The Scots College

Student Number: _____

Teacher: _____

2023 **Year 12 Assessment Task 2**

Mathematics Extension 1

Date: Monday 1st May, 2023

General Instructions

- Reading time – 5 minutes
- Working time – 1 hour 30 minutes
- Write using black pen only.
- Calculators approved by NESA may be used.
- A reference sheet is provided separately.
- Show all relevant mathematical reasonings and/or calculations.
- One A4 page of double-sided hand-written notes can be utilised.

Total marks: 52

- **Section I** (8 marks)
  - Allow about 10 minutes for this section.
  - Attempt all questions.
  - Answer on the multiple-choice answer sheet provided.

- **Section II** (44 marks)
  - Allow about 1 hour 20 minutes for this section.
  - Attempt all questions, providing mathematical reasoning/calculations where necessary.
  - Start each question in a **new** booklet.

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Section I

8 marks
Attempt Questions 1–8.
Allow about 10 minutes for this section.

Use the multiple-choice answer sheet for Questions 1 – 8. Circle the correct response.

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1. Which of the following expressions is equivalent to $2\cos 7x \sin 2x$?

(A) $\sin 9x - \sin 5x$

(B) $\cos 5x + \cos 9x$

(C) $\sin 5x + \sin 9x$

(D) $\cos 9x - \cos 5x$

2. What is the angle between the vectors $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 18 \end{pmatrix}$?

(A) $\theta = \cos^{-1}\left(\frac{52}{65}\right)$

(B) $\theta = \cos^{-1}\left(\frac{-52}{65}\right)$

(C) $\theta = \cos^{-1}\left(\frac{33}{65}\right)$

(D) $\theta = \cos^{-1}\left(\frac{-33}{65}\right)$

3. The radius of a sphere is increasing at a rate of 6 centimetres per minute. What is the rate of increase of the volume of the sphere, in cubic centimetres per minute, when the radius is 7 centimetres?

(A) 112π

(B) 144π

(C) 1168π

(D) 1176π

4. A body has displacement of $4\underset{\sim}{i} + \underset{\sim}{j}$ metres at a particular time. The body moves with constant velocity and three seconds later its displacement is $-5\underset{\sim}{i} + 13\underset{\sim}{j}$ metres.

Which of the following represents the velocity vector in ms^{-1} ?

(A) $2\underset{\sim}{i} + 6\underset{\sim}{j}$

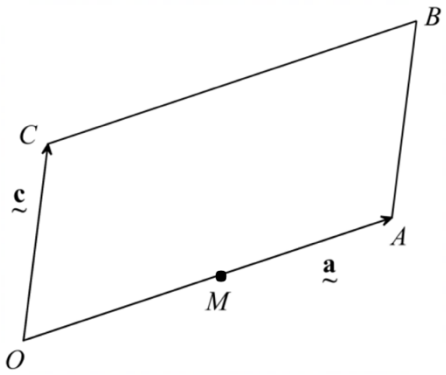
(B) $-2\underset{\sim}{i} + 2\underset{\sim}{j}$

(C) $4\underset{\sim}{i} - 3\underset{\sim}{j}$

(D) $-3\underset{\sim}{i} + 4\underset{\sim}{j}$

5. In the diagram, *OABC* is a parallelogram. The vector $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OC} = \underset{\sim}{c}$.

M is the midpoint of $\overrightarrow{OA}$. Which of the following expressions is represented by the vector $\overrightarrow{MC}$?



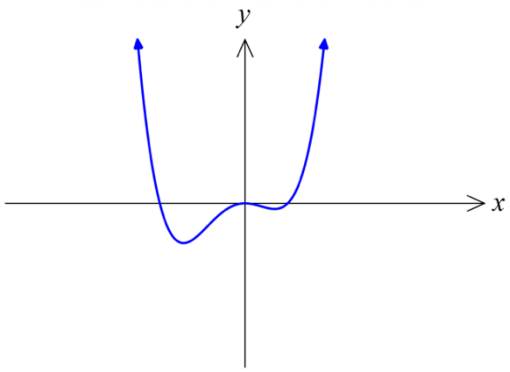
(A) $\underset{\sim}{a} + \frac{1}{2}\underset{\sim}{c}$

(B) $\underset{\sim}{c} - \frac{1}{2}\underset{\sim}{a}$

(C) $\frac{1}{2}\underset{\sim}{a} + \underset{\sim}{c}$

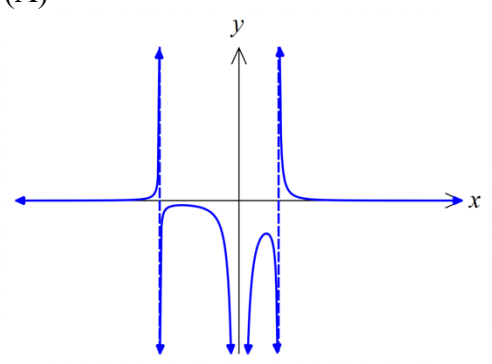
(D) $\frac{1}{2}\underset{\sim}{a} - \underset{\sim}{c}$

6. The diagram shows the graph of $f(x)$

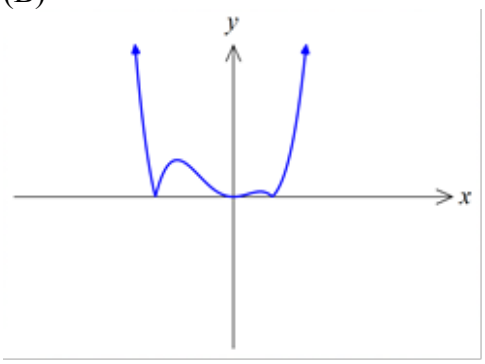


Which graph best represents $y = f(|x|)$?

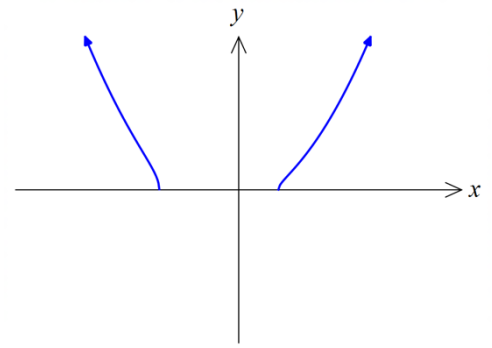
(A)



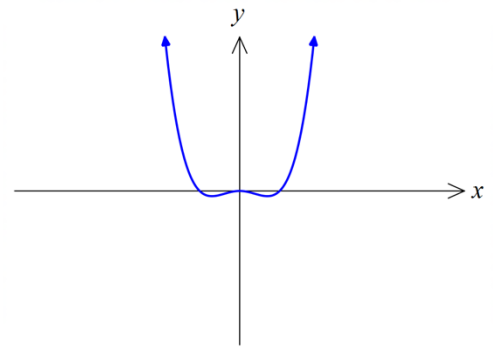
(B)



(C)



(D)



7. A and B are the points $(-4, 1)$ and $(5, 13)$ respectively. What is the unit vector in the direction of $\overrightarrow{AB}$?

(A) $4\underset{\sim}{i} + 3\underset{\sim}{j}$

(B) $\frac{3}{5}\underset{\sim}{i} + \frac{4}{5}\underset{\sim}{j}$

(C) $9\underset{\sim}{i} + 12\underset{\sim}{j}$

(D) $\frac{2}{5}\underset{\sim}{i} - \frac{4}{5}\underset{\sim}{j}$

8. What is the coefficient of x^3 in the expansion of $(2 - 3x)^7$?

(A) -864

(B) 864

(C) $-15\,120$

(D) $15\,120$

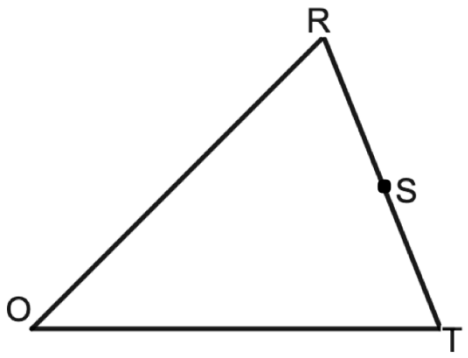
Section II
44 marks

Attempt Questions 9, 10 and 11 **writing your answers in separate booklets for each question**.
Allow about 1 hour 20 minutes for this section.

Question 9 (14 marks) Start a new answer booklet. **Mark**

(a) Use the substitution $t = \tan\frac{x}{2}$ to show that $\sec x + \tan x = \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$ **3**

(b) S is a point on the side RT of ΔORT. **3**
Given that OS is the perpendicular bisector of RT, prove using vectors that ΔORT is isosceles.



Not to Scale

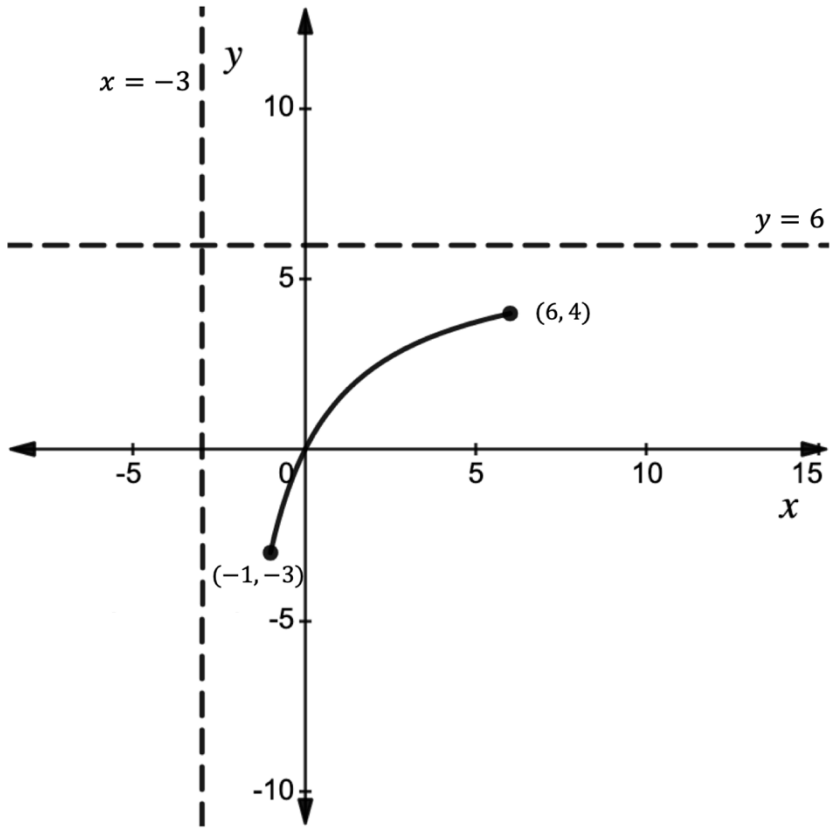
(c) A car is driving with velocity of $\left(7\underset{\sim}{i} - 5\underset{\sim}{j}\right) ms^{-1}$

(i) Find the exact speed of the car. **1**

(ii) Find the bearing the car is travelling on to the nearest degree. **2**

Question 9 continues on page 8

(d) The diagram below shows a sketch of the graph $y = f(x)$,
where $f(x) = \frac{6x}{x+3}$ with domain $-1 \leq x \leq 6$



(i) Copy or trace this diagram into your booklet. **2**
On the same set of axes, sketch the graph of the inverse function, $y = f^{-1}(x)$.

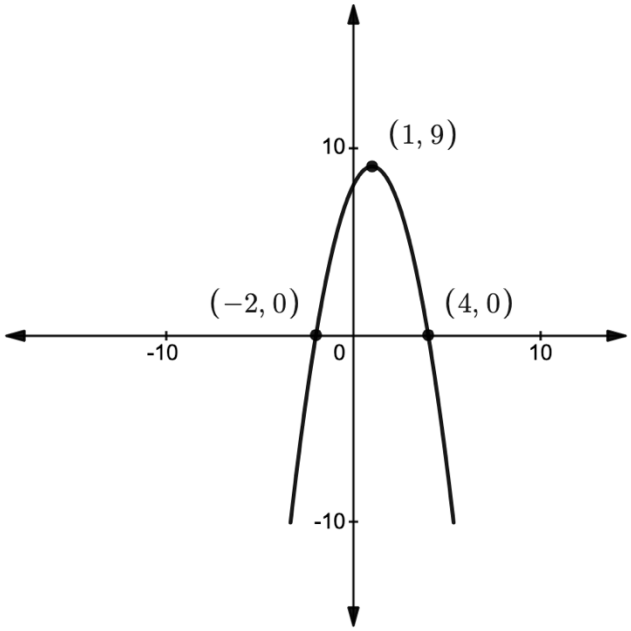
(ii) Find the coordinates of any points of intersection of the two curves $y = f(x)$ and $y = f^{-1}(x)$. **1**

(iii) Find an expression for $y = f^{-1}(x)$ in terms of x. **2**

Question 10 (14 marks) Start a **new** answer booklet. **Mark**

(a) The diagram shows the graph of $f(x) = -x^2 + 2x + 8$ **3**

In the answer booklet, sketch $y = \frac{1}{|f(x)|}$



(b) Prove by mathematical induction that for all integers $n \geq 1$, **3**

$9^{n+2} - 4^n$ is divisible by 5.

(c) Given that $f(x) = 5\sin x + 12\cos x$

(i) Express $5\sin x + 12\cos x$ in the form $R\sin(x + \alpha)$, where $R > 0$ and $0° \leq \alpha \leq 90°$ **3**

(ii) Find the maximum value of $5\sin x + 12\cos x$. **1**

(iii) Hence, find the smallest positive value of x, for which this maximum occurs. **1**

(d) If $\underset{\sim}{a} = -3\underset{\sim}{i} + \underset{\sim}{j}$ and $\underset{\sim}{b} = -m\underset{\sim}{i} - \underset{\sim}{j}$ are vectors, where $m \in R$. **3**
Find the value(s) of m if vector $\underset{\sim}{b}$ is perpendicular to vector $\underset{\sim}{a} - \underset{\sim}{b}$

Question 11 (16 marks) Start a **new** answer booklet. **Mark**

(a) A curve has the parametric equations

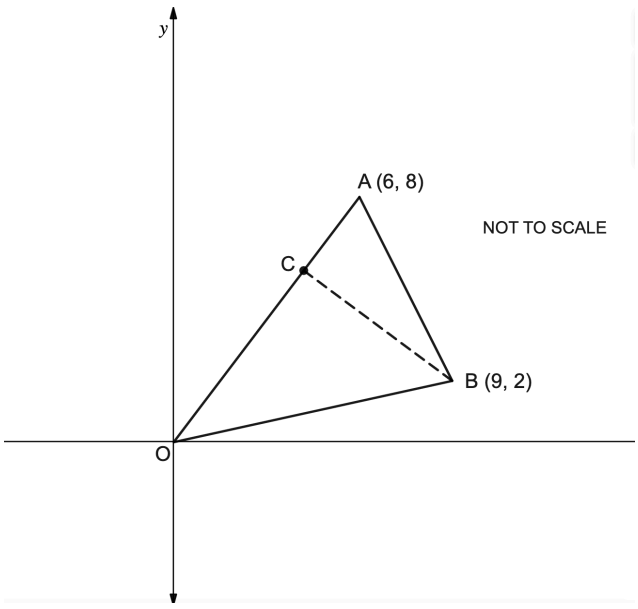
$$x = \sin^2 t, \qquad y = \sin 2t, \qquad 0 < t < \pi$$

(i) Show that $\frac{dy}{dx} = 2\cot 2t$ **3**

(ii) Find a cartesian equation for the curve **3**

(b) The coefficient of x in the expansion of $\left(x + \frac{1}{ax^2}\right)^7$ is $\frac{3}{7}$. Find all possible values of a. **3**

(c) The diagram below shows ΔOAC, where $\overrightarrow{OA} = 6\underset{\sim}{i} + 8\underset{\sim}{j}$ and $\overrightarrow{OB} = 9\underset{\sim}{i} + 2\underset{\sim}{j}$.

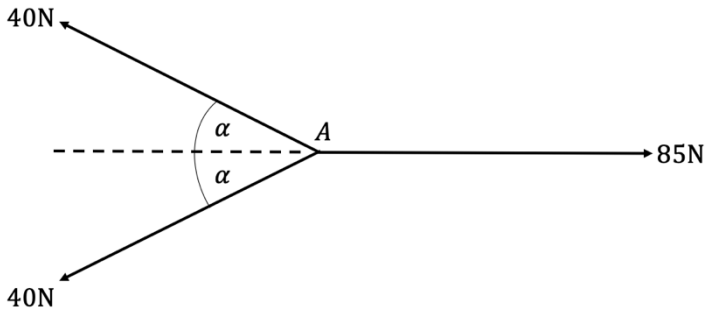


(i) By considering the projection of $\overrightarrow{OB}$ onto $\overrightarrow{OA}$, find the coordinates of C **2**

(ii) Hence, calculate the exact area of ΔOAB **2**

Question 11 continues on page 11

(d) Three forces of magnitude 85N, 40N and 40N act at a point A, as shown on the diagram below. The angle α is such that $\cos\alpha = \frac{3}{4}$



Find the magnitude and direction of the resultant vector. **3**

End of Examination ☺

Student Number: Solutions

Teacher: _____

Year 12 Mathematics Extension 1

Section I Multiple Choice Answer Sheet

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word ~~correct~~ and drawing an arrow as follows.

A ⊗ B ⊗ C ○ D ○

Question 1	**A** ●	**B** ○	**C** ○	**D** ○
Question 2	**A** ●	**B** ○	**C** ○	**D** ○
Question 3	**A** ○	**B** ○	**C** ○	**D** ●
Question 4	**A** ○	**B** ○	**C** ○	**D** ●
Question 5	**A** ○	**B** ●	**C** ○	**D** ○
Question 6	**A** ○	**B** ○	**C** ○	**D** ●
Question 7	**A** ○	**B** ●	**C** ○	**D** ○
Question 8	**A** ○	**B** ○	**C** ●	**D** ○

Question 9

a) LHS = $\sec x + \tan x$

$= \frac{1+t^2}{1-t^2} + \frac{2t}{1-t^2}$ (1)

$= \frac{t^2+2t+1}{1-t^2}$

$= \frac{(t+1)^2}{(1-t^2)(1+t)}$.

$= \frac{t+1}{1-t}$ $(\frac{1}{2})$

RHS = $\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$.

$= \frac{\tan \pi/4 + \tan \frac{x}{2}}{1 - \tan \pi/4 \tan \frac{x}{2}}$ (1)

$= \frac{1+t}{1 - 1 \times t}$

$= \frac{1+t}{1-t}$ $(\frac{1}{2})$.

$=$ LHS

b). Let $\overrightarrow{OR} = \underset{\sim}{r}$, $\overrightarrow{OT} = \underset{\sim}{t}$

$\overrightarrow{RT} = \underset{\sim}{t} - \underset{\sim}{r}$

$OS = t - \frac{1}{2}\overrightarrow{RT}$

$= t - \frac{1}{2}(\underset{\sim}{t} - \underset{\sim}{r})$

$= \frac{1}{2}(\underset{\sim}{t} + \underset{\sim}{r})$. } (1).

$\overrightarrow{OS} \perp \overrightarrow{RT}$ $\quad \overrightarrow{OS} \cdot \overrightarrow{RT} = 0$

$\frac{1}{2}(t+r) \cdot (t-r) = 0$

$\underset{\sim}{t}\underset{\sim}{t} - \underset{\sim}{t}\underset{\sim}{r} + \underset{\sim}{t}\underset{\sim}{r} - \underset{\sim}{r}\underset{\sim}{r} = 0$ } (1).

$t.t - \underset{\sim}{r}.\underset{\sim}{r} = 0$

$|t|^2 - |\underset{\sim}{r}|^2 = 0$

$|t| = |r|$ } (1).

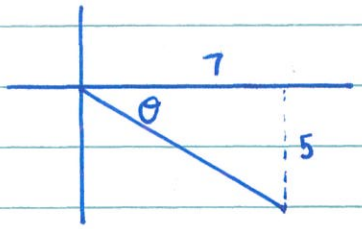
$\therefore$ $\triangle ORT$ is isosceles since $OT = OR$.

c) $7\underset{\sim}{i} - 5\underset{\sim}{j}$

i) speed $= \sqrt{7^2 + (-5)^2}$

$= \sqrt{74}$ (1).

ii).



$\tan\theta = \frac{5}{7}$

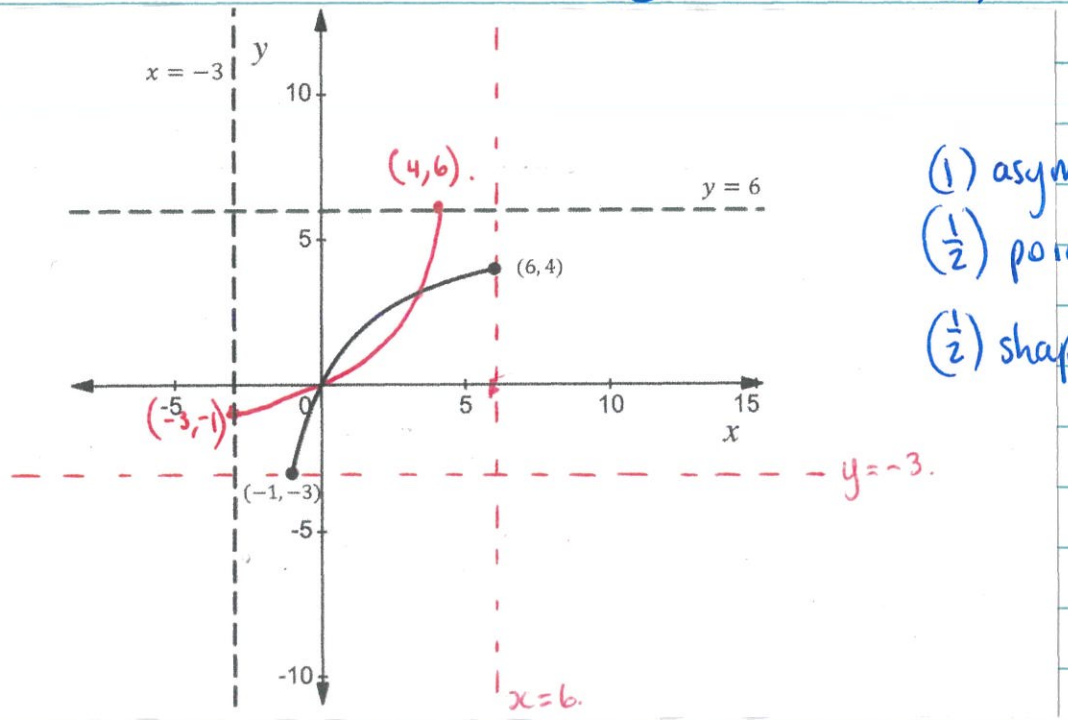
$\theta = 35.537677779$ (1)

$= 36°$ (nearest degree).

bearing $= 90 + 36$

$= 126°\,T$ (1).

d) i)



(1) asymptotes.

($\frac{1}{2}$) points

($\frac{1}{2}$) shape.

ii) $\frac{6x}{x+3} = x.$

$6x = x^2 + 3x$

$x^2 - 3x = 0$

$x(x-3) = 0$

$x = 0, 3.$

pt $(0,0)$, $(3,3)$. (1).

iii) $f^{-1}(x)$: $x = \frac{6y}{y+3}.$

$xy + 3x = 6y.$

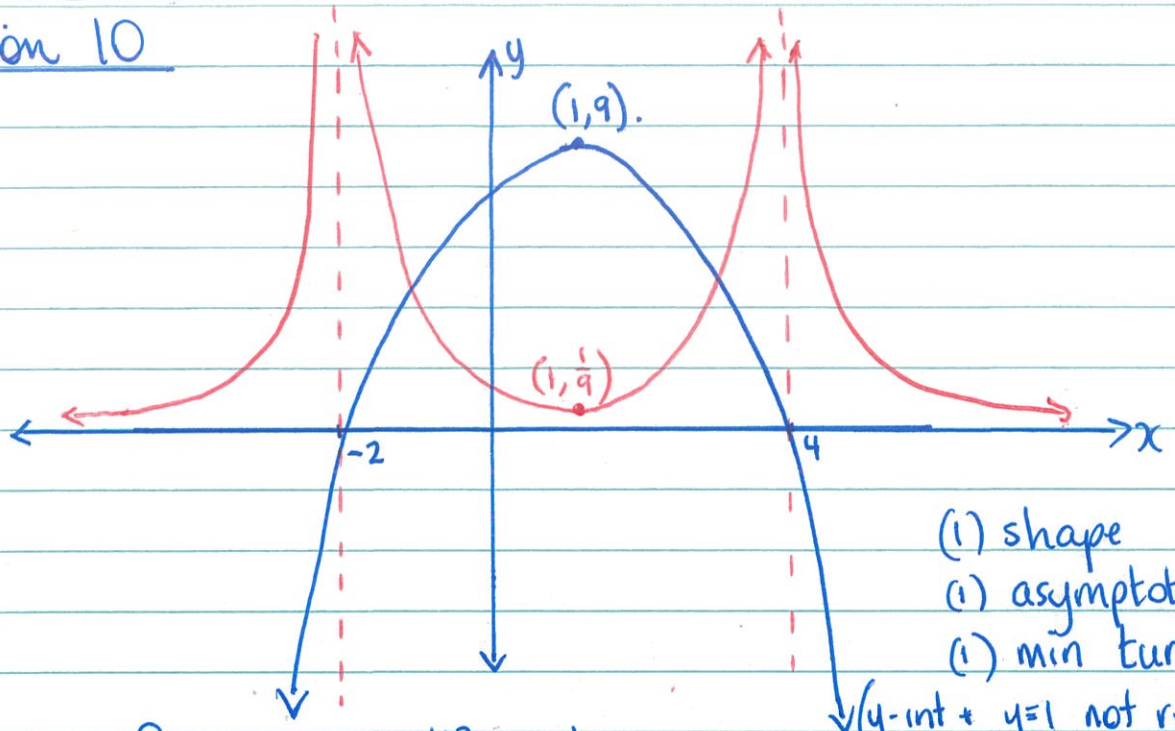
$3x = 6y - xy$

$3x = y(6-x).$ (1)

$y = \frac{3x}{6-x}$ (1)

Question 10

a)



(1) shape
(1) asymptotes.
(1) min turning pt.
(y-int & y=1 not required)

b) * test true for $n=1$ $\quad 9^{1+2} - 4^1 = 729 - 4$ (1)

$= 725$ which is divisible by 5.

$\therefore$ true for $n=1$

* Assume true for $n=k$

$9^{k+2} - 4^k = 5M$ where M is an integer (½)

* Prove true for $n=k+1$

$$9^{k+1+2} - 4^{k+1} = 9^{k+3} - 4^{k+1} \quad (\tfrac{1}{2})$$
$$= 9^{k+2} \cdot 9^1 - 4^{k+1}$$
$$= (5M + 4^k)9 - 4^{k+1} \text{ from the assumption}$$
$$= 5.9M + 9.4^k - 4.4^k$$
$$= 5.9M + 4^k(9-4)$$
$$= 5.9M + 4^k.5$$
$$= 5(9M + 4^k) \text{ which is divisible by 5 (1)}$$

since M & k are integers

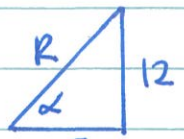
$\therefore$ Statement proved true by Mathematical induction

(-½) missing 'from assumption' or equivalent
(-½) missing 'as M & k are integers'

c) i) $5\sin x + 12\cos x = R\sin x\cos\alpha + R\cos x\sin\alpha$.

$R\cos\alpha = 5$ $\quad$ $R\sin\alpha = 12$

$\cos\alpha = \frac{5}{R}$ $\quad$ $\sin\alpha = \frac{12}{R}$



(1) working towards

$R^2 = 12^2 + 5^2$

$R = 13$ (1).

$\tan\alpha = \frac{12}{5}$

$\alpha = 67.38013505$

$= 67°$ (nearest degree). (1)

$5\sin x + 12\cos x = 13\sin(x + 67°)$.

ii) Maximum value is 13. (1).

iii). $13\sin(x+67) = 13$.

$\sin(x+67) = 1$

$x + 67 = 90°$

$x = 23°$ (1).

d). $\underset{\sim}{a} - \underset{\sim}{b} = \begin{pmatrix} -3 \\ 1 \end{pmatrix} - \begin{pmatrix} -m \\ -1 \end{pmatrix}$.

$= (-3+m)\underset{\sim}{i} + 2\underset{\sim}{j}$ (1).

$\underset{\sim}{b}(\underset{\sim}{a} - \underset{\sim}{b}) = 0$

$-m(-3+m) + (-1 \cdot 2) = 0$ (1).

$3m - m^2 - 2 = 0$

$m^2 - 3m + 2 = 0$

$(m-2)(m-1) = 0$

$m = 1$ or 2. (1).

Question 11

a) i) $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$.

$y = \sin 2t$ $\quad x = \sin^2 t$

$\frac{dy}{dt} = 2\cos 2t$ (1) $\quad \frac{dx}{dt} = 2\sin t \cos t$

$= \sin 2t$ (1)

$= 2\cos 2t \times \frac{1}{\sin 2t}$

$= \frac{2\cos 2t}{\sin 2t}$ (1).

$= 2\cot 2t$

$\sin t = \sqrt{x}$ $-\frac{1}{2}$ if no mention of why it is positive

$\cos t = \sqrt{1-x}$ must mention domain as to why it's positive

ii). $x = \sin^2 t$ $\quad y = 2\sin t \cos t$

$\cos t = \frac{y}{2\sin t}$

$\cos^2 t = \frac{y^2}{4\sin^2 t}$

$= \frac{y^2}{4x}$ (1).

$\sin^2 t + \cos^2 t = 1$.

$x + \frac{y^2}{4x} = 1$. (1).

$4x^2 + y^2 = 4x$.

$y^2 = 4x - 4x^2$. OR $4x^2 - 4x + y^2 = 0$.

(1).

b). ${}^{7}C_{k}\,(x)^{7-k}\left(\frac{1}{ax^2}\right)^{k}$.

$x^{7-k} \times (x)^{-2k}$

$7-k+(-2k)=1$ (M1)

$3k=6$

$k=2$. (1).

${}^{7}C_{2}\,(x)^{5}.\left(\frac{1}{ax^2}\right)^{2} = \frac{3}{7}x$

$\frac{21 \cdot 1}{a^2} = \frac{3}{7}$ } (1).

$a^2 = \frac{21 \times 7}{3}$

$a^2 = 49$

$a = \pm 7$ (1).

(2) for $a=7$.

c) i). $\text{proj}_{OA}\,OB = \frac{9 \cdot 6 + 2 \cdot 8}{\sqrt{6^2+8^2}^{\,2}}\,(6\underset{\sim}{i} + 8\underset{\sim}{j})$. ($\frac{1}{2}$) (1).

$= \frac{70}{100}\,(6\underset{\sim}{i} + 8\underset{\sim}{j})$. ($\frac{1}{2}$).

$= \frac{21}{5}\underset{\sim}{i} + \frac{28}{5}\underset{\sim}{j}$. ($-\frac{1}{2}$) if left like this. ($\frac{1}{2}$)

$\therefore$ coordinates of C $(4.2, 5.6)$ or $\left(\frac{21}{5}, \frac{28}{5}\right)$. (M1) ($\frac{1}{2}$).

ii). $|\vec{BC}| = \sqrt{(9-4\cdot2)^2 + (2-5\cdot6)^2}$

$= \sqrt{36}$

$= 6$ (1)

$|\vec{OA}| = \sqrt{6^2+8^2}$

$= 10$ $(\frac{1}{2})$

Area $\triangle AOB = \frac{1}{2} \times 10 \times 6$

$= 30u^2$. $(\frac{1}{2})$.

d). $\sum F$ horizontal $= -40\cos\alpha + (-40\cos\alpha) + 85$

$= 2\times(-40\times\frac{3}{4}) + 85$

$= -60 + 85$

$= +25$ (1).

vertical $= 40\sin\alpha + (-40\sin\alpha) + 0$

$= 0$. (1)

∴ resultant vector has a magnitude of 25N to the right/eas

(1).

$\frac{1}{2}$ mag + $\frac{1}{2}$ direction

bearing T° required $(-\frac{1}{2})$

0° not accepted. $(-\frac{1}{2})$.

(-1) no vertical resolution working out shown or mention that they cancel each other out.