Student Name:
Teacher:

St George Girls High School

Mathematics Extension 1

2021 Year 11 HSC Assessment Task 1

General Instructions

- Reading time – 5 minutes
- Working time – 70 minutes
- Write using black pen.
- Calculators approved by NESA may be used.
- A reference sheet is provided.
- For questions in **Section I**, use the multiple-choice answer sheet provided at the back of this exam paper.

  For questions in **Section II:**
  - Answer the questions in the **writing booklets** provided.
  - Start each question in a new writing booklet.
  - Show relevant mathematical reasoning and/or calculations.
  - Marks may not be awarded for incomplete or poorly presented solutions, or where multiple solutions are provided.

Total marks: 45

Section I – 5 marks (pages 3 – 4)

- Attempt Questions 1–5
- Allow about 8 minutes for this section

Section II – 40 marks (pages 5 – 10)

- Attempt Questions 6–11
- Allow about 62 minutes for this section

Q1-5	/5
Q6	/7
Q7	/7
Q8	/7
Q9	/6
Q10	/7
Q11	/6
TOTAL	**/45**
	%

Section I - Multiple Choice

5 marks
Attempt Questions 1 – 5
Allow about 8 minutes for this section
Circle the correct answer for each question in this booklet.

1. In how many ways can the letters of the word CAMBRIDGE be arranged?

(A) $8!$

(B) ${}^{9}C_{8}$

(C) $9!$

(D) ${}^{9}C_{9}$

2. The expansion $(a+b)^{100}$ has

(A) 99 terms.

(B) 100 terms.

(C) 101 terms.

(D) 102 terms.

3. Three marbles are drawn at random from a bag that contains 5 identical blue marbles, 6 identical green marbles and 3 identical red marbles.
Which expression below gives the correct probability that exactly two blue marbles are drawn?

(A) $\dfrac{{}^{5}C_{2}}{{}^{14}C_{3}}$

(B) $\dfrac{{}^{5}C_{2} \times {}^{9}C_{1}}{{}^{14}C_{3}}$

(C) $\dfrac{2}{5} \times \dfrac{1}{9}$

(D) $\dfrac{2}{14} \times \dfrac{1}{9}$

4. Each member of a club is to write three different numbers, from 1 to 10 inclusive, on a card and place it in a bag.
What is the smallest number of members in the club that guarantees that at least two members wrote the same combination of three numbers, in any order on their cards?

(A) 121

(B) 120

(C) 720

(D) 721

5. Out of 8 divers, 5 were selected to represent Australia in the olympics. Three of these five divers were placed 1st, 2nd and 3rd.
In how many ways could this process be carried out?

(A) $\frac{8!}{5!3!}$

(B) $\frac{8!}{5!}$

(C) $\frac{8!}{3!2!}$

(D) $\frac{8!}{3!3!}$

End of Multiple-Choice Section I

Section II

41 marks
Attempt Questions 6 – 11
Allow about 62 minutes for this section

Marks

Question 6 (7 marks) - **Start a NEW Writing Booklet.**

a) There are only 5 people allowed to line up outside the MIMCO shop due to the current COVID restrictions. If there are 10 people on their way to shop at MIMCO, in how many possible ways could this line be formed? **1**

b) A closed box contains twelve balls numbered 1, 2, 3, ...,12.

(i) How many unordered selections of 4 balls can be made from the box? **1**

(ii) What is the probability that a selection of 4 balls chosen at random contains only even numbered balls? **1**

c) There are 74 students from Year 7,8,9,10,11 and 12 sitting in the library. Explain why there must be at least 13 students from at least one of the year groups in the library. **2**

d) Simplify $\frac{(n+1)!}{n^2+n}$. **2**

Question 7 (7 marks) - **Start a NEW Writing Booklet.** **Marks**

a) (i) Use Pascal's triangle to write out the expansion of $(3x - \frac{2}{x^2})^4$. **2**
(You DO NOT need to simplify your expansion)

(ii) Hence write down the term that contains x^2. **1**

b) A Netball competition has 12 clubs with a large number of players in each club. How many players from these clubs must Melissa know in order to know at least 20 players from at least one club. **1**

c) Consider the set $\{M, N, O, P, Q, R, S, T\}$.

(i) How many subsets can be made from this set? **1**

(ii) How many subsets with at least three letters do not contain the letter R? **2**

Question 8 (7 marks) - **Start a NEW Writing Booklet.** **Marks**

a) Find the coefficient of x^5 in $(3-2x^3)(1+2x)^8$. **3**

b) The six Year 12 Extension 1 Maths teachers (Mrs Iosifidis, Mrs Chand, Mr Moncrieff, Mrs Moncrieff, Mr Cowper and Ms Dodd) are to sit at a round table to discuss how to mark the recent exam.

(i) In how many ways can the teachers be arranged? **1**

(ii) In how many ways can they be arranged if Mrs Moncrieff insists on sitting next to Mr Moncrieff on his right-hand side? **1**

(iii) What is the probability that Mr Cowper sits between Mr and Mrs Moncrieff? **2**

Question 9 (6 marks) - **Start a NEW Writing Booklet.** **Marks**

a) (i) How many arrangements of the letters for the surname IOSIFIDIS are possible ? **1**

(ii) How many of the arrangements in part (i) will the S's be separated? **2**

b) Using the fact that $(1+x)^3(1+x)^8 = (1+x)^{11}$, show that

$${}^3C_0\,{}^8C_3 + {}^3C_1\,{}^8C_2 + {}^3C_2\,{}^8C_1 + {}^3C_3\,{}^8C_0 = {}^{11}C_3\,.$$ **3**

Question 10 (7 marks) - **Start a NEW Writing Booklet.** **Marks**

a) In a group there are 12 boys and 8 girls. In how many ways can a committee of five be chosen if

(i) there are no restrictions. **1**

(ii) there are to be three boys and two girls in the committee. **1**

(iii) at least one boy and one girl are to be included in the committee. **2**

b) Positive integers less than 700 are formed from the digits 2,4,6,7 and 9 with no repetitions allowed. If one of these integers is chosen at random, find the probability that it is odd. **3**

Question 11 (6 marks) - **Start a NEW Writing Booklet.** **Marks**

a) A team of k people are to be selected from a total of n applicants.
Let Matilda be one of these applicants.

(i) In how many ways can the team be selected if Matilda is on the team? **1**

(ii) In how many ways can the team be selected if Matilda is not on the team? **1**

(iii) Without using factorial notation, use the scenario described above to explain the identity

$$^{n}C_{k} = {}^{n-1}C_{k-1} + {}^{n-1}C_{k}\,.$$ **2**

b) St George Girls High School has a garden in the shape of an equilateral triangle with each side measuring 2 km. There are 5 students who like to hide in the garden as far away from each other as possible.
Prove that no matter how hard they try to get away from each other, there are still always two of these students within 1 km of each other. **2**

End of Examination

Student Name: ____________________ **Class Teacher:** ____________________

Section I

Year 11 HSC Assessment Task 1 2021

Mathematics Extension 1

Multiple-choice Answer Sheet - Questions 1 – 5

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ~~●~~ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows:

correct

A ~~●~~ B ~~●~~ C ○ D ○

1. A ○ B ○ C ○ D ○

2. A ○ B ○ C ○ D ○

3. A ○ B ○ C ○ D ○

4. A ○ B ○ C ○ D ○

5. A ○ B ○ C ○ D ○

Yr11 HSC Assessment Task 1 2021 - SOLUTIONS

1. $9!$ C

2. 101 C

3. $\dfrac{{}^5C_2 \times {}^9C_1}{{}^{14}C_3}$ B

No restrictions ${}^{14}C_3$

${}^5C_2$ to see all ways to choose 2 blue

then $14-5=9$

${}^9C_1$ for the third marble.

4. ${}^{10}C_3$ ways to pick three different numbers (order not important)

$=120$

So worst case 120 members all pick different combinations and then the next member must pick a combination that is the same as another member

$\therefore 120+1$

$=121$ A

5.

$${}^8C_5 \times {}^5P_3$$

$$= \frac{8!}{5!(8-5)!} \times \frac{5!}{(5-3)!}$$

$$= \frac{8!}{3!\,2!}$$ C

C C B A C

MATHEMATICS EXTENSION 1 – QUESTION 6

a) $^{10}P_5 = 10 \times 9 \times 8 \times 7 \times 6$

$= 30\,240$ (1)

b) i $^{12}C_4 = \dfrac{12 \times 11 \times 10 \times 9}{4!}$

$= 495$ (1)

ii Selections with only even numbers $= {}^6C_4$
$=15$

$\therefore P(\text{only even}) = \dfrac{15}{495}$

$= \dfrac{1}{33}$ (1)

c) Consider 72 students over the six grades. If they were evenly distributed, there would be 12 from each year ~~gr~~ group. The 73rd and 74th students must each belong to one of those year groups so there must be at least 13 students in at least one year group.

OR

Consider the 74 students as the pigeons and the 6 year groups as the pigeonholes. By the pigeonhole theorem, there must be at least $\left\lceil \frac{74}{6} \right\rceil$ students in at least one year group.

$\left\lceil \dfrac{74}{6} \right\rceil = \left\lceil 12\tfrac{1}{3} \right\rceil$

$= 13$

$\therefore$ there must be at least 13 students from at least one of the year groups.

Full solution ②

Partial explanation, or showing that $74 \div 6 = 12\frac{1}{3}$, but without explanation of <u>why</u> that should be rounded up to 13 ①

In general this question was not well done. Notation for the ceiling function was poor; square brackets are not the same thing. Many students did not <u>explain</u>; simply finding $74 \div 6$ and rounding up with no explanation as to <u>why</u> did not attract full marks.

d) $$\frac{(n+1)!}{n^2+n} = \frac{(n+1)\,n!}{(n+1)\,n}$$

$$= \frac{n(n-1)!}{n}$$

$$= (n-1)!$$

Full solution ②

Some correct "unrolling" of factorials ①

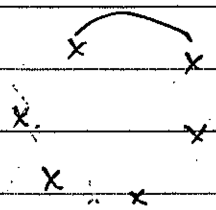
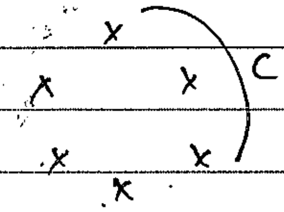
MATHEMATICS EXTENSION 1 – QUESTION 7

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a) $\left(3x-\frac{2}{x}\right)^4$		1 mark correct row pascal's Δ.
(i) 1 1 1 1 2 1 1 3 3 1 1 4 6 4 1		
$= (3x)^4 + 4(3x)^3\left(-\frac{2}{x}\right) + 6(3x)^2\left(-\frac{2}{x}\right)^2 + 4(3x)\left(-\frac{2}{x}\right)^3 + \left(-\frac{2}{x}\right)^4$		
or $= {}^4C_0(3x)^4 + {}^4C_1(3x)^3\left(-\frac{2}{x}\right) + {}^4C_2(3x)^2\left(-\frac{2}{x}\right)^2 + {}^4C_3(3x)\left(-\frac{2}{x}\right)^3 + {}^4C_4\left(-\frac{2}{x}\right)^4$		1 mark correct exponsion (no need to simplify)
(ii) ${}^4C_1(3x)^3\left(\frac{-2}{x}\right)$		
$= 4 \times 27\cancel{x^3}^{\,x^2} \times -\frac{2}{\cancel{x}}$		
$= -216x^2$		
b) $20-1=19$		
$19 \times 12 = 228$		
$228+1=229$		
c) (i) $^{8} = 256$	1 mark	correct answer
(ii) $2^7 - ({}^7C_0 + {}^7C_1 + {}^7C_2)$	1 mark	2^7
$= 99$	1 mark	c rrect answe
OR		
$C_3 + {}^7C_4 + {}^7C_5 + {}^7C_6 + {}^7C_7$	1 mark	addition
$= 99$	1 mark	correct answer
OR		
$2^8 - ({}^8C_0 + C_1 + {}^8C_2) - ({}^7C_2 + {}^7C_3 + {}^7C_4 + {}^7C_5 + {}^7C_6 + {}^7C_7)$	1 mark	for each subtraction.
$= 99$		

MATHEMATICS EXTENSION 1 – QUESTION 8

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a) $(3-2x^3)\left[\binom{8}{0}+\binom{8}{1}(2x)+\binom{8}{2}(2x)^2+\binom{8}{3}(2x)^3\right.$	1 mk	
$\left.+\binom{8}{4}(2x)^4+\binom{8}{5}(2x)^5+\binom{8}{6}(2x)^6+\binom{8}{7}(2x)^7+\binom{8}{8}(2x)^8\right]$		
Terms with x^5 are:		
$3\times\binom{8}{5}(2x)^5 + -2x^3\times\binom{8}{2}(2x)^2$		
But we need the coefficient of x^5		
$\therefore\ (3)\times\underbrace{\binom{8}{5}2^5}_{\frac{1}{2}\text{ mk}} + \underbrace{(-2)\binom{8}{2}2^2}_{\frac{1}{2}\text{ mk}}$		
$= 5376 - 224$	$\frac{1}{2}$ mk	for showing the understanding that the coefficients are added together.
$= 5152$ — $\frac{1}{2}$ mk		
or General term $=\binom{n}{k}a^{n-k}b^k$		
Need terms with x^5		
$\therefore\ (3-2x^3)\times\binom{8}{k}(1)^{8-k}(2x)^k$		
$3\times\binom{8}{5}(1)^{8-5}(2)^5x^5$	— 1 mk	
$+(-2)\binom{8}{2}(1)^{8-2}(2)^2x^2$		
Coefficient of x^5		
$=3\times\underbrace{\binom{8}{5}2^5}_{\frac{1}{2}\text{ mk}} + \underbrace{(-2)\binom{8}{2}2^2}_{\frac{1}{2}\text{ mk}}$		
$= 5376 - 224$ — $\frac{1}{2}$ mk		
$= 5152$ — $\frac{1}{2}$ mk		

MATHEMATICS EXTENSION 1 – QUESTION 8

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
b) i) $5! = 120$	1mk	Generally well done
ii)  $4! = 24$	1mk	Quite well done.
iii)  $3! \times 2! = 12$ Mr and Mrs Moncrieff can swap positions $4 - 1 = 3$ different groups 'fix' one group as the other teachers are going to be arranged relative to this 'fixed' group.	1mk	Generally well done. However, a few students forgot to calculate the probability
$\therefore$ probability $= \frac{12}{120} = \frac{1}{10}$	1mk	

MATHEMATICS EXTENSION 1 – QUESTION 9

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a) (i) 9 letters, 4 I's and 2 S's		
Number of arrangements $= \frac{9!}{4!2!}$		This part was
		well done by
$= 7560$	1	most students
(ii) Find the number of arrangement where the S's are together.		
8 letters, 4 I's		
Number of arrangements $= \frac{8!}{4!}$	1	
$= 1680$		The majority
Number of arrangements where the		of students
S's are separated $= 7560 - 1680$		did this well.
$= 5880$	1	
b) RHS $= {}^{11}C_0 + {}^{11}C_1 x + {}^{11}C_2 x^2 + {}^{11}\ \ x^3 + \ldots + {}^{11}C_{11} x^{11}$		
The coefficient of x^3 is ${}^{11}C_3$	1	1 mark for
LHS $= (1+x)^3(1+x)^8$		coefficient of x^3
$(1+x)^3 = {}^3C_0 + {}^3C_1 x + {}^3C_2 x^2 + {}^3C_3 x^3$		
$(1+x)^8 = {}^8C_0 + {}^8C_1 x + {}^8C_2 x^2 + {}^8C_3 x^3 + \ldots + {}^8C_8 x^8$	1	
LHS $= ({}^3C_0 + {}^3C_1 x + {}^3C_2 x^2 + {}^3C_3 x^3)({}^8C_0 + {}^8C_1 x + {}^8C_2 x^2 + {}^8C_3 x^3 + \ldots)$		1 mark for showing
$= {}^3C_0\,{}^8C_0 + {}^3C_0\,{}^8C_1 x + {}^3C_0\,{}^8C_2 x^2 + {}^3C_0\,{}^8C_3 x^3 + \ldots + {}^3C_1 x\,{}^8C_2 x^2 + \ldots$		the expansion
$\ldots + {}^3C_2 x^2\,{}^8C_1 x + \ldots + {}^3C_3 x^3\,{}^8C_0 + \ldots)$		
Equating coefficients of x^3 the coefficient	1	1 mark for
of x^3 in LHS is ${}^3C_0\,{}^8C_3 + {}^3C_1\,{}^8C_2 + {}^3C_2\,{}^8C_1 + {}^3C_3\,{}^8C_0$		equating coefficients
$\therefore {}^3C_0\,{}^8C_3 + {}^3C_1\,{}^8C_2 + {}^3C_2\,{}^8C_1 + {}^3C_3\,{}^8C_0 = {}^{11}C_3$		

EXT 1 HSC Task 1 - Nov 2021

MATHEMATICS EXTENSION 1 – QUESTION 10

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
(a) (i) ${}^{20}C_5 = 15\,504$	1	Done well
(ii) ${}^{12}C_3 \times {}^{8}C_2 = 6160$	1	Done well
(iii) A committe of five is made up of:		
1 Boy and 4 Girls: Nº of ways $= {}^{12}C_1 \times {}^{8}C_4 = 840$	1mk	Some student added the subsets instead of multiply.
2 Boys, 3 Girls: Nº of ways $= {}^{12}C_2 \times {}^{8}C_3 = 3696$		
3 Boys, 2 Girls: Nº of ways $= {}^{12}C_3 \times {}^{8}C_2 = 160$		
4 Boys, 1 Girl: Nº of ways $= {}^{12}C_4 \times {}^{8}C_1 = 3960$		
Total number of ways $= 840 + 3696 + 160 + 3960$ $= 14\,656$	1 mark	2
Alternatively, Number of ways $= {}^{20}C_5 - (5 \text{ Girls or } 5 \text{ Boys})$		Many student attempted this method but some made errors along the ways.
$= 15\,504 - ({}^{8}C_5 + {}^{12}C_5)$ 1 1		
$= 15\,504 - (56 + 792)$ $= 14\,656$		
		2

EXT 1 HSC Task 1 – NOV 2021

MATHEMATICS EXTENSION 1 – QUESTION 10

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
(b) For the total number of ways of selecting positive integers < 700 from the digits 2, 4, 6, 7 and 9 we look at: 3 cases.		
Case 1: 3 digits $\underset{2 \text{ or } 4 \text{ or } 6}{\boxed{3}} \times \overbrace{\underbrace{\boxed{4} \times \boxed{3}}_{^4P_2}}^{\text{remaining}} = 36$	1 mark (Cases 1–3)	½ mark each for at most 2 cases
Case 2: 2 digits $\underset{2,4,6,7 \text{ or } 9}{\boxed{5}} \times \underset{\text{remaining}}{\boxed{4}} = 20$		
Case 3: 1 digit: $\boxed{5} = 5$		NOTE: Many students only considered a 3 digit number < 700 and not a 2 digit or 1 digit number.
$\therefore n(S) = 36 + 20 + 5$		
$= 61$ ways		
For $n(E)$, Now for the odd numbers, $n(E)$ the last digit is either 7 or 9.		
Case 1: 3 digits $\underset{2 \text{ or } 4 \text{ or } 6}{\boxed{3}} \times \underset{3 \text{ other}}{\boxed{3}} \times \underset{7 \text{ or } 9}{\boxed{2}} = 18$	½	
Case 2: 2 digits $\underset{^4P_1}{\boxed{4}} \times \underset{7 \text{ or } 9}{\boxed{2}} = 8$	½	
Case 3: 1 digit: $\underset{7}{\boxed{1}}$ or $\underset{9}{\boxed{1}} = 2$	½	
$\therefore n(E) = 18 + 8 + 2$		
$= 28$		
$\therefore P(\text{odd} < 7) = \dfrac{28}{61}$	½	3

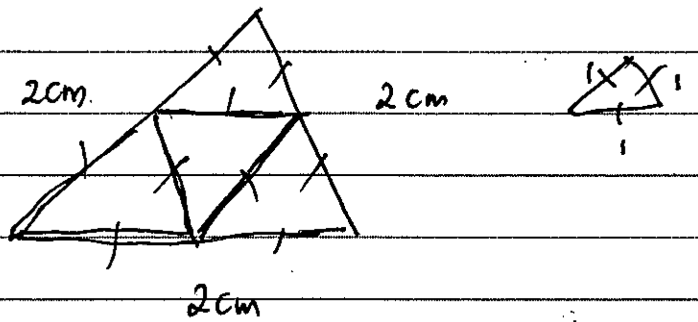
HSC task 1 - 2021

MATHEMATICS EXTENSION 1 – QUESTION 11		
pg 1		
SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
(a) team of k people with n applicants.		
Including Matilda.		
(i) if team of k is chosen with Matilda		
on the team. as Matilda has 1 spot		
select $k-1$ from $n-1$ applicants		
not Matilda		
$\therefore {}^{n-1}C_{k-1}$	1	well done
(ii) if team of k with Matilda NOT on		
the team... select k members from $n-1$ applicants	1	well done
$\therefore {}^{n-1}C_k$		
(iii) Prove ${}^nC_k = {}^{n-1}C_{k-1} + {}^{n-1}C_k$		
Without using factorials & the scenario.		
$LHS = {}^nC_k$		
Using the scenario we could consider the		
LHS to be equivalent to choosing k		
applicants from n players when choosing		
the team		
Matilda being on the team and not on the	1	need to show this understanding
team are mutually exclusive and complementary events		
So...		
If k players are chosen from n applicants,		
either Matilda could be in the team, or not.		
If Matilda is in the team		
$k-1$ players need to be chosen from $n-1$		
players ie ${}^{n-1}C_{k-1}$ as Matilda is		
already on the team.		

HSC task 1 - 2021

MATHEMATICS EXTENSION 1 – QUESTION 11

pg 2

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
If Matilda is not on the team		
k players need to be chosen from $n-1$ players as Matilda is NOT included ie $^{n-1}C_k$		Some students didn't show enough detail.
So combining Matilda in or not in.		
$^nC_k = {^{n-1}C_{k-1}} + {^{n-1}C_k}$.		
(b) 	1	sensible partioning showing an understanding of pigeonhole principle.
step 1: Break up into 4 equilateral triangles of length 1cm for each side.		
Placing 5 students into 4 particians, by pigeonhole principle $\lceil \frac{5}{4} \rceil = \lceil 1\frac{1}{4} \rceil = 2$		
$\therefore$ at least 2 students will lie in one of the smaller equilateral triangles, within 1 km of each other.		