SYDNEY BOYS HIGH SCHOOL

2019

YEAR 11
TERM 2
COHORT
TASK 1

Mathematics Extension

General Instructions

- Reading time – 5 minutes
- Working time – 90 minutes
- Write using black pen.
- NESA approved calculators may be used.
- A reference sheet is provided with this paper.
- Leave your answers in the simplest exact form, unless otherwise stated.
- Marks may **NOT** be awarded for messy or badly arranged work.
- In Questions 11–17, show ALL relevant mathematical reasoning and/or calculations.

Total Marks: 76

Section I – 10 marks (pages 2–4)

- Attempt Questions 1–10
- Allow about 15 minutes for this section.

Section II – 66 marks (pages 5–11)

- Attempt Questions 11–17
- Allow about 1 hour and 15 minutes for this section.

Examiner: *External Examiner*

THIS PAPER IS NOT TO BE REMOVED FROM THE EXAM HALL

Section I – Multiple Choice

10 Marks
Attempt question 1−10
Allow approximately 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1−10

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1 Which of the following is equal to $\left(2\sqrt{3}+3\sqrt{2}\right)^2$?

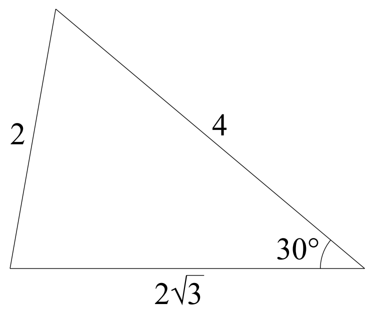
(A) 30

(B) $30+6\sqrt{6}$

(C) $4\sqrt{3}+9\sqrt{2}$

(D) $30+12\sqrt{6}$

2 What is the area of the triangle drawn below?



(A) 4 u^2

(B) $2\sqrt{3}$ u^2

(C) $4\sqrt{3}$ u^2

(D) $2\sqrt{6}$ u^2

3 Which of the following is equal to $\cot x \times \sec x$?

(A) $\sin x$

(B) $\operatorname{cosec} x$

(C) $\cos x$

(D) $\sec x$

4 Which of the following is neither odd nor even?

(A) $f(x) = \sin x$

(B) $f(x) = \dfrac{x}{x^2+3}$

(C) $f(x) = -\sqrt{4-x^2}$

(D) $f(x) = \dfrac{1}{x} + 3$

5 Which of the following is the result when $2\log a + \log b + \log b$ is simplified?

(A) $2\log(ab)$

(B) $\log(2a^2b)$

(C) $\log(a^2 + 2b)$

(D) $\log(2a + 2b)$

6 Which of the following is the correct factorisation of $8x^3 + 27$?

(A) $(2x + 3)(4x^2 + 6x + 9)$

(B) $(2x + 3)(4x^2 - 6x + 9)$

(C) $(2x - 3)(4x^2 + 6x + 9)$

(D) $(2x - 3)(4x^2 - 6x + 9)$

7 A_n is the nth term in a sequence.
Which one of the following does NOT define a geometric sequence?

(A) $A_{n+1} = 4,\ A_0 = 4$

(B) $A_{n+1} = n,\ A_0 = 1$

(C) $A_{n+1} = -A_n,\ A_0 = 5$

(D) $A_{n+1} = 4A_n,\ A_0 = 2$

8 Siobhan invests \$1200 for two years. Interest is calculated at the rate of 3.35% p.a., compounding monthly.
What is the future value of her investment in two years time?

(A) \$2646.26

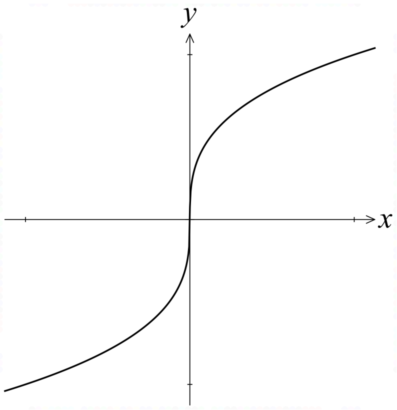
(B) \$1283.03

(C) \$1281.75

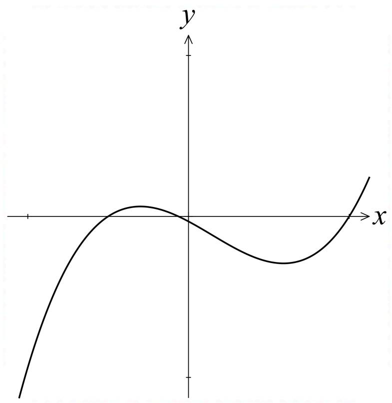
(D) \$1206.71

9 Which of the graphs below would be classified as one-to-many?

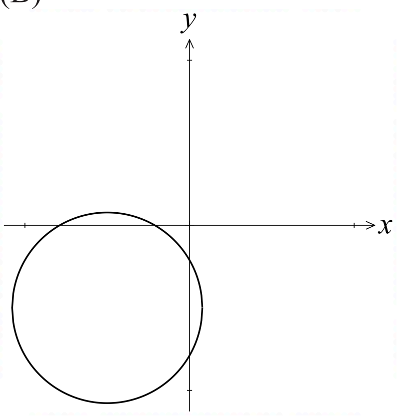
(A)



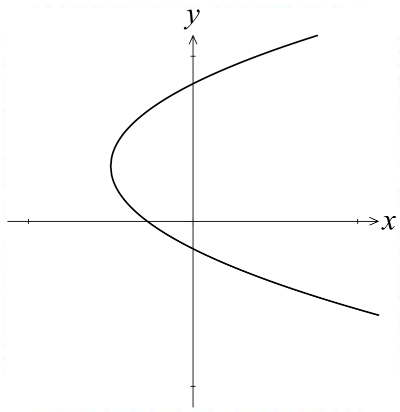
(C)



(B)



(D)



10 Johnson took out a loan for \$20 000 to buy a new sound system.
The contract required that he pay the loan over 5 years with monthly instalments of \$421.02.
After 2 ½ years, Johnson calculated that he owed the following amount of money:

$$\$\left[20\,000(1.008)^{30} - \frac{421.02(1.008^{30}-1)}{0.008}\right]$$

How much was the interest rate per annum charged by the bank for this reducing balance loan?

(A) 1.008% (B) 0.096% (C) 9.6% (D) 12.095%

Section II

75 marks
Attempt Questions 11–17
Allow about 1 hour and 15 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11–17, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (12 Marks) Use a SEPARATE writing booklet.

(a) Evaluate

(i) $|-5|-|7|$. **1**

(ii) $\log_4 64$. **1**

(b) What is the exact value of $\sin 240°$? **1**

(c) Simplify fully

(i) $\dfrac{x}{x^2+3x}$. **1**

(ii) $\sqrt{2y}\left(\sqrt{2y}+\dfrac{1}{\sqrt{2y}}\right)$. **1**

(d) Rationalise the denominator of $\dfrac{7}{3-\sqrt{2}}$. **2**

(e) Consider the points $M(-4, 4)$ and $N(6, -3)$ in the number plane.

(i) Write down the coordinates of the midpoint of MN. **1**

(ii) What is the exact length of the interval MN? **1**

(iii) Determine the gradient of the interval MN. **1**

(f) (i) Given an arithmetic series with the first term 5 and a common difference of 4, what is the eleventh term? **1**

(ii) Given a geometric series with the first term 5 and a common ratio of $\dfrac{2}{3}$, what is the limiting sum? **1**

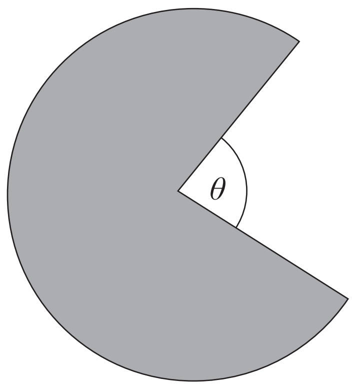
Question 12 (12 Marks) Use a SEPARATE writing booklet.

(a) Write down the domain of $y = \sqrt{x+3}$. **1**

(b) Simplify $3^{4n} \times 9^{2n} \times 27^{3n}$. **2**

(c) Sketch the polynomial $p(x) = (x-1)^2(x-3)$, clearly showing intercepts. **2**

(d) The logo, for a company that makes chocolate, is a sector of a circle of radius 2 cm, shown as shaded in the diagram.
The area of the logo is 3π cm^2.



(i) Find, in radians, the value of the angle θ, as indicated on the diagram. **1**

(ii) Find the total length of the perimeter of the logo. **1**

(e) Evaluate $\sum_{k=3}^{5} k!$ **2**

(f) Solve $|\ 3x + 2\ | = 4$ **2**

(g) What is the angle of inclination of the line $y = -2x + 1$? **1**
Express your answer to the nearest degree.

Question 13 (8 Marks) Use a SEPARATE writing booklet.

(a) Show that, for all values of x, that the expression below is an integer. **2**

$$(3\sin x + \cos x)^2 + (\sin x - 3\cos x)^2$$

(b) (i) Find the discriminant of $3x^2 - 6x + 3$. **1**

(ii) Hence, or otherwise, show that $y = 11x + 5$ is a tangent to the curve $y = 3x^2 + 5x + 8$. **1**

(c) The radioactive decay of a substance is given by $R = 1000e^{-ct}$, $t \geq 0$

(i) How many atoms were there initially? **1**

It takes 5730 years for half of the substance to decay.

(ii) Find the value of c to three significant figures. **2**

(iii) How many atoms will be left after 17 190 years? **1**

Question 14 (9 Marks) Use a SEPARATE writing booklet.

(a) If the function g is defined as $g(x)=3-4x$, solve the inequality **2**

$$g^2(x)>0$$

(b) Solve $2\sin^2 2\theta-\sin 2\theta-1=0$ for $[0,\pi]$ **2**

(c) The length of daylight, $L(t)$, is defined as the number of hours from sunrise to sunset, can be modelled by the equation

$$L(t)=12+2\cos\left(\frac{2\pi t}{366}\right),$$

where t is the number of days after 21 December 2019, for $0\le t\le 366$.

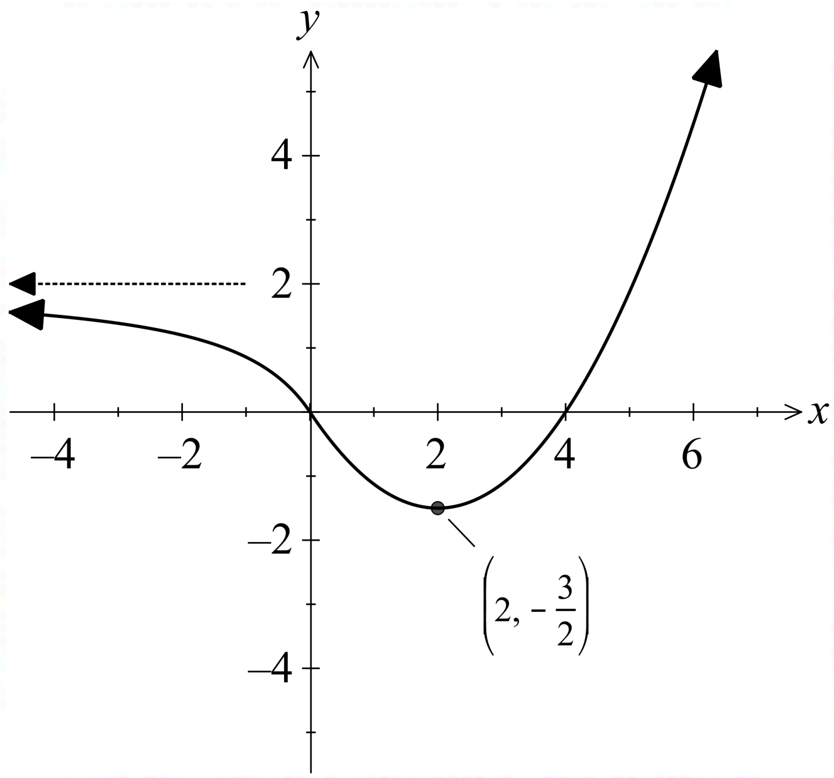
(i) Find the length of daylight on 21 December 2019. **1**

(ii) What is the shortest and longest length of daylight? **2**

(iii) What are the two values of t for which the daylight is 11 hours? **2**

Question 15 (7 Marks) Use a SEPARATE writing booklet.

The graph of $y = f(x)$ is drawn below.



The graph has a minimum value at $\left(2, -\frac{3}{2}\right)$ and passes through the origin.

As $x \to -\infty$, $y \to 2$ and as $x \to \infty$, $y \to \infty$.

On the sheet provided sketch the following transformations.

(a) $y = f(-x)$ **1**

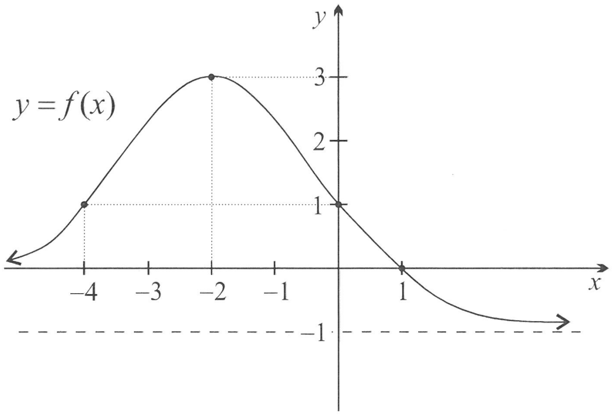
(b) $y = 2f(x) - 1$ **2**

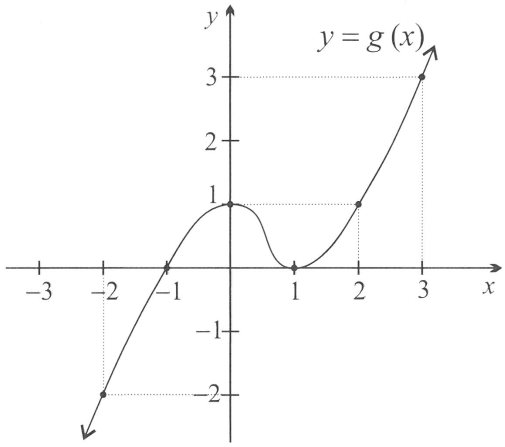
(c) $y = f(2x) - 1$ **2**

(d) $y = f(2x - 1)$ **2**

Question 16 (8 Marks) Use a SEPARATE writing booklet.

(a) The graphs of $y = f(x)$ and $y = g(x)$ are drawn below.

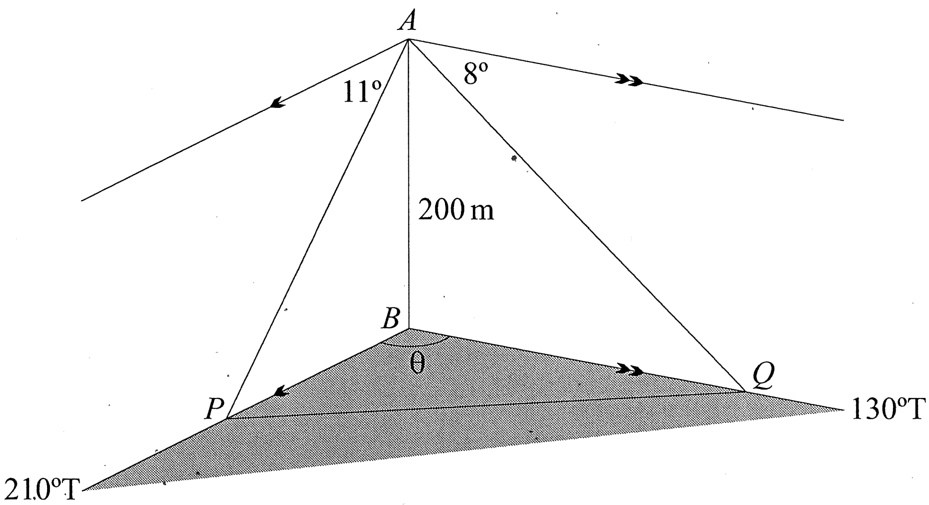




(i) Determine the domain and range of $f\left(g(x)\right)$. **2**

(ii) How many solutions are there to the equation $f\left(g(x)\right) = 0$? **1**

(b)



The HMAS Choy is enroute to carry out its latest mission in Cairns.
It is first observed from the top of a 200 m cliff, *AB* at an angle of depression of 8° when it is at the point *Q*. Ten minutes later it is observed at point *P* with an angle of depression of 11°.
Let $\angle PBQ = \theta$. The bearing of *Q* from *B* is 130° T and the bearing of *P* from *B* is 210° T.

(i) Show that $PQ^2 = 200^2(\tan^2 79° + \tan^2 82° - 2\tan 79° \tan 82° \cos\theta)$. **3**

(ii) Find the speed of the HMAS Choy in km/h correct to three significant figures. **2**

Question 17 (10 Marks) Use a SEPARATE writing booklet.

(a) An artist posted a song online. Each day there were $2^n + 3n$ downloads, where n is the number of days after the song was posted.

(i) Find the number of downloads on each of the first 3 days after the song was posted. **2**

(ii) What is the total number of times the song was downloaded in the first 20 days after it was posted? **2**

(b) Eric's reducing balance loan of \$125 000 over 20 years attracts interest at 10.2% p.a. (compounding quarterly).

Repayments of \$3678.16 per quarter are made.

Five years into the loan the interest rate is increased to 10.8% p.a. (compounding quarterly), but the quarterly repayment remains unchanged.

(i) Find the amount outstanding when the rate changes. **3**

(ii) Find the new total length term of Eric's loan. **3**

End of paper

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SUGGESTED SOLUTIONS

MC QUICK ANSWERS

1 D

2 B

3 B

4 D

5 A

6 B

7 B

8 B

9 D

10 C

X1 Y11 HY 2019 Multiple choice solutions

Mean (out of 10): 8.07

1. $(2\sqrt{3}+3\sqrt{2})^2$

$= 12 + 12\sqrt{6} + 18$

$= 30 + 12\sqrt{6}$ D

A	2
B	9
C	0
D	163

2. Area $= \frac{1}{2} \times 4 \times 2\sqrt{3} \sin 30^\circ$

$= 2\sqrt{3}\ u^2$ B

A	2
B	155
C	17
D	0

3. $\cot x \times \sec x$

$= \frac{\cos x}{\sin x} \times \frac{1}{\cos x}$

$= \frac{1}{\sin x}$

$= \operatorname{cosec} x$ B

A	9
B	162
C	3
D	0

4. A $\sin(-x) = -\sin x$ ODD

B $\frac{-x}{(-x)^2+3} = \frac{-x}{x^2+3}$ ODD

C $-\sqrt{4-(-x)^2} = -\sqrt{4-x^2}$ EVEN

D $\frac{1}{-x}+3 \neq \frac{1}{x}+3$

NEITHER

$\frac{1}{-x}+3 \neq -(\frac{1}{x}+3)$

D

A	7
B	22
C	12
D	133

5. $2\log a + \log b + \log b$

$= 2(\log a + \log b)$

$= 2\log(ab)$ A

A	140
B	9
C	21
D	3

6. $8x^3+27$

$= (2x)^3 + 3^3$

$= (2x+3)((2x)^2 - 3(2x) + 3^2)$

$= (2x+3)(4x^2-6x+9)$ B

A	5
B	167
C	1
D	1

7. A $4, 4, 4, \ldots$ $a=4, r=1$

GEOMETRIC

B $1, 2, 3, \ldots$ ARITHMETIC

C $5, -5, 5, \ldots$ $a=5, r=-1$

GEOMETRIC

D $2, 8, 32, \ldots$ $a=2$ $r=4$

GEOMETRIC B

A	45
B	96
C	23
D	9

8. Future value $= 1200 \times R^{24}$ $(R = 1 + \frac{3.35}{1200})$

$= 1283.034\ldots$

$\approx \$1283.03$ B

A	6
B	155
C	8
D	5

9. A 1 to 1
B many to many
C many to one
D one to many D

A	4
B	16
C	37
D	117

10. 0.008 is the monthly interest rate
∴ 0.096 is the yearly interest rate
9.6% C

A	19
B	14
C	133
D	8

Sydney Boys High School, Year 11, Mathematics Extension 1, Cohort Task 1, 2019

Question 11 (12 Marks)

The average mark was very high (as it should be). A small amount of students missed a question and thus lost a mark for this. Be wary of this in your trial and HSC! A few (2 or 3 perhaps) copied the wrong information from the question into their workbooks. If this resulted in a wrong evaluation, then half a mark was deducted for being careless – this is an exam, be meticulous and don't make these silly mistakes in your trial or HSC.

(a) Evaluate

(i) $|-5|-|-7|$
$=5-7$
$=-2$

This was generally well done by students. There was a small minority that treated the numbers like pronumerals and looked for multiple solutions. If you wrote multiple solutions you lost the full mark. If you wrote only one solution which was wrong but still had the second line, then you were given half a mark otherwise only the correct answer got full marks.

(ii) $\log_4 64 = 3$ Four to the power of three equals sixty-four. This is a quick calculation in one's head. One could have found this on the calculator using a change of base to either e or 10, e.g. $\dfrac{\ln 64}{\ln 4}$ or $\dfrac{\log_{10} 64}{\log_{10} 4}$ in your calculator gives 3.

(b) $\sin 240^\circ$
$=-\sin 60^\circ$
$=-\dfrac{\sqrt{3}}{2}$

Sine is only positive in the 1st and 2nd quadrants. Since this is in the 3rd quadrant, then the final answer will be negative. The equivalent angle for 240 degrees in the first quadrant is 60 degrees (240 – 180).

Some students failed to realise that the answer is negative, which was a deduction of a full mark. If you got the exact value wrong but gave the correct sign, only half a mark was taken off.

(c) Simplify fully

(i)

$$\frac{x}{x^2+3x}$$

$$=\frac{x}{x(x+3)}$$

$$=\frac{1}{x+3}$$

$$\text{or } (x+3)^{-1}$$

This was generally well done by students. Some chose to state that the answer is $x + 3$, which is an abomination and was swiftly rebuked with a deduction of full marks.

(ii)

$$\sqrt{2y}\left(\sqrt{2y}+\frac{1}{\sqrt{2y}}\right)$$

$$=\left(\sqrt{2y}\right)^2+\frac{\sqrt{2y}}{\sqrt{2y}}$$

(iii)

$$=2y+1$$

Well done by all who attempted it. If you got one term wrong and it was just a simple mistake then only half a mark was deducted. If both terms were wrong then there were no marks awarded.

(d) Rationalise the denominator:

$$\frac{7}{3-\sqrt{2}}$$

$$=\frac{7}{3-\sqrt{2}}\times\frac{3+\sqrt{2}}{3+\sqrt{2}}$$

$$=\frac{7\left(3+\sqrt{2}\right)}{3^2-\left(\sqrt{2}\right)^2}$$

$$=\frac{7\left(3+\sqrt{2}\right)}{9-2}$$

$$=3+\sqrt{2}$$

Too many students did not choose to multiply top and bottom by the conjugate of the denominator for this expression. This resulted in a deduction of 1 mark. If you left a surd in the denominator of your expression then you clearly have not done what the question asked and this was also a deduction of one mark. Small arithmetic errors had half a mark deducted but using the wrong sign in the denominator for either the product of squares or the product of factors of the difference of two squares resulted in a loss of 1 mark. Not everyone simplified this at the end and HSC markers may not be as generous as I am (thank me later). Please make it a habit to simplify all your answers whether the question asks to or not (it states that all answers should be simplified as much as possible on the front cover of this and every other exam).

(e) M (-4, 4) & N (6, -3), find the midpoint, length and gradient of interval MN.

(i) Midpoint of (x_1, y_1) & $(x_2, y_2) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= \left(\frac{-4+6}{2}, \frac{4+(-3)}{2}\right)$

$= \left(1, \frac{1}{2}\right)$

If you got the sign wrong here then you lost full marks, since it showed you don't understand how this formula works. Adding incorrectly the right numbers lost you half a mark for each mistake. Choosing the wrong numbers for your *x* & *y* lost you half a mark each.

(ii) $MN = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ *

$= \sqrt{(6-(-4))^2 + (-3-4)^2}$ *

$= \sqrt{10^2 + (-7)^2}$

$= \sqrt{149} units$

The wrong equation lost you the full mark, e.g. using addition within the brackets rather than the difference. Arithmetic errors other than that cost you half a mark each.

(iii) Gradient of $MN = \frac{y_2 - y_1}{x_2 - x_1}$

$= -\frac{7}{10}$ * from (ii)

See the guide for (ii) above. Some students used run over rise which was the wrong equation. One thing that was incredibly annoying (grrrr..) was seeing the negative sign in the denominator of the gradient. STOP DOING THIS!

(f) (i) AP with $a = 5$ and $d = 4$, find T_{11}.

$T_n = a + (n-1)d$	This was done very well actually. Not many did
$T_{11} = 5 + (11-1)4$	anything wrong here. If you didn't apply the formula
$= 45$	properly you lost the whole mark. If you made a silly arithmetic error like: 5 + 40 = 54 (which happened a few times) then you only lost half a mark.

(ii) GP with $a = 5$ and $r = \dfrac{2}{3}$, find the limiting sum.

$S_\infty = \dfrac{a}{1-r}$	This was also well done. Pretty much the same guidelines apply
$= \dfrac{5}{1-\frac{2}{3}}$	here as for (i) of this question.
$= \dfrac{5}{\frac{1}{3}}$	
$= 15$	

QUESTION 12

a) $x : x \geq -3$ no marks for $x \geq 3$ 1

b) $3^{4n} \times 9^{2n} \times 27^{3n}$

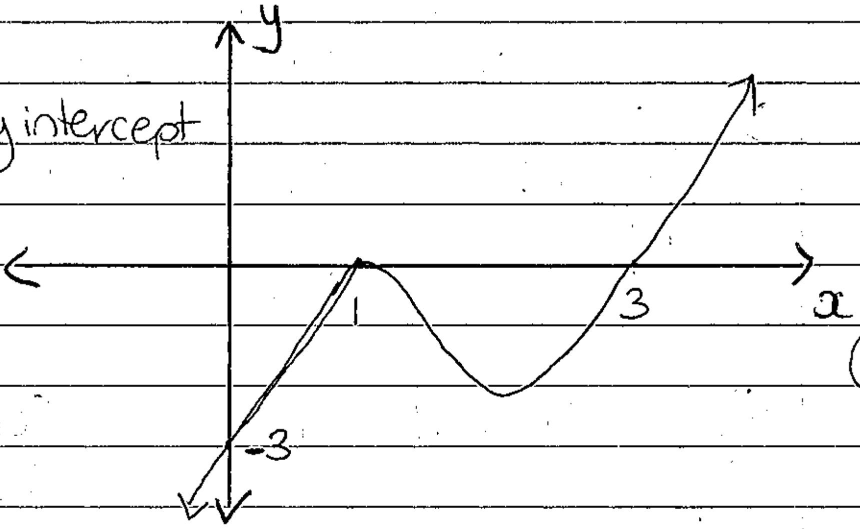
$= 3^{4n} \times (3^2)^{2n} \times (3^3)^{3n}$ 2

$= 3^{4n} \times 3^{4n} \times 3^{9n}$

$= 3^{17n}$

c)

-1 for no y intercept



2

d $r = 2cm$ $A = 3\pi cm^2$

i) area of circle radius 2: $A = \pi \times 2^2$

$= 4\pi$

shaded area $= 3\pi$

$\therefore$ % of whole circle $= \frac{3\pi}{4\pi}$

$= 75\%$

$2\pi^c \times 75\% = \frac{6\pi}{4}$ ½ for $3\pi/2$

$= \frac{3\pi^c}{2}$ $\theta = \frac{\pi}{2}$ 1

$2\pi - \frac{3\pi}{2} = \frac{\pi}{2}$

iii) $P = \frac{3}{4}$ of circumference $+ 2 + 2$

Circumference $= 2\pi r$

$= 2 \cdot \pi \cdot 2$

$= 4\pi$

Generally well done.

$P = \left(\frac{3}{4} \times 4\pi\right) + 2 + 2$ (1)

$= 3\pi + 4$ cm

approx (13.42 cm) → exact value preferred

e) $\sum_{k=3}^{5} k! = 3! + 4! + 5!$ (2)

$= 150$

f) $|3x + 2| = 4$

Some Ss didn't find both solutions

$3x + 2 = 4$, $3x + 2 = -4$

$3x = 2$, $3x = -6$ (2)

$x = 2/3$ OR $x = -2$

g) $y = -2x + 1$

$\therefore m = -2$

$\therefore \tan\theta = -2$

lots of Ss don't know what nearest degree means.

$\therefore \theta = -63.4349\ldots$ (1)

$= 117^\circ$ to the positive x axis

½ for 63°

0 for -63°

Question 13

a) $(3\sin x+\cos x)^2+(\sin x-3\cos x)^2=9\sin^2 x+6\sin x\cos x+\cos^2 x+\sin^2 x-6\sin x\cos x+9\cos^2 x$

$$=10(\sin^2 x+\cos^2 x)$$

$$=10$$

$\therefore$ Integer value for any x.

Comments

Generally well done.

Mistakes included not expanding the squares correctly, or showing for a specific x-value only.

① for expansion & ① for simplification.

b) i) $\Delta=b^2-4ac$

$$=(-6)^2-4\times3\times3$$

$$=0$$

Comments

Well done.

Lots of students evaluated discriminant as $\sqrt{b^2-4ac}$. Since $\sqrt{0}=0$, this didn't affect answer, but correct your misunderstanding.

ii) Solving simultaneously,

$$11x+5=3x^2+5x+8$$

$$3x^2-6x+3=0$$

$\therefore$ Since $\Delta=0$ (shown above), the functions' simultaneous solution has a double root. i.e. Line touches curve.

Comments

Many students factorised & solved $3x^2-6x+3=0$ which is unnecessary. Some wasted even more time using Calculus to find the tangent at $x=1$.

Question 13 Continued

c) i) At $t = 0$,

$R = 1000e^{(-c)\times 0}$

$= 1000$

Comments

Overall well done, but too many students getting $1000e \approx 2718$, demonstrating a lack of understanding of indices.

ii) At $t = 5730$, $R = 500$

$500 = 1000e^{-5730c}$

$0.5 = e^{-5730c}$

$\log_e 0.5 = -5730c$ (or $\log_e 2 = 5730c$)

$c = \dfrac{\log_e 0.5}{-5730}$

$\approx 1.209680943 \times 10^{-4}$

$\approx 1.21 \times 10^{-4} = 0.000121$

Comments

Worst attempted part of section. Many took $R = \frac{1}{2}$. Many tried to take $\log_{1000e}(-5730c)$, which is wrong two ways. Some didn't know how to round to 3 significant figures.

① For correctly arranging exponential equation into logarithmic equation.

* ① For evaluating for c correctly. No penalty given for rounding errors.

iii) At $t = 17190$

$R = 1000e^{-1.21\times10^{-4}\times17190}$

$= 125$

No ~~er~~ penalty for rounding error.

Some students noticed that $17190 = 3 \times 5730$

Hence $R = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times 1000$.

Question 14

(a) $g(x) = 3 - 4x$, $g^2(x) > 0$

$[\text{Note: } g^2(x) = g(g(x))]$

$$\begin{aligned}\therefore \quad g^2(x) &= g(g(x)) \\ &= 3 - 4(3 - 4x) \\ &= 3 - 12 + 16x \\ &= -9 + 16x\end{aligned}$$

$$\begin{aligned}\therefore \quad -9 + 16x &> 0 \\ 16x &> 9 \\ x &> \frac{9}{16}\end{aligned}$$

$\boxed{2}$

note $\longrightarrow g^2(x) \neq [g(x)]^2$.

Comments:
Done very poorly.
Most students squared the entire function.

(b) $2\sin^2 2\theta - \sin 2\theta - 1 = 0 \qquad [0, \pi]$

Let $x = \sin 2\theta$

$$\begin{aligned}\therefore \quad 2x^2 - x - 1 &= 0 \\ \frac{(2x-2)(2x+1)}{2} &= 0 \\ (x-1)(2x+1) &= 0\end{aligned}$$

$$\therefore \quad x = 1, -\frac{1}{2}$$

$\therefore \quad \sin 2\theta = 1 \quad$ or $\quad \sin 2\theta = \frac{-1}{2}$

$2\theta = \sin^{-1}(1)$

$2\theta = \frac{\pi}{2}, \frac{5\pi}{2}, \ldots \qquad 2\theta = \sin^{-1}\left(\frac{-1}{2}\right)$

$\theta = \frac{\pi}{4}, \frac{5\pi}{4}, \ldots \qquad 2\theta = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{19\pi}{6}, \ldots$

$\theta = \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \ldots$

$\therefore \quad \theta = \frac{\pi}{4}, \frac{7\pi}{12}, \frac{11\pi}{12} \qquad [0, \pi] \qquad \boxed{2}$

Comments:
Done reasonably well.
However, most students either forgot to look at the domain or left answers in degrees. And factorised the quadratic incorrectly

(c) $L(t) = 12 + 2\cos\left(\frac{2\pi t}{366}\right)$

(i) On 21 December 2019: $t = 0$.

$L(0) = 12 + 2\cos\left(\frac{2\pi(0)}{366}\right)$

$= 12 + 2\cos(0)$

$= 12 + 2(1)$

$= 14$ hours. $\boxed{1}$

(ii) amplitude $= 2$.

$\therefore$ longest $= 12 + 2$

$= 14$ hours

$\therefore$ Shortest $= 12 - 2$

$= 10$ hours $\boxed{2}$

(iii) $$11 = 12 + 2\cos\left(\frac{2\pi t}{366}\right)$$

$$-1 = 2\cos\left(\frac{2\pi t}{366}\right)$$

$$\cos\left(\frac{2\pi t}{366}\right) = -\frac{1}{2}$$

$$\frac{2\pi t}{366} = \cos^{-1}\left(\frac{-1}{2}\right)$$

$$\frac{2\pi t}{366} = \frac{2\pi}{3}, \frac{4\pi}{3}$$

$t = 122, 244$ days. $\boxed{2}$

Comments:

Done poorly.

(i) Most students recognised that the 21st of December was $t = 0$.

(ii) Some students did not calculate the correct amplitude to the graph, as well as students working out the 'when' not the 'how long'.

(iii) Most students forgot to either convert the answers to radians first then solve. As they tried to multiple 2 different measurements together (radians and degrees).

Question 15

<table>
<tr><td colspan="3">

General Comments:

- Candidates whom received full marks in this question were able to sketch clearly all the key features (listed in the Marker's Comments) as shown in the solution.
- Candidates are encouraged to draw in pencil first if they are not sure of the solution, before going over with black pen once they are satisfied with their solution. Any candidates whom left their final solution in pencil will NOT be able to appeal for any marks for that question.
- Candidates whom tested points using the original graph, to generate the points for the new graphs were generally successful in this question.
- However, many candidates still need to review their understanding of the transformations of graphs.

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<tr><td>Question/ Marks</td><td>Solution</td><td>Marker's Comments</td></tr>
<tr><td>

15a)

1

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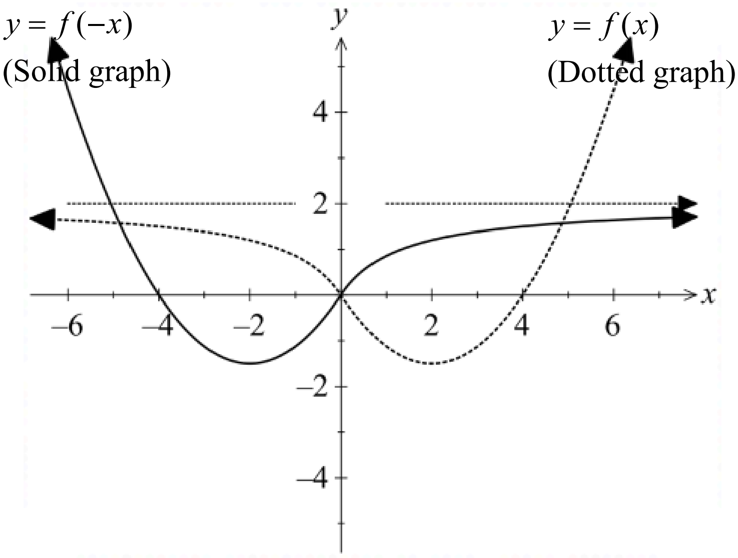

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- This was done well by majority of candidates.
- Candidates whom were not successful in this question were reflecting the original graph on the *x*-axis rather the *y*–axis. Candidates should understand the difference between $y = -f(x)$ and $y = f(-x)$.

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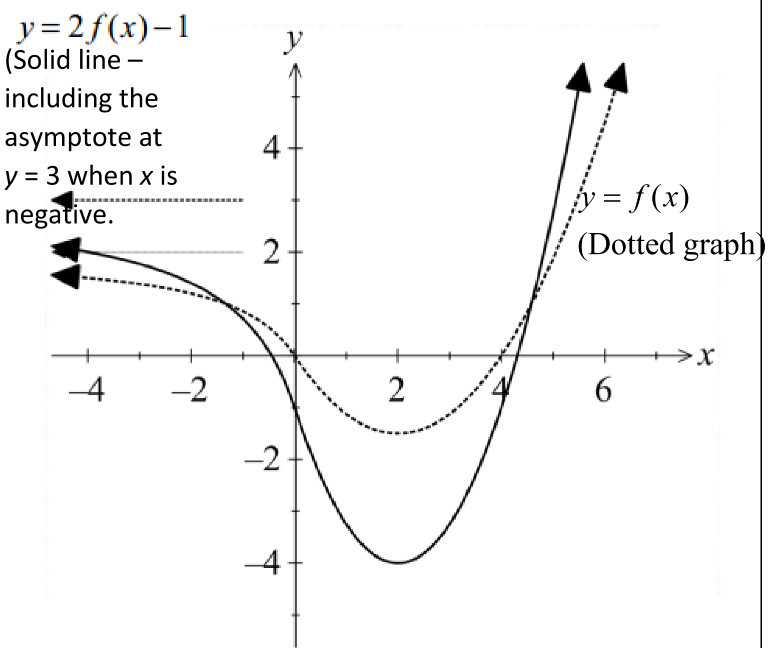
15b)

2

</td><td>



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- The minimum key features that candidates need to show to achieve full marks:

❖ Asymptote at $y = 3$ when $x \leq -1$.
❖ *y*-intercept at -1.
❖ Turning point near (2,-4).
❖ *x*-intercept near -0.5 and also 4.2.
❖ Correct shape as shown to the left particularly how it intersects the original graph.

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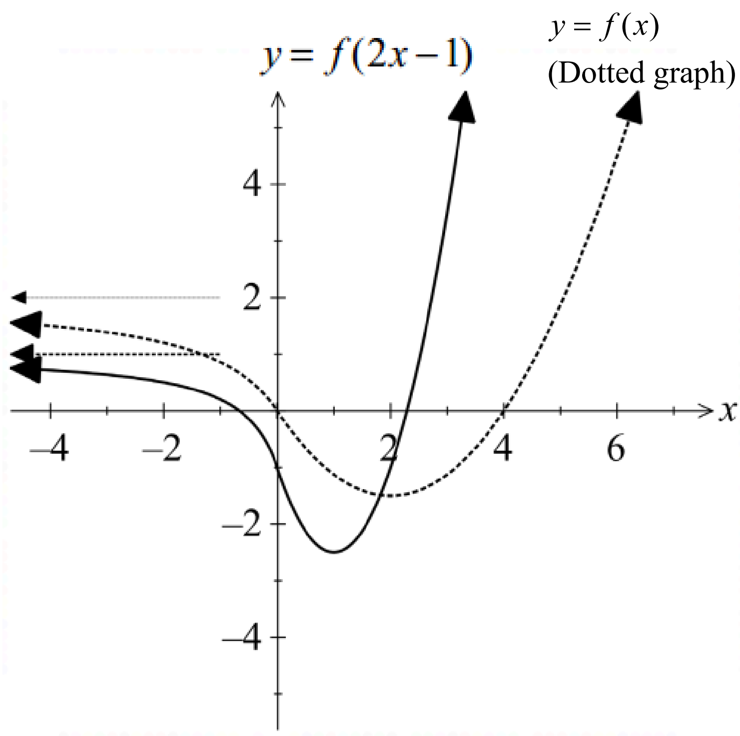
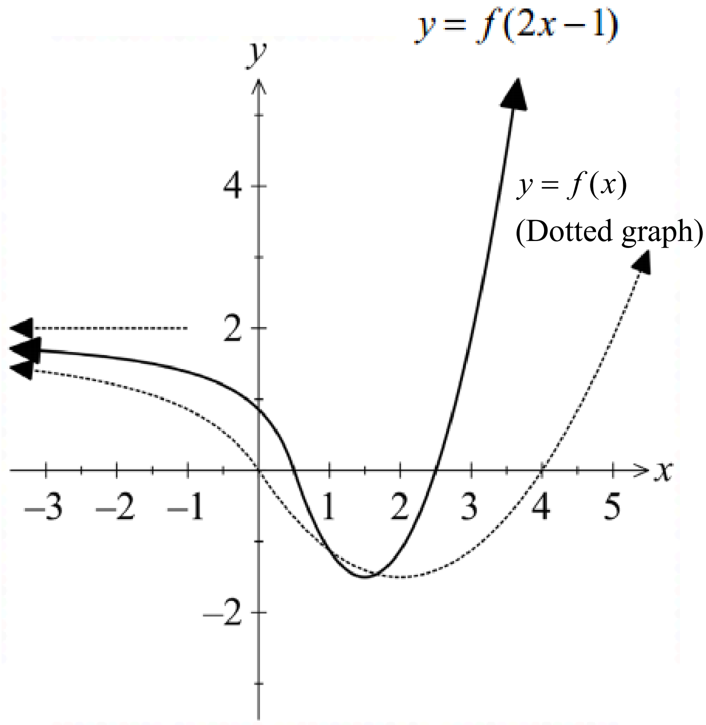
15c) **2**		- The minimum key features that candidates need to show to achieve full marks: ❖ Asymptote at $y = 1$ when $x \leq -1$. ❖ y-intercept at -1. ❖ Turning point near (1,-2.2). ❖ x-intercept near -1 and also 2. ❖ Correct shape as shown to the left particularly how it intersects the original graph.
15d) **2**		- The minimum key features that candidates need to show to achieve full marks: ❖ Asymptote at $y = 2$ when $x \leq -1$. ❖ y-intercept is near 1. ❖ Turning point near (1.4,-1.8). ❖ x-intercept near 0.5 and also 2.5. ❖ Correct shape as shown to the left. No marks were deducted if the intersection shown in the left was not shown.

16) a) i) $y = f(x)$ D: all real x

R: $-1 < y \leq 3$

$y = g(x)$ D: all real x

R: all real y

$y = f(g(x))$ D: all real x OR $(-\infty, \infty)$

R: $-1 < y \leq 3$ OR $(-1, 3]$

Note: For a real number a to be in the domain of $f(g(x))$, a must be in the domain of $g(x)$, and $g(a)$ must be in the domain of $f(x)$.

The range of $f(g(x))$ is the range of $f(x)$ when it is restricted just to the range of $g(x)$.

ii) $f(g(x)) = 0$

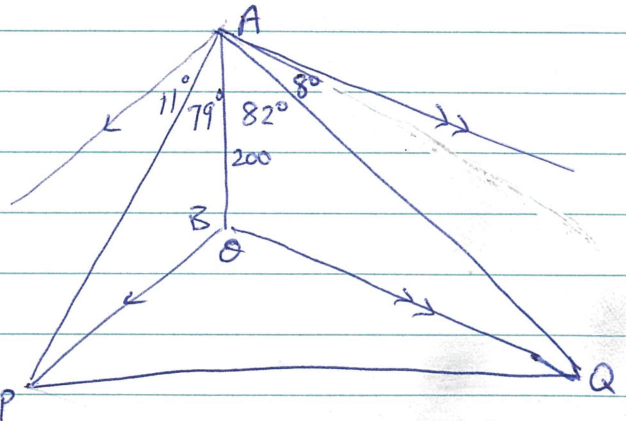
since $f(1) = 0$

consider $g(x) = 1$

$y = 1$ & $y = g(x)$ intersect in two places (when $x = 0, 2$)

$\therefore$ 2 solutions

b) i)



$\tan 82 = \dfrac{BQ}{200}$

$BQ = 200 \tan 82$

similarly

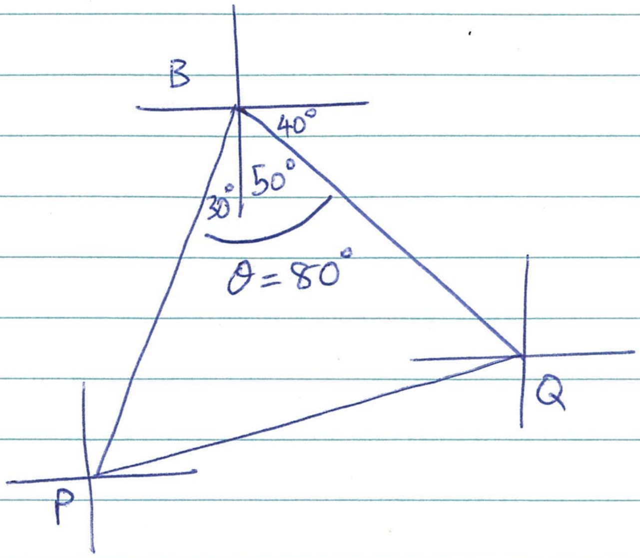
$BP = 200 \tan 79$

$PQ^2 = BP^2 + BQ^2 - 2(BP)(BQ)\cos\theta$

$= (200 \tan 79)^2 + (200 \tan 82)^2 - 2(200 \tan 79)(200 \tan 82)\cos\theta$

$= 200^2 (\tan^2 79 + \tan^2 82 - 2 \tan 79 \tan 82 \cos\theta)$

ii)



$$PQ^2 = 200^2(\tan^2 79 + \tan^2 82 - 2\tan 79 \tan 82 \cos 80)$$

$$PQ = 200\sqrt{\tan^2 79 + \tan^2 82 - 2\tan 79 \tan 82 \cos 80}$$

$$\approx 1604.767883 \text{ m}$$

$$\text{speed} = 1.604767883 \text{ km} / 10 \text{ minutes}$$

$$\approx 9.63 \text{ km/h}$$

COMMENT:

Very few students gained full marks in Q16.
Students had trouble with the range of $f(g(x))$ & the number of solutions to $f(g(x)) = 0$.

Given b)i) is a show that, students can't expect to gain 3 marks for stating

$$PQ^2 = (200\tan 79)^2 + (200\tan 82)^2 - 2(200\tan 79)(200\tan 82)\cos\theta$$

before stating $PQ^2 = 200^2(\tan^2 79 + \tan^2 82 - 2\tan 79 \tan 82 \cos\theta)$

For b)ii) θ needed to be found using knowledge of bearings. A separate diagram with N, S, E, W lines helps.

Question 17 Solutions

(a) An artist posted a song online. Each day there were $2^n + 3n$ downloads, where n is the number of days after the song was posted.

(i) Find the number of downloads on each of the first 3 days after the song was posted. **2**

Let $T_n = 2^n + 3n$

$$T_1 = 2^1 + 3\times 1$$
$$= 5$$

$$T_2 = 2^2 + 3\times 2$$
$$= 10$$

$$T_3 = 2^3 + 3\times 3$$
$$= 17$$

The number of downloads on each of the first three days is 5, 10 and 17

Comment:

Most students didn't read this question carefully.
Students who calculated all three downloads (or showed evidence) before adding were generally not penalised.
However some students only wrote T_3 or only S_3 and these students were penalised.

(a)	(ii)	What is the total number of times the song was downloaded in the first 20 days after it was posted?	**2**

$$\text{Let } S_n = \sum_{i=1}^{n} T_n = \sum_{i=1}^{n}\left(2^n + 3n\right)$$

$$S_{20} = \sum_{i=1}^{20}\left(2^n + 3n\right) = \sum_{i=1}^{20} 2^n + \sum_{i=1}^{20} 3n$$

$\sum_{i=1}^{20} 2^n$: This is a geometric series with $a = 2$, $r = 2$ and $n = 20$

$$\sum_{i=1}^{20} 2^n = \frac{2(2^{20}-1)}{2-1} = 2(2^{20}-1) \qquad \left[S_n = \frac{a(r^n-1)}{r-1}\right]$$

$\sum_{i=1}^{20} 3n$: This is an arithmetic series with $a = 3$, $d = 3$, $n = 20$ and $l = 60$

$$\sum_{i=1}^{20} 3n = \frac{20}{2}\left(3+60\right) = 630 \qquad \left[S_n = \frac{n}{2}\left(a+l\right)\right]$$

$$\therefore S_{20} = 2(2^{20}-1) + 630 = 2\,097\,780$$

The song had a total of 2 097 780 downloads.

Comment:

Some students resorted to listing all twenty terms and then adding – this would be an unrealistic method for 200 downloads.

Despite the fact that most students could see that the sequence is not arithmetic nor geometric, there were repeated attempts to use wrong formulae to calculate the sum.

(b) Eric's reducing balance loan of $125 000 over 20 years attracts interest at 10.2% p.a. (compounding quarterly).

Repayments of $3678.16 per quarter are made.

Five years into the loan the interest rate is increased to 10.8% p.a. (compounding quarterly), but the quarterly repayment remains unchanged.

(i) Find the amount outstanding when the rate changes. **3**

20 years = 80 quarters
5 years = 20 quarters
10.2% p.a = 2.55% per quarter

Let $R = 1 + 2.55\% = 1.0255$
Let A_n = the amount owing after n quarters.
$A_0 = 125\,000$

$$\begin{aligned} A_1 &= A_0 R - 3678.16 \\ &= 125\,000R - 3678.16 \end{aligned}$$

$$\begin{aligned} A_2 &= A_1 R - 3678.16 \\ &= (125\,000R - 3678.16)R - 3678.16 \\ &= 125\,000R^2 - 3678.16(1+R) \end{aligned}$$

$$\begin{aligned} A_3 &= A_2 R - 3678.16 \\ &= \left[125\,000R^2 - 3678.16(1+R)\right]R - 3678.16 \\ &= 125\,000R^3 - 3678.16(1+R+R^2) \end{aligned}$$

$$\boxed{\begin{aligned} A_{20} &= 125\,000R^{20} - 3678.16(1+R+R^2+...+R^{19}) \\ &= 125\,000R^{20} - 3678.16 \times \frac{R^{20}-1}{R-1} \end{aligned}}$$

$$\begin{aligned} &\doteqdot 112\,402.983 \\ &\doteqdot 112\,402.98 \end{aligned}$$

The amount outstanding was $112 402.98

Comment:
This question is very similar to the question provided in the sample questions.

Students are expected to show development of the formulae from their knowledge of series. As a result if students jumped straight to the results in the shaded rectangle above, they were significantly penalised.
Students who assumed that $A_n = (A_{n-1} - 3678.16) \times 1.0255$ were not able to get full marks.
Many students realised that the rate needs to be changed to quarters, but then didn't change the corresponding years. That said some students don't know to convert to quarters.
Many students are rounding too soon and this might have caused inaccurate answers and hence marks.

(b)	(ii)	Find the new total length term of Eric's loan.	**3**

At the start of the 21st quarter Eric owes \$112 402.98
10.8% p.a = 2.7% per quarter

Let $r = 1 + 2.7\% = 1.027$
Let B_n = the amount owing after n quarters.
$B_0 = 112\,402.98$

The pattern will follow in a similar way as in part (i)
i.e. $B_n = B_0 r^n - 3678.16\left(1 + r + r^2 + ... + r^{n-1}\right)$

$$\therefore B_n = B_0 r^n - 3678.16 \times \frac{r^n - 1}{r - 1}$$

To find the new term of the loan we need to solve $B_n = 0$

$$\therefore B_0 r^n - 3678.16 \times \frac{r^n - 1}{r - 1} = 0$$
$$\therefore B_0(r-1)r^n - 3678.16(r^n - 1) = 0$$
$$\therefore \left[B_0(r-1) - 3678.16\right]r^n = -3678.16$$

$$\therefore r^n = \frac{3678.16}{3678.16 - B_0(r-1)}$$
$$\doteqdot 5.71782526$$

$$n \doteqdot \frac{\ln 5.71782526}{\ln r}$$
$$\doteqdot 65.4452763$$

This means that he will pay off the loan after a further 66 quarters
This means the loan has a term of 86 quarters or $21\frac{1}{2}$ years.

Note – the last quarter will not need a full repayment of \$3678.16.

Comment:

ecf ≡ error carried forward: Many students benfitted from this in that they were only penalised once for the wrong answer provided for A_{20} in part (i). Fresh errors were penalised though or if the error made in part (i) over simplified the problem in part (ii).

The development of the formulae was done in part (i) and didn't need to be repeated. Students could assume the structure of the formulae.

Some students don't realise that there are exactly 66 payments after the change in interest rate and not an approximation.

End of solutions