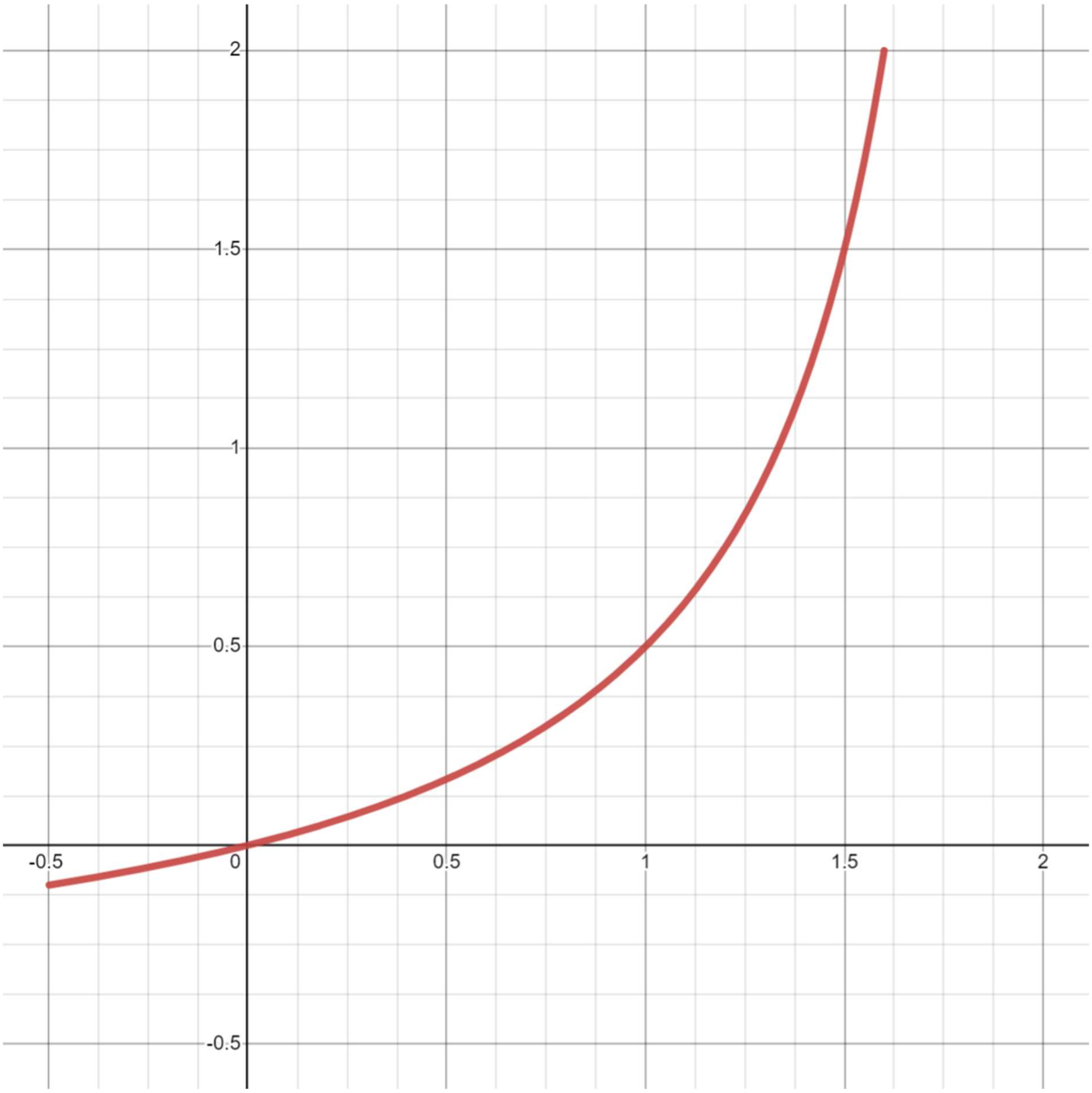
(iii) Find an expression for $y = f^{-1}(x)$ in terms of x. **2**

End of paper

STUDENT NUMBER: ______________________________ TEACHER:_______________

Question 13 (b) (2 marks) Answer this question on this piece of paper.

(b) The graph of $f(x)$, $-0.5 \le x \le 1.6$ is shown below. **2**



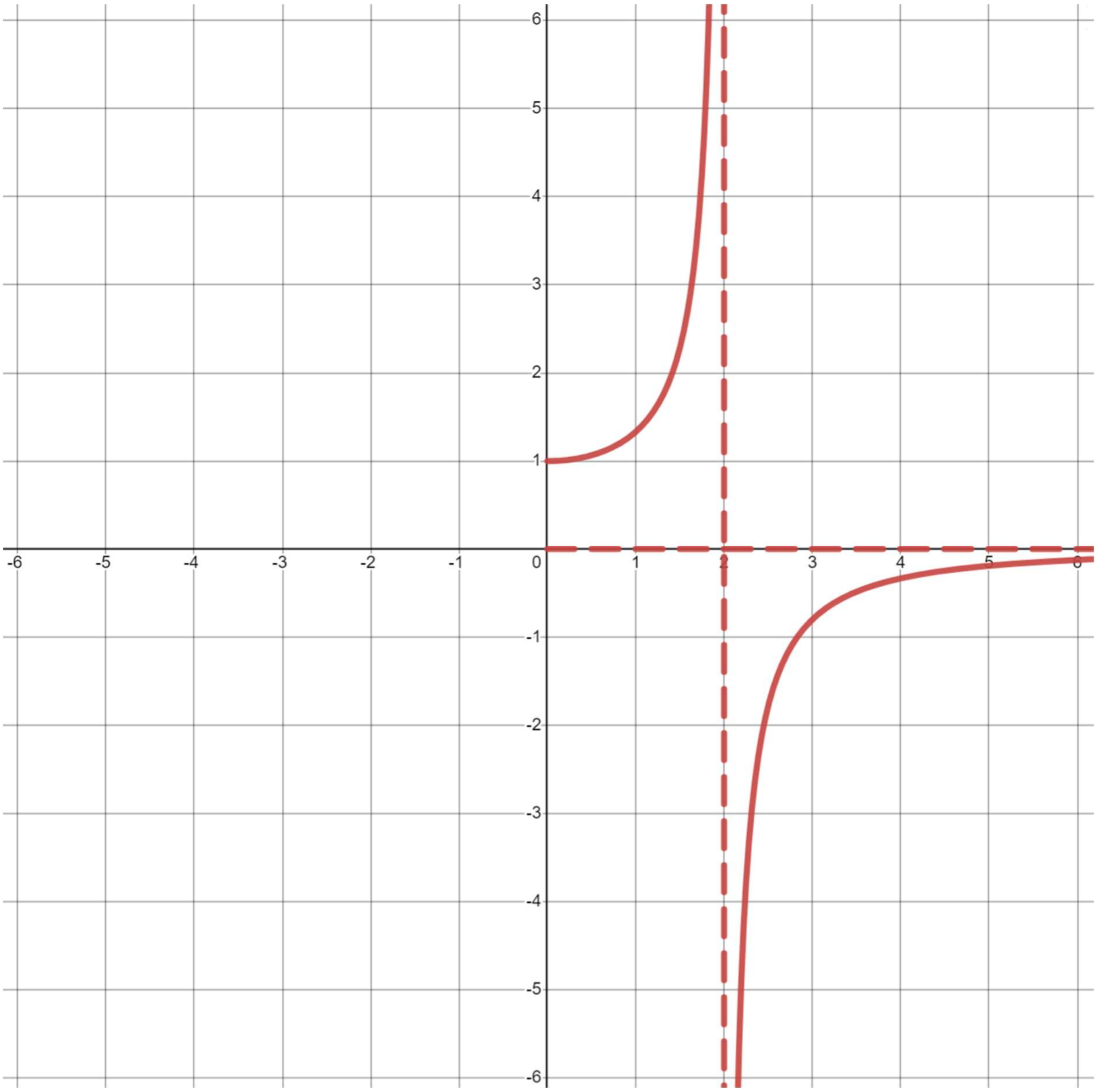
On the axes above, sketch the graph of $f^{-1}(x)$.

STUDENT NUMBER: ______________________________ TEACHER:_______________

Question 14(c)(i) (2 marks) Answer this question on this piece of paper.

(c) (i) The diagram below shows a sketch of the graph $y = f(x)$, where **2**

$f(x) = \frac{4}{4 - x^2}$, $x \geq 0,\ x \neq 2.$



On the same set of axis, sketch the graph of the inverse function, $y = f^{-1}(x)$.

Now complete parts (ii) and (iii) from the question booklet.

2023

PRELIMINARY ASSESSMENT TASK 1

Extension 1 Mathematics

General Instructions

- Reading time – 10 minutes
- Working time – 90 minutes
- Write using black pen only
- For questions in Section II, show relevant mathematical reasoning and/or calculations.
- Calculators approved by NESA may be used
- DO NOT USE LIQUID PAPER OR CORRECTION TAPE

Total Marks – 61

Section I: 10 marks

- Attempt questions 1-10, using the answer sheet provided.
- Allow about 15 minutes for this section

Section II: 51 marks

- Attempt questions 11-14, answer the questions in the booklets provided.
- Allow about 75 minutes for this section

	M/C	11	12	13	14	TOTAL
Inequalities	1-5 /5	/13	/14			/32
Inverse Functions	6-10 /5			/11	/13	/29
TOTAL						/61

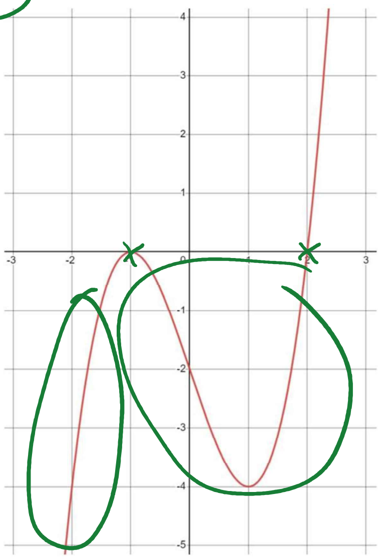
STUDENT NUMBER: ____________________________ TEACHER:______________

Section I

10 marks
Attempt Questions 1 to 10
Use the multiple-choice answer sheet for Questions 1 to 10

---

1. Given the graph of $y = (x-2)(x+1)^2$, which of the following is a solution to the inequation $(x-2)(x+1)^2 < 0$?



(A) $x < 2$

(B) $x \leq 2$

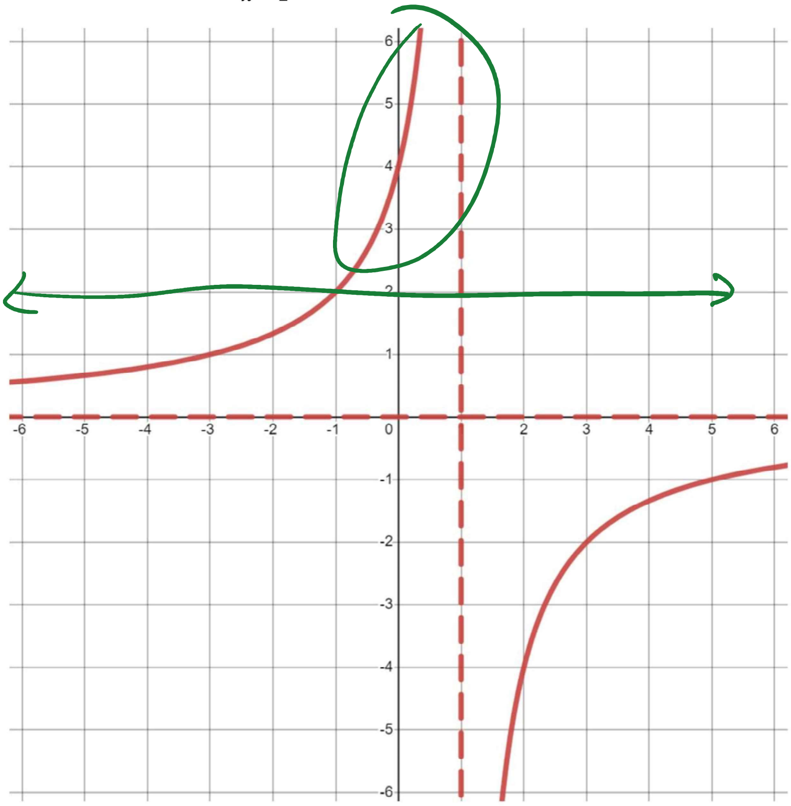
(C) $x < -1, -1 < x < 2$

(D) $x > 2$

2. Which solution below matches the diagram shown?

(A) $[-\infty, -2] \cup [4, \infty]$
(B) $(-\infty, -2] \cup [4, \infty)$
(C) $(-\infty, -2) \cup (4, \infty)$
(D) $[-\infty, -2) \cup (4, \infty]$

3. The graph of $y = -\frac{4}{x-1}$ is shown below.



Use this to solve the inequality $\frac{4}{x-1} < -2$

$\times -1 \qquad \times -1$

$-\frac{4}{x-1} > 2$

NOTE: Multiply by -1 so the LHS of the inequality matches the graph given. Ensure you flip the inequality sign.

(A) $x < 1, \ x > 1$

(B) $1 < x < 3$

(C) $-1 < x < 1$

(D) $x < 1, \ x > 3$

4. Solve $3 - \frac{1+3x}{2} \geq -14$.

$-\frac{1+3x}{2} \geq -17$

$1+3x \leq 34$

$3x \leq 33$

$x \leq 11$

(A) $x \leq 11$

(B) $x \geq 11$

(C) $x \leq -11$

(D) $x \geq -11$

5. The points of intersection of $y = \sqrt{4 - x^2}$ and $y = 3x - 2$ are:

(A) $x = 0$ and $x = \frac{5}{6}$

(B) $x = 0$ and $x = \frac{6}{5}$

(C) $x = \dfrac{6}{5}$

(D) $x = 0$ and $x = \sqrt{\frac{4}{5}}$

$$(\sqrt{4-x^2})^2 = (3x-2)^2 \rightarrow$$

NOTE: When you square both sides you may introduce a new solution, so must check by substitution.

$$4 - x^2 = 9x - 12x + 4$$
$$0 = 10x^2 - 12x$$
$$= 2x(5x-6)$$
$$x = 0 \text{ or } \frac{6}{5}$$
$$\therefore x = \frac{6}{5}$$

TEST $x=0$

$LHS = \sqrt{4-0^2} = 2$ $RHS = 3(0) - 2 = -2$

$LHS \neq RHS$

TEST $x = \frac{6}{5}$

$LHS = \sqrt{4 - (\frac{6}{5})^2} = \frac{8}{5}$ $RHS = 3(\frac{6}{5}) - 2 = \frac{8}{5}$

$LHS = RHS$

6. Given a set of coordinates or ordered pairs from a function, the method to finding its inverse is to:

(A) Reverse all the positive and negative signs.

(B) Reciprocate all the x and y values.

(C) Switching all x values with their associated y value.

(D) All of the above.

7. The function $f(x) = 2x^2 - 5x - 3$ will have an inverse function for the domain:

(A) $D: [\frac{5}{4}, \infty)$

(B) $D: \left[-\frac{1}{2}, 3\right]$

(C) $D: \left[-3, \frac{1}{2}\right]$

(D) $D: [-\frac{5}{4}, \infty)$

$$\text{Vertex} \rightarrow x = \frac{-b}{2a}$$
$$= \frac{-(-5)}{2(2)}$$
$$= \frac{5}{4}$$

8. Given $h(x) = \sqrt{5 - x}$, which of the following are the domain and range for $h^{-1}(x)$:

(A) $x > 0, y < 5$

(B) $x \geq 0, y \leq 5$

(C) $x < 5, y > 0$

(D) $x \leq 5, y \geq 0$

$h(x) \rightarrow D: x \leq 5$
$R: y \geq 0$
$\therefore h^{-1}f(x) \rightarrow R: y \leq 5$
$D: x \geq 0$

9. Given $f(x) = x^2 + 7x - 2$, for $x \leq -\frac{7}{2}$, find $f^{-1}(-2)$

(A) -7

(B) 0

(C) $-\frac{291}{40}$

(D) 12

$-2 = x^2 + 7x - 2$
$0 = x^2 + 7x$
$= x(x+7)$
$x = 0$ or -7

10. Which of the following functions is its own inverse.

(A) $y = 2x$

(B) $y = \frac{2}{x}$

(C) $y = x^2$

(D) $y = x^3$

(A) $y = 2x \rightarrow x = 2y$
$y = \frac{x}{2}$ ×

(B) $y = \frac{2}{x} \rightarrow x = \frac{2}{y}$
$yx = 2$
$y = \frac{2}{x}$ ✓

End of Section I

Q11)

(a) Solve $\frac{2x+4}{3} - \frac{4x-7}{2} \geq 5$ **2**

$$\frac{2(2x+4)-3(4x-7)}{6} \geq 5$$

$$4x+8-12x+21 \geq 30$$

$$-8x+29 \geq 30$$

$$-8x \geq 1$$

$$x \leq -\frac{1}{8}$$

(b) (i) Sketch the graphs of $y = x^2$ and $y = -x + 2$ using the domain restriction [-3,3] for both graphs. **3**

pts. of intersection

$$x^2 = -x+2$$

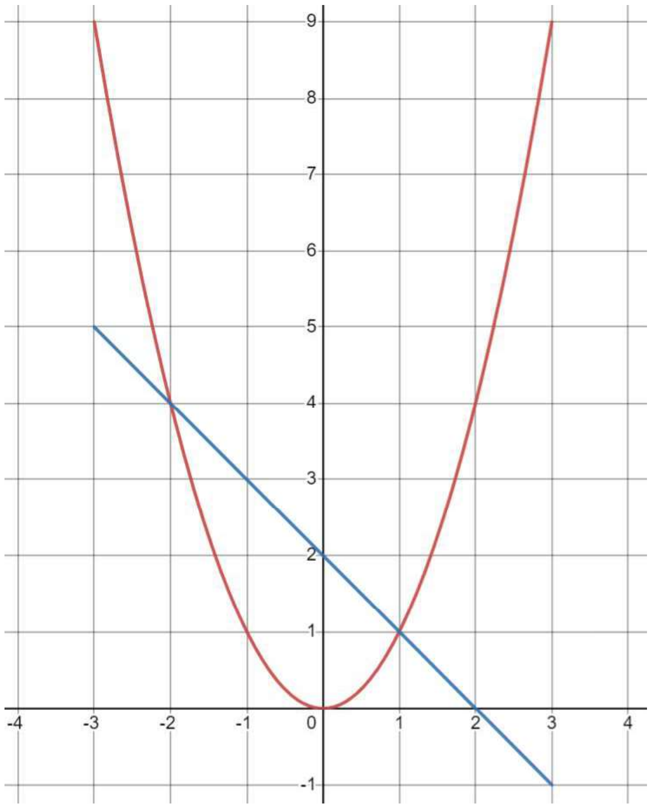
$$x^2+x-2=0$$

$$(x+2)(x-1)=0$$

$$x=-2 \text{ or } 1$$

$y=(-2)^2$
$=4$
$(-2,4)$

$y=(1)^2$
$=1$
$(1,1)$



(ii) Explain how the graphs you have sketched in part (i) can be used to solve the inequality $x^2 + x - 2 < 0$. **2**

$x^2 + x - 2 < 0$ can be rearranged to $x^2 < -x + 2$. Now the LHS is x^2 and the RHS is $-x + 2$. These are the same as the graphs sketched in part (i). Finding where the LHS is less than the RHS is equivalent to finding where the graph of the LHS (x^2) is below the graph of the RHS ($-x + 2$).

The points of intersection of the two graphs are important as they are the exact points where the parabola changes from being above to below the straight line (or vice versa).

(iii) Sketch the graphs of $y = x^2 + x - 2$, identifying all important features. **3**

x-int when $y=0$

$0 = x^2 + x - 2$

$x = 1, -2$ (from part i)

y-int $= -2$

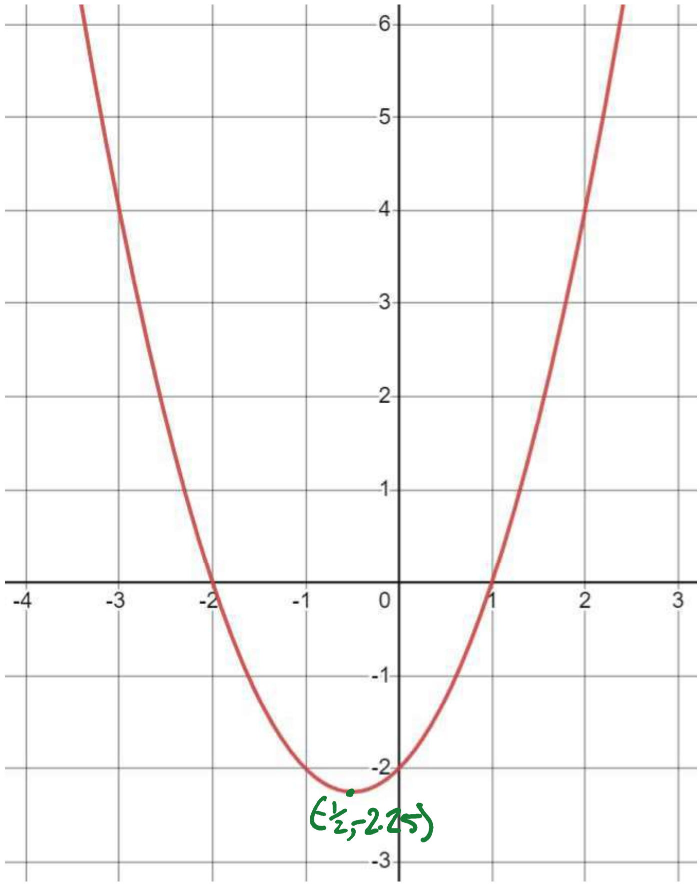
vertex $\rightarrow x = \frac{1+-2}{2}$

$= -\frac{1}{2}$

$y = (-\frac{1}{2})^2 + (-\frac{1}{2}) - 2$

$= -2.25$

$(-0.5, -2.25)$



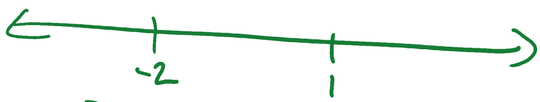
(iv) Explain how the graph you have sketched in part (iii) can be used to solve the inequality $x^2 + x - 2 < 0$. **1**

$x^2 + x - 2 < 0$ can be solved by drawing the graph of $y = x^2 + x - 2$ and finding the x-values for which the graph is below the x-axis.

(v) Use the algebraic method (also known as the cases method, testing points method or critical points method) to solve the inequality $x^2 + x - 2 < 0$ **2**

$x^2 + x - 2 = 0$

$x = -2, 1$ (from part i)



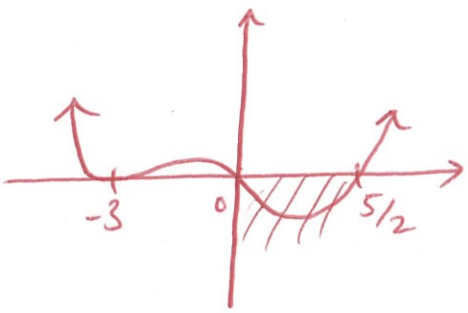
TEST	$x = -3$	$x = 0$	$x = 2$
	$(-3)^2 + (-3) - 2$	$(0)^2 + (0) - 2$	$(2)^2 + (2) - 2)$
	$= 4$	$= -2$	$= 4$
	> 0	< 0	> 0
	✗	✓	✗

$\therefore -2 < x < 1$

(12)

a) $3x(2x-5)(x+3)^2 < 0$

$x = 0,\ 5/2,\ -3$ are intercepts for the polynomial



from the graph

$(0, 5/2)$

NOTE: if not in interval notation, mark is lost.

→ Done well.

b) By selecting specific x-values, you can substitute them into the inequality and see which option matches.

Test $x=0$ $\quad LHS = |3(0)-2|$ $\quad RHS = |0+4|$

$= |-2| \qquad = 4$

$= 2$

$2 < 4$

$LHS < RHS$

Since A is the only option which includes $x=0$, we can select this as the correct answer.

NOTE: Done poorly.

Graphing the inequality and seeing which graph is above the other is still using inequality knowledge.

If you sketch it without using an inequality knowledge will get you the marks. You need to think about what "demonstrate" and "explain with clear detail" means.

c) Real roots when $\Delta \geq 0$ NOTE:

$b^2-4ac \geq 0$

$k^2-4(1)(-2+k) \geq 0$

$k^2+8-4k \geq 0$

Find the discriminant of the expression k^2+8-4k

$\Delta_2 = (-4)^2-4(1)(8)$

$= -16$

since $\Delta_2 < 0$ and coefficient of expression is positive

$\Rightarrow$ positive definite graph

which means for every k value, expression is above 0

$\therefore$ for all real k, $x^2+kx-2+k$ has real roots.

d) $\left|\frac{x}{2}-2\right|>4$

$\frac{x}{2}-2>4$ or $\frac{x}{2}-2<-4$

$\frac{x}{2}>6$ $\qquad$ $\frac{x}{2}<-2$

$x>12$ $\qquad$ $x<-4$

$(-\infty,-4)\cup(12,\infty)$

NOTE: A LOT of algebra mistakes! $6\times 2=3$? $-2\times 2=-1$?

e) $\frac{3}{x-2} \leq 4$, $x \neq 2$

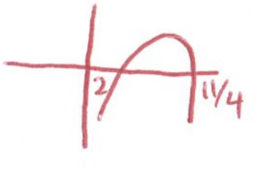
$3(x-2) \leq 4(x-2)^2$

$3(x-2)-4(x-2)^2 \leq 0$

$(x-2)[3-4(x-2)] \leq 0$

$(x-2)[3-4x+8] \leq 0$

$(x-2)(-4x+11) \leq 0$



$\therefore x<2$, $x \geq 11/4$

f) $\frac{2}{|x+4|} \leq 1$

$2 \leq |x+4|$

$2 \leq (x+4)$ or $-2 \geq x+4$

$-2 \leq x$ $\qquad$ $-6 \geq x$

$\therefore x \geq -2$, $x \leq -6$

Q13)

(a) Given the function $f(x) = 2x - 5$, show that, its inverse function is $f^{-1}(x) = \frac{x+5}{2}$. **2**

$y = 2x - 5$

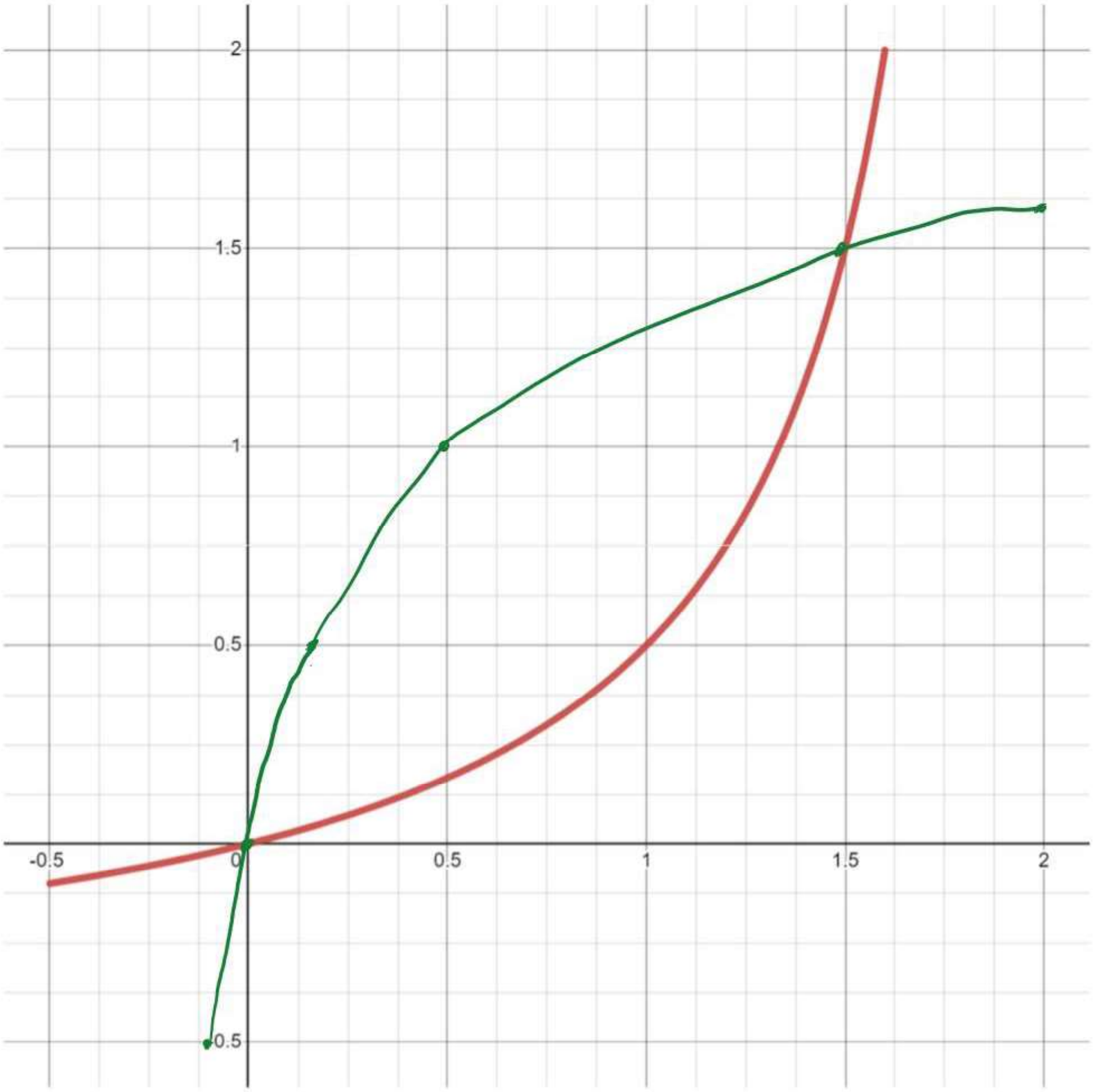
$x = 2y - 5$

$\frac{x+5}{2} = y$

$\therefore f^{-1}(x) = \frac{x+5}{2}$

Generally well done, though a lot of students forgot the last line.

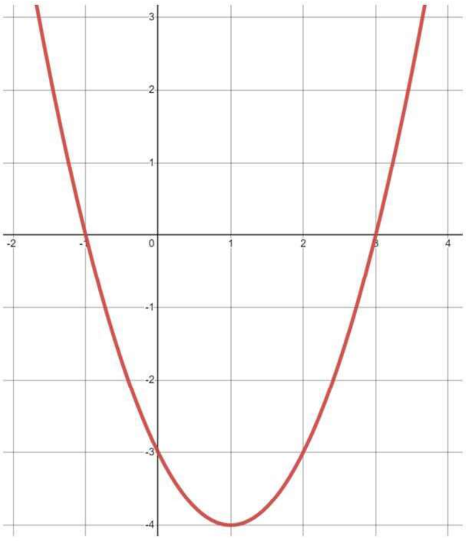
(b) The graph of $f(x)$, $-0.5 \leq x \leq 1.6$ is shown below. **2**



On the axes above, sketch the graph of $f^{-1}(x)$.

Since you were given specific values on y = f(x) you were expected to find the corresponding coordinates on the inverse function. For example (1, 0.5) becomes (0.5, 1) on the inverse function.

(c) The graph of $y = x^2 - 2x - 3$ is shown below. **4**



Students didn't discuss, a lot of students didn't write full sentences. Students lost a mark if they didn't mention effectiveness in their answers.

Anne, Benny, Carol and Dane were asked to restrict the domain of the function, so that its inverse function could be calculated. The domain restrictions they each decided on are shown below.

Anne: $x \leq 1$ Benny: $x \geq 3$ Carol: $x \geq -1$ Dane: $x \geq 1$

Discuss the effectiveness of each students' domain restriction.

Anne's restriction includes the entire left side of the parabola, up to the vertex. This interval will pass the horizontal line test and hence will have an inverse function. This interval is effective as it is the maximum interval which will allow an inverse function.

Benny's restriction will also pass the horizontal line test, so will also have an inverse function. However, it is a small interval compared to Anne's. So it is effective but not as effective as Anne's and Dane's restrictions.

Carol's restriction will not pass the horizontal line test, so will not have an inverse function. Carol can only find an inverse relation. Therefore, it is not an effective restriction.

Dane's restriction is similar to Anne's, but includes the entire right side of the parabola after the vertex. This interval is also the maximum interval which will pass the horizontal line test and hence all an inverse function. Dane's restriction is as effective Anne's but it involves the positive x values of the parabola.

(d) Given the function $g(x) = \sqrt{3x+2} - 6$ for $x \geq -\frac{2}{3}$, find $g^{-1}(x)$. **3**

$y - \sqrt{3x+2}$, $x \geq -\frac{2}{3}$ $y \geq 0$

$x = \sqrt{3y+2}$, $y \geq -\frac{2}{3}$ $x \geq 0$

$x^2 = 3y+2$

$\frac{x^2-2}{3} = y$

$g^{-1}(x) = \frac{x^2-2}{3}$, $x \geq 0$

Students didn't read the question completely, they either didn't find the inverse function or they didn't find the domain.
2 marks for the correct inverse function.
1 mark for the correct domain.

Question 14

(a) The 2022, Year 11, Term 1, Mathematics Extension 1 examination included the following multiple choice question: **3**

> The function $f(x) = -2x^2 + 4x + 2$, where $x \geq 1$, has inverse $f^{-1}(x)$.
> For what value of a will $f(a) = f^{-1}(a)$?
>
> (A) $\frac{\sqrt{3}+1}{2}$ ≈ 1.37 (B) 2
>
> (C) $\frac{2+\sqrt{2}}{2}$ ≈ 1.71 (D) $\frac{5}{2}$

Demonstrate and explain with clear detail, how the correct solution could have been found without calculating $f^{-1}(x)$.

Let $f(h) = k$

Then $f^{-1}(k) = h$ (definition of inverse function)

When $f(a) = f^{-1}(a)$,

$k = h$

Thus $f(a) = a$

1

Now, substitute the possible answers into $f(x)$ and find the one where the result is equal to the value substituted.

(A) $f\left(\frac{\sqrt{3}+1}{2}\right) = -2\left(\frac{\sqrt{3}+1}{2}\right)^2 + 4\left(\frac{\sqrt{3}+1}{2}\right) + 2$

$f(1.37) = 3.7$

$\neq \frac{\sqrt{3}+1}{2}$

(B) $f(2) = -2(2)^2 + 4(2) + 2$

$= 2$

(C) $f\left(\frac{2+\sqrt{2}}{2}\right) = -2\left(\frac{2+\sqrt{2}}{2}\right)^2 + 4\left(\frac{2+\sqrt{2}}{2}\right) + 2$

$f(1.71) = 3$

$\neq \frac{2+\sqrt{2}}{2}$

(D) $f\left(\frac{5}{2}\right) = -2\left(\frac{5}{2}\right)^2 + 4\left(\frac{5}{2}\right) + 2$

$= -\frac{1}{2}$

$\neq \frac{5}{2}$

1

$\therefore$ (B) 2 is the solution 1

Question 14

(a) The 2022, Year 11, Term 1, Mathematics Extension 1 examination included the following multiple choice question: **3**

> The function $f(x) = -2x^2 + 4x + 2$, where $x \geq 1$, has inverse $f^{-1}(x)$.
> For what value of a will $f(a) = f^{-1}(a)$?
>
> (A) $\frac{\sqrt{3}+1}{2}$ (B) 2
>
> (C) $\frac{2+\sqrt{2}}{2}$ (D) $\frac{5}{2}$

Demonstrate and explain with clear detail, how the correct solution could have been found without calculating $f^{-1}(x)$.

Alternative:

* Most students have used this method

① The graphs of $y = f(x)$ and $y = f^{-1}(x)$ intersect on the line $y = x$.
Thus $f(x) = f^{-1}(x)$ when $y = x$.
ie. $f(a) = a$

* Poor explanation

* Explained but did not demonstrate by showing the steps and the answer.

Now, either repeat the steps from the previous method or
solve $y = f(x)$ and $y = x$ simultaneously to find the points of intersection.

$-2x^2 + 4x + 2 = x$

$2x^2 - 3x - 2 = 0$

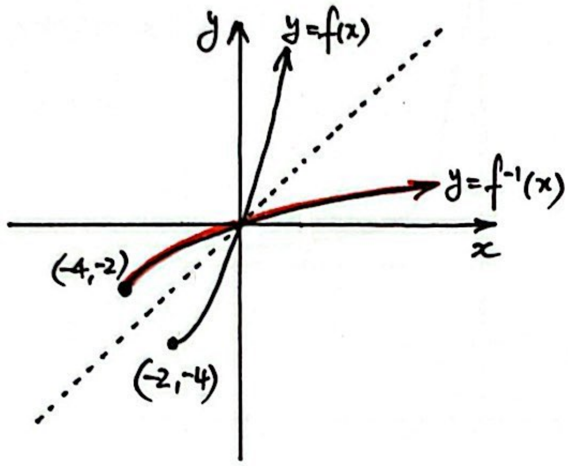
$(2x+1)(x-2) = 0$

① $x = -\frac{1}{2}$ or 2

① $\therefore x = 2$ (since $x \geq 1$)

$\therefore a = 2$

(b) Let $f(x) = x^2 + 4x$ for $x \geq -2$. This function has an inverse, $f^{-1}(x)$. Find an expression for $f^{-1}(x)$ and state its domain **4**



$y = x^2 + 4x$, $x \geq -2$ and $y \geq -4$

① $x = y^2 + 4y$, $y \geq -2$ and $x \geq -4$

$x + 4 = y^2 + 4y + 4$

① $x + 4 = (y+2)^2$

$y + 2 = \pm\sqrt{x+4}$

$y = -2 \pm \sqrt{x+4}$

① $\therefore f^{-1}(x) = -2 + \sqrt{x+4}$ (since $y \geq -2$)

① Domain : $x \geq -4$.

* Found the expression for $f^{-1}(x)$ but did not state the domain.

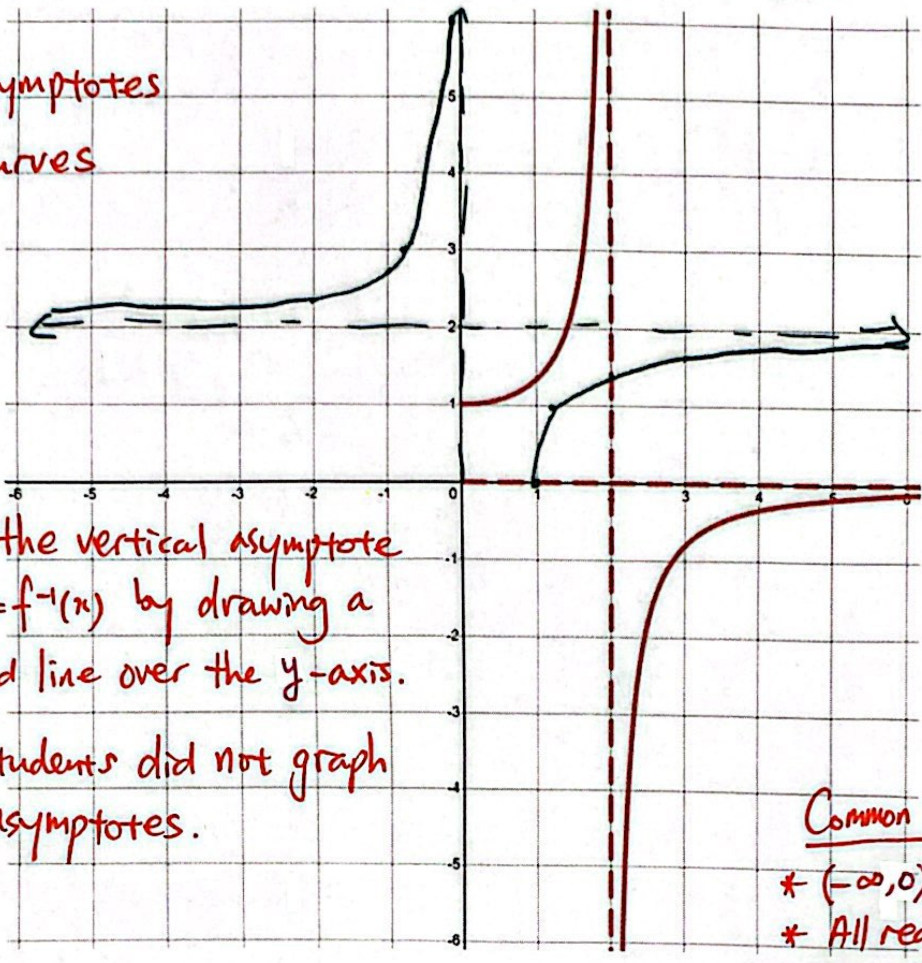
* $y = -2 \pm \sqrt{x+4}$
$\Downarrow$
$f^{-1}(x) = -2 + \sqrt{x+4}$
with either no reason or $x \geq -2$ instead.

* $f^{-1}(x) = -2 \pm \sqrt{x+4}$?

(c) (i) The diagram below shows a sketch of the graph $y = f(x)$, where $f(x) = \frac{4}{4-x^2}$, $x \geq 0$ **2**

① Asymptotes

① Curves



* Show the vertical asymptote of $y=f^{-1}(x)$ by drawing a dotted line over the y-axis.

* Few students did not graph any asymptotes.

Common mistakes for (ii)

* $(-\infty,0)\cup(0,\infty)$

* All real numbers, $x\neq 0$

* $(-\infty,0)\cup(1,\infty)$

* Accepted $x<0$, $x\geq 1$

On the same set of axis, sketch the graph of the inverse function, $y = f^{-1}(x)$

Poorly done

(ii) State the domain of $f^{-1}(x)$ **2**

① ① $(-\infty,0)\cup[1,\infty)$

Alternatively, $x<0$ or $x\geq 1$

(iii) Find an expression for $y = f^{-1}(x)$ in terms of x. **2**

$$y=\frac{4}{4-x^2}, \quad x\geq 0$$

$$x=\frac{4}{4-y^2}, \quad y\geq 0$$

$$x(4-y^2)=4$$

$$4-y^2=\frac{4}{x}$$

$$y^2=4-\frac{4}{x}$$

① $y=\pm\sqrt{4-\frac{4}{x}}$

① $\therefore f^{-1}(x)=\sqrt{4-\frac{4}{x}}$ (since $y\geq 0$)

Common mistakes for (iii)

* $y^2=4-\frac{4}{x}$

$\Downarrow$

$y=\sqrt{4-\frac{4}{x}}$ with no reason.

Did not show $\pm$ sign.

* $y=\sqrt{\frac{4}{x}-4}$?

* $y=\pm\sqrt{4-\frac{4}{x}}$

$\Downarrow$

$y=\sqrt{4-\frac{4}{x}}$ since $x\geq 0$?