SYDNEY BOYS HIGH SCHOOL

2020

YEAR 11

TERM 2

Cohort Task #1

Mathematics Extension

General Instructions

- Reading time – 5 minutes
- Working time – 1.5 hours
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided with this paper
- Marks may **NOT** be awarded for messy or badly arranged work
- In Questions 11–29, show ALL relevant mathematical reasoning and/or calculations

Total Marks: 58

Section I – 10 marks (pages 2–4)
- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 48 marks (pages 5–27)
- Attempt Questions 11–29
- Allow about 1 hour and 15 minutes for this section

Examiner:

AMG

NESA Number									

Name: ____________________ Class: 11Ma __

Section I – Multiple Choice

10 Marks
Attempt question 1−10
Allow approximately 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1−10

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1 Which of the following expressions is equivalent to $\left(3\sqrt{5}-5\right)^2$?

A. 20

B. 70

C. $20-30\sqrt{5}$

D. $70-30\sqrt{5}$

2 Which of the following expressions is equivalent to $\frac{x^6}{y^6} \div x^2 y^{-3}$?

A. $\frac{x^3}{y^2}$

B. $\frac{x^4}{y^2}$

C. $\frac{x^4}{y^3}$

D. $\frac{x^4}{y^9}$

3 What are the solutions to $|\, x - 3 \,| = 6$?

A. $x = -3$ or 9

B. $x = 3$ or -9

C. $x = -3$ or -9

D. $x = 3$ or 9

4 The graph of the function f passes through the point $(-2, 7)$.

If $h(x) = f\left(\frac{x}{2}\right) + 5$, the graph of the function h must pass through which of the following points?

A. $(-1, -12)$

B. $(-1, 19)$

C. $(-4, 12)$

D. $(-4, -14)$

5 Which expression is equivalent to $\log_a 2m - \log_a m$?

A. $\dfrac{\log_a 2m}{\log_a m}$

B. $\log_a m$

C. $\log_a 2m$

D. $\log_a 2$

6 The value of a compound interest investment, in dollars, after n years, V_n, can be modelled by the recurrence relation shown below

$$V_0 = 100\,000, \quad V_{n+1} = 1.01V_n$$

What is the interest rate, per annum, for this investment?

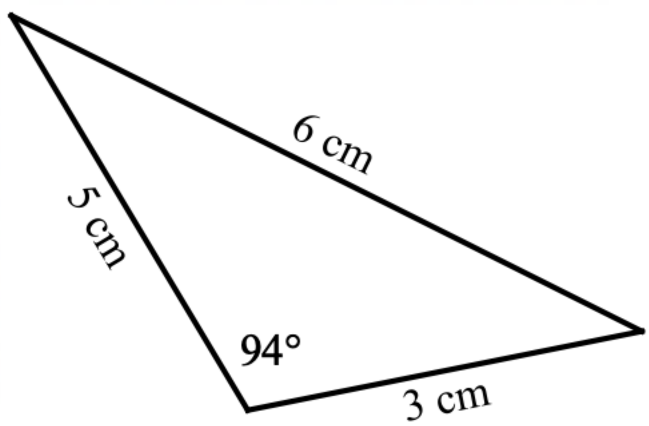
A. 0.101%

B. 1%

C. 1.01%

D. 101%

7 Which is the correct expression for the area of the triangle below?



A. $\frac{1}{2} \times 5 \times 3 \times \cos 94°$

B. $\frac{1}{2} \times 5 \times 6 \times \cos 94°$

C. $\frac{1}{2} \times 5 \times 3 \times \sin 94°$

D. $\frac{1}{2} \times 5 \times 6 \times \sin 94°$

8 If $f(x)=\dfrac{1}{4x^3}$ what is $f'(x)$ equal to?

A. $-\dfrac{3}{4x^4}$

B. $\dfrac{3}{4x^4}$

C. $-\dfrac{12}{4x^4}$

D. $\dfrac{12}{4x^4}$

9 What is the natural domain of the function $f(x)=\dfrac{1}{\sqrt{4-x^2}}$?

A. $[-2,2]$

B. $(-\infty,-2]\cup[2,\infty)$

C. $(-2,2)$

D. $(-\infty,-2)\cup(2,\infty)$

10 Let $f(x)=x^2+3x-4$ and $g(x)=|\,x\,|$.

What is the correct expression for $f(g(x))$?

A. $f(g(x))=\begin{cases} x^2+3x-4, & x\ge 0 \\ x^2-3x-4, & x<0 \end{cases}$

B. $f(g(x))=\begin{cases} x^2-3x-4, & x\ge 0 \\ x^2+3x-4, & x<0 \end{cases}$

C. $f(g(x))=\begin{cases} x^2+3x-4, & x\le -4 \text{ or } x\ge 1 \\ -x^2-3x+4, & -4<x<1 \end{cases}$

D. $f(g(x))=\begin{cases} -x^2-3x+4, & x\le -4 \text{ or } x\ge 1 \\ x^2+3x-4, & -4<x<1 \end{cases}$

NESA Number										Q11-14	

YEAR 11 Mathematics Extension

TERM 2 Cohort Task #1

Part A

Section II

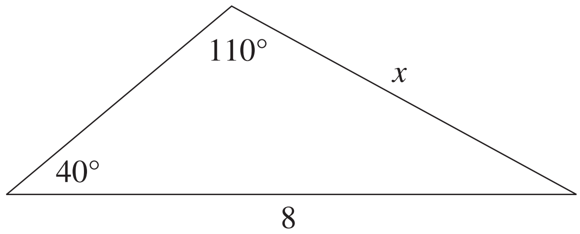
Part A 9 marks
Attempt Questions 11–14

Answer each question in the space provided. A blank page is provided at the end of each question to allow rewriting of a part.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (2 marks)

Using the sine rule, find the value of x correct to one decimal place. **2**



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Question 12 (2 marks)

What is the limiting sum of the following geometric series? **2**

$$2000 - 1200 + 720 - 432 \ldots$$

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Part A continues on page 7

Question 13 (4 marks)

Differentiate with respect to x:

(a) $y = x^4 - 6x - 5$ **1**

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(b) $y = \left(2x + 7\right)^2$ **1**

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(c) $y = 2\sqrt{x}$ **1**

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(d) $y = \dfrac{5x - 1}{x}$ **1**

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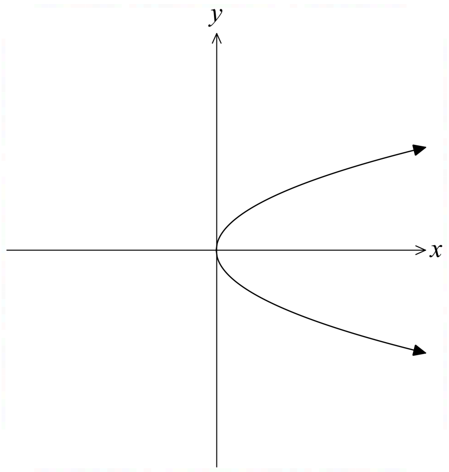
Question 14 (1 mark)

Classify the graph shown below as one-to-one, many-to-one, one-to-many or many-to-many. **1**



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Use this space to re-write any questions for Part A.

End of Part A

NESA Number										Q15-19	

YEAR 11 Mathematics Extension

TERM 2 Cohort Task #1

Part B

Section II

Part B **9 marks**
Attempt Questions 15–19

Answer each question in the space provided. A blank page is provided at the end of each question to allow rewriting of a part.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 15 (2 marks)

In an arithmetic series, the third term is 8 and the twentieth term is 59. **2**

Find the 50^{th} term.

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Question 16 (2 marks)

Solve the equation $2\sin\theta + 1 = 0$ for $0 \le \theta \le 2\pi$. Answers should be left in exact form. **2**

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Question 17 (2 marks)

Find the angle of inclination, to the positive direction of the x-axis, of the tangent to the curve $y = \dfrac{x}{\sqrt{x+1}}$ at the point where $x = 1$. **2**

Give your answer correct to the nearest degree.

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Part B continues on page 11

Question 18 (1 mark)

Consider the following amortisation table for a reducing balance loan. **1**

Payment number	Payment	Interest	Principal reduction	Balance
0	0.00	0.00	0.00	300 000.00
1	1050.00	900.00	150.00	299 850.00
2	1050.00	899.55	150.45	299 699.55
3	1050.00	899.10	150.90	299 548.65

The annual interest rate for this loan is 3.6%. Interest is calculated immediately before each payment. For this loan, calculate the frequency of the repayments i.e. is the loan repaid weekly, fortnightly, monthly, quarterly, half yearly or yearly or something else?

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Question 19 (2 marks)

If $f(x)=\sqrt{4-x}$ and $g(x)=x^2$, find the domain and range of $f(g(x))$. **2**

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Use this space to re-write any questions for Part B.

End of Part B

NESA Number											Q20-24	

YEAR 11 Mathematics Extension

TERM 2 Cohort Task #1

Part C

Section II

Part C **10 marks**
Attempt Questions 20–24

Answer each question in the space provided. A blank page is provided at the end of each question to allow rewriting of a part.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 20 (2 marks)

Simplify $\dfrac{\left(\tan^2\theta-1\right)\left(\sin\theta\cos\theta+1\right)}{\cos^2\theta\left(\tan^3\theta-1\right)}$. **2**

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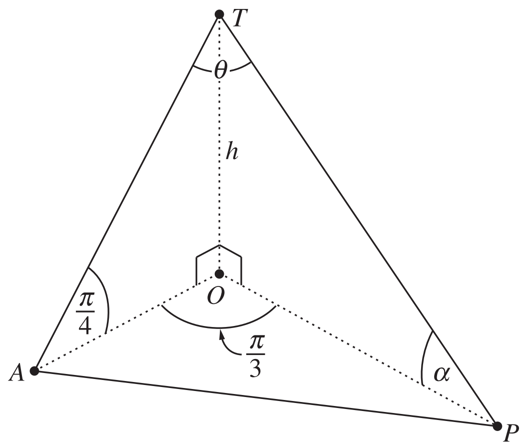
Part C continues on page 15

Question 21 (4 marks)



Consider the diagram, which shows a vertical tower OT of height h metres, a fixed point A, and a point P such that $\angle AOP = \dfrac{\pi}{3}$ radians. The angle of elevation of T from A is $\dfrac{\pi}{4}$ radians. Let the angle of elevation of T from P be α radians and let $\angle ATP = \theta$ radians.

Show that $\cos\theta = \dfrac{1}{\sqrt{2}}\sin\alpha + \dfrac{1}{2\sqrt{2}}\cos\alpha$ **4**

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Question 23 (3 marks)

(a) Use the formula **2**

$$f'(-3) = \lim_{h \to 0} \frac{f(-3+h) - f(-3-h)}{2h}$$

to find $f'(-3)$ where $f(x) = x^2 + 2x + 3$.

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(b) Hence, find the equation of the normal to the curve $y = f(x)$ at the point $(-3, 6)$. **1**

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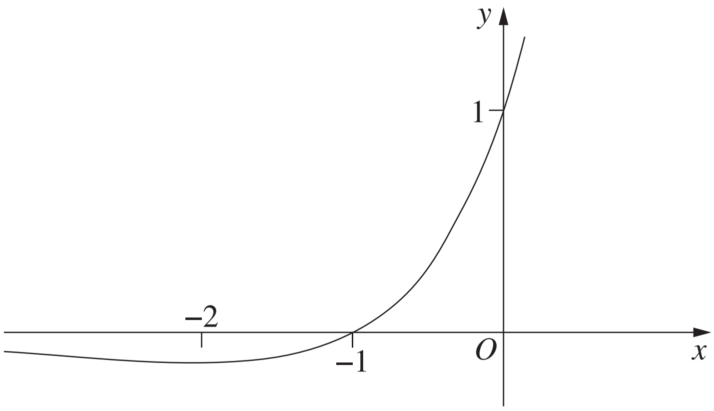
Question 24 (1 mark)

The diagram below shows the graph of $y = e^x(1+x)$. **1**



How many solutions are there to the equation $e^x(1+x) = 1 - x^2$

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Use this space to re-write any questions for Part C.

End of Part C

NESA Number										Q25-27	

YEAR 11 Mathematics Extension

TERM 2 Cohort Task #1

Part D

Section II

Part D **9 marks**
Attempt Questions 25–27

Answer each question in the spaces provided. A blank page is provided at the end of each question to allow rewriting of a part.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 25 (3 marks)

The population of a country grew exponentially between 1920 and 2020. **3**
This population can be modelled by the equation $P(t) = 92e^{kt}$, where $P(t)$ is the population of the country in millions, t is the time in years after 1920 and k is a positive constant.
The population of the country in 1970 was 184 million.

Assuming that this model continues to be valid after 2020, estimate the population of the country in 2030 to the nearest million.

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Question 26 (3 marks)

Let $x = \tan\phi + \cot\phi$ and $y = \sin\phi + \cos\phi$.

By finding xy^2, or otherwise, find an equation linking x and y without ϕ. **3**

Question 27 (3 marks)

Markham, the Mayor of Gaynforde, wants to redevelop a block of land to assist with social distancing for his family. **3**
Markham will need to borrow \$350 000 to make this happen.
Interest on this loan will be charged at the rate of 4.9% per annum, compounding fortnightly.
After three years of equal fortnightly repayments, the balance of Markham's loan will be \$262 332.33

It can be shown that the amount, $\$A_n$, owing after the nth repayment is given by

$$A_n = 350\,000R^n - M\left(1 + R + R^2 + \ldots + R^{n-1}\right),$$ (Do NOT show this)

where R is related to the interest rate and $\$M$ is the fortnightly repayment.

What is the value of each fortnightly repayment that Markham will make?
Round your answer to the nearest cent. [Take 1 year = 26 fortnights]

Use this space to re-write any questions for Part D.

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End of Part D

NESA Number										Q28-29	

YEAR 11 Mathematics Extension

TERM 2 Cohort Task #1

Part E

Section II

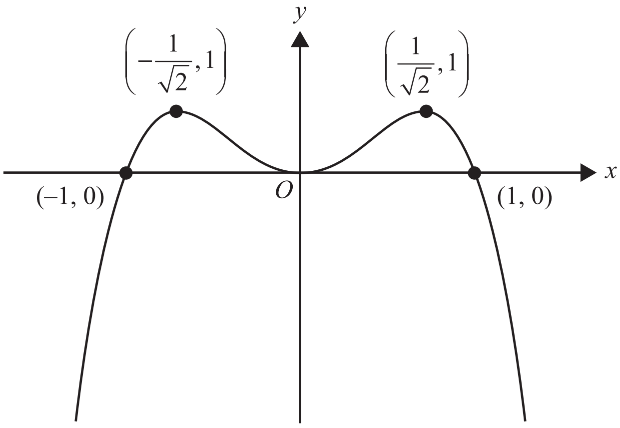
Part E 11 marks
Attempt Questions 28–29

Answer each question in the spaces provided. A blank page is provided at the end of each question to allow rewriting of a part.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 28 (4 marks)

The function g is a polynomial function of degree 4.
Part of the graph of g is shown below. The graph of g touches the x-axis at the origin.



(a) What is the rule for g? **1**

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(b) Let the function G be defined such that $G(x) = \log_e(g(x)) - \log_e(x^3 + x^2)$. **3**
What is the domain and range of G?

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Question 29 (7 marks)

(a) By completing the square, find in terms of θ, the minimum value as x varies of **2**

$$9x^2 - 12x\cos\theta + 4$$

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(b) Similarly, find the maximum value as x varies of $12x^2\sin\theta - 9x^4$. **2**

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Question 29 continues on page 27

Question 29 (continued)

(c) Hence determine the values of x and θ, for $0 < \theta < \pi$, that satisfy the equation **3**

$$9x^4 + \left(9 - 12\sin\theta\right)x^2 - 12x\cos\theta + 4 = 0.$$

End of paper

Use this space to re-write any questions for Part E.

End of Part E

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2020 | YEAR 11
TERM 2 TASK 1

Mathematics Extension

SUGGESTED SOLUTIONS

MC QUICK ANSWERS

1 D

2 C

3 A

4 C

5 D

6 B

7 C

8 A

9 C

10 A

2020 Y11 Ext HY Multiple choice solutions

Mean (out of 10): 8.23

1. $(3\sqrt{5}-5)^2$

$= 9\times 5 - 30\sqrt{5} + 25$

$= 70 - 30\sqrt{5}$ D

A	1
B	0
C	14
D	176

2. $\frac{x^6}{y^6} \div x^2 y^{-3}$

$= \frac{x^6}{y^6} \div \frac{x^2}{y^3}$

$= \frac{x^6}{y^6} \times \frac{y^3}{x^2}$

$= \frac{x^4}{y^3}$ C

A	4
B	3
C	167
D	17

3. $|x-3| = 6$

$\therefore x-3 = 6$ or -6

$\therefore x = 9$ or -3 A

A	185
B	2
C	1
D	3

4. $f(-2) = 7$

$\therefore f\left(\frac{-4}{2}\right) = 7$

$\therefore f\left(-\frac{4}{2}\right) + 5 = 12$

$\therefore h(-4) = 12$

$\therefore (-4, 12)$ lies on h C

A	22
B	14
C	145
D	9

5. $\log_a 2m - \log_a m$

$= \log_a\left(\frac{2m}{m}\right)$

$= \log_a 2$ D

A	14
B	18
C	2
D	157

6. As $V_{n+1} = 1.01\,V_n$

$V_{n+1} = 101\%\,V_n$

$\therefore$ Interest rate is 1% B

A	5
B	168
C	13
D	5

7. Area $= \frac{1}{2}ab\sin C$

$= \frac{1}{2}\times 5\times 3\times \sin 94^\circ$ C

A	7
B	1
C	183
D	0

8. $f(x) = \frac{1}{4x^3}$

$= \frac{1}{4} x^{-3}$

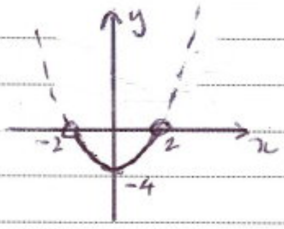
$\therefore f'(x) = \frac{1}{4} \times -3 \times x^{-4}$

$= -\frac{3}{4x^4}$ (A)

A	152
B	12
C	23
D	4

9. $4 - x^2 > 0$

$\therefore x^2 - 4 < 0$



$\therefore -2 < x < 2$

OR $(-2, 2)$ (C)

A	33
B	10
C	127
D	21

10. $g(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

$\therefore f(g(x)) = |x|^2 + 3|x| - 4$

$= x^2 + 3|x| - 4$

$= \begin{cases} x^2 + 3x - 4 & \text{if } x \geq 0 \\ x^2 - 3x - 4 & \text{if } x < 0 \end{cases}$ (A)

A	112
B	14
C	40
D	24

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Part A
Sample Solutions

Questions with no individual comment were generally well done.

Section II

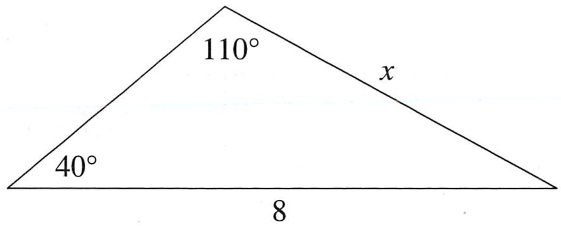
Part A **9 marks**
Attempt Questions 11–14

Answer each question in the space provided. A blank page is provided at the end of each question to allow rewriting of a part.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (2 marks)

Using the sine rule, find the value of x correct to one decimal place. **2**



$$\frac{x}{\sin 40^\circ} = \frac{8}{\sin 110^\circ}$$

$$x = \frac{8\sin 40^\circ}{\sin 110^\circ}$$

$x = 5.47232\ldots$

$= 5.5$ (1dp)

Question 12 (2 marks)

What is the limiting sum of the following geometric series? **2**

$$2000 - 1200 + 720 - 432 \ldots$$

$a = 2000$ $S_n = \frac{a}{1-r}$

$r = -0.6$ $= \frac{2000}{1+0.6} = 1250$

Most mistakes were made by not recognising 'r' was negative

Part A continues on page 7

Question 13 (4 marks)

Differentiate with respect to x:

(a) $y = x^4 - 6x - 5$ **1**

$\frac{dy}{dx} = 4x^3 - 6$

(b) $y = (2x+7)^2$ **1**

$\frac{dy}{dx} = 2(2x+7) \times 2$

$= 4(2x+7)$

Many Ss forgot to multiply by differential of brackets.

(c) $y = 2\sqrt{x}$ **1**

$y = 2x^{1/2}$

$\frac{dy}{dx} = \frac{1}{2} \cdot 2x^{-1/2}$

$\frac{dy}{dx} = \frac{1}{\sqrt{x}}$

(d) $y = \frac{5x-1}{x}$ $= \frac{5x}{x} - \frac{1}{x} \Rightarrow = 5 - x^{-1}$ **1**

$\frac{dy}{dx} = 0 - -1 \times x^{-2}$

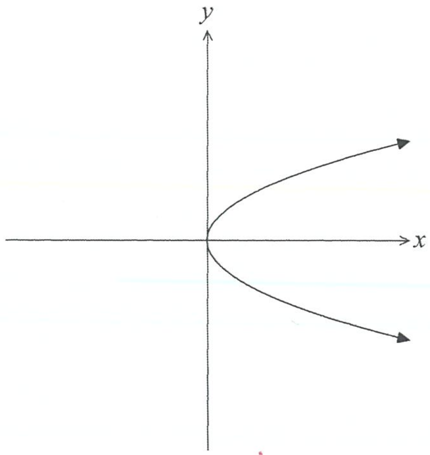
$= x^{-2}$

$= \frac{1}{x^2}$

Those that did it by product rule often forgot to subtract all of 2nd & ended up with negative answer.

Question 14 (1 mark)

Classify the graph shown below as one-to-one, many-to-one, one-to-many or many-to-many. **1**



one to many

YEAR 11 Mathematics Extension

TERM 2 Cohort Task #1

Part B
Sample Solutions

Section II

Part B (9 marks)

Question 15

$T_3 = a + 2d = 8$
$T_{20} = a + 19d = 59$

$17d = 51 \quad \therefore a = 2$

$T_{50} = 2 + 49 \times 3$
$T_{50} = 149$

Criteria	**Mark**
Correct solution with working out	2
Provide correct solution for the first term and the common difference	1

Markers Comments: Generally well-done. Some candidates got confused with the substitution.

Question 16

$\sin\theta = -\frac{1}{2}$

Sine is negative in the third and fourth quadrant.

$\theta = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$
$\theta = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

Criteria	**Mark**
Correct solutions with working out	2
Provide the answer of 210° and 330° with correct working out	$1\frac{1}{2}$
Recognise the basic angle is $\frac{\pi}{6}$	$\frac{1}{2}$

Markers Comments: Moderately well-done. Candidates cannot disregard the negative sign.

Question 17

Using the quotient rule:

$$\frac{dy}{dx}=\frac{\sqrt{x+1}\times(1)-x\times\frac{1}{2}(x+1)^{-\frac{1}{2}}(1)}{x+1}$$

$$\frac{dy}{dx}=\frac{\sqrt{x+1}-\frac{x}{2\sqrt{x+1}}}{x+1}$$

At $x=1$

$$\frac{dy}{dx}=\frac{\sqrt{2}-\frac{1}{2\sqrt{2}}}{2}$$

$$=\frac{4-1}{2\sqrt{2}}\times\frac{1}{2}$$

$$\therefore\frac{dy}{dx}=m=\frac{3}{4\sqrt{2}}$$

$$m=\tan\theta$$

$$\therefore\tan\theta=\frac{3}{4\sqrt{2}}$$

$$\theta=28^{\circ}$$

Alternate Solution

Using the product rule:

$$y=x(x+1)^{-\frac{1}{2}}$$

$$\frac{dy}{dx}=x\left(-\frac{1}{2}(x+1)^{-\frac{3}{2}}(1)\right)+(x+1)^{-\frac{1}{2}}(1)$$

At $x=1$,

$$\frac{dy}{dx}=m=-\frac{1}{2}\times2^{-\frac{3}{2}}+2^{-\frac{1}{2}}$$

Criteria	**Mark**
Correct solution with working out	2
Correct differentiation but incorrect answer to the substitution of $x=1$ but able to use $m=\tan\theta$ to find the angle of inclination.	$1\frac{1}{2}$
Correct differentiation but did not substitute $x=1$ correctly and did not find the angle of inclination correctly.	1
Understand $m=\tan\theta$ and able to find the angle using $\tan^{-1}$	$\frac{1}{2}$

Markers Comments: Candidates found it difficult to differentiate correctly. Candidate should not spend time to simplify the term into a single fraction for this question, as this causes more errors. Simply substitute $x=1$ suffice to find the gradient.

Question 18

$A = P(1+r)^n$ The first payment is n = 1

A = Total amount = \$300 900 for n = 1

$$\therefore \$300900 = \$300000(1+r)^1$$
$$1.003 = 1+r$$
$$\therefore r = 0.003$$
$$3.6\% \quad \therefore r = 0.036 \div n$$
$$0.036 \div n = 0.003$$
$$\therefore n = 12$$

Hence, this is a MONTHLY repayment.

Criteria	**Mark**
Correct solution and working out.	1
Only correctly answer without any working out	$\frac{1}{2}$

Markers Comments: Not very well done. Some candidates were confused with the table given. It may be less confusing to just use one row and verify answer the second payment.

Question 19

$$f(g(x)) = \sqrt{4 - x^2}$$

Positive semi-circle centre at the origin and radius equals to 2 units.

$D: -2 \le x \le 2$ or $D: [-2, 2]$

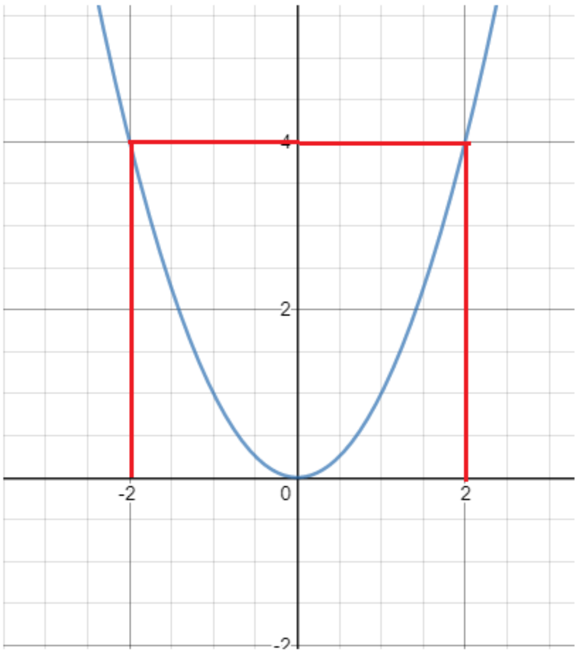
$R: 0 \le y \le 2$ $R: [0, 2]$

Alternative method:

$f(x) = \sqrt{4 - x}$

$D: x \le 4$

f



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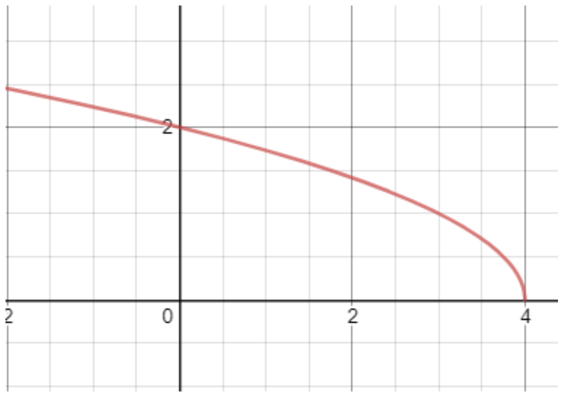

$0 \le y \le 2$

$-2 \le x \le 2$

Criteria	**Mark**
Correct solutions for both the domain and range	2
Only provided the correct solution for the domain	1
Only provided the correct solution for the range	1
Recognition that $f(g(x)) = \sqrt{4 - x^2}$	$\frac{1}{2}$

Markers Comments: Not very well done. Candidate got confused when they tried to complete the question graphically. It seems to be easier for the candidates who recognised $f \circ g$ is the positive of a semi-circle.

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TERM 2 Cohort Task #1

Part C
Sample Solutions

20. $$\frac{(\tan^2\theta-1)(\sin\theta\cos\theta+1)}{\cos^2\theta(\tan^3\theta-1)}$$

$$=\frac{(\tan\theta-1)(\tan\theta+1)(\sin\theta\cos\theta+1)}{\cos^2\theta(\tan\theta-1)(\tan^2\theta+\tan\theta+1)}$$

$$=\frac{(\tan\theta+1)(\sin\theta\cos\theta+1)}{\cos^2\theta(\tan^2\theta+\tan\theta+1)}$$

$$=\frac{(\tan\theta+1)(\sin\theta\cos\theta+1)}{(\sin^2\theta+\sin\theta\cos\theta+\cos^2\theta)}$$

$$=\frac{(\tan\theta+1)(\sin\theta\cos\theta+1)}{(\sin\theta\cos\theta+1)}$$

$$=(\tan\theta+1)$$

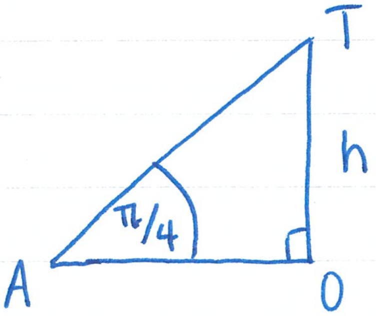
(OR) $$=\frac{\sin\theta+\cos\theta}{\cos\theta}$$

$$=\sec\theta(\sin\theta+\cos\theta)$$

Comments:

This Question was very poorly done.

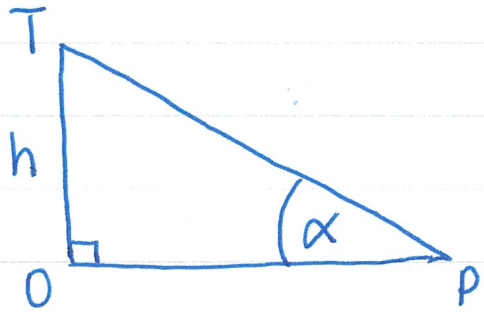
21.



$AO = OT$

$= h$ (angles of isosceles triangle AOT).

$AT = \sqrt{2}h.$

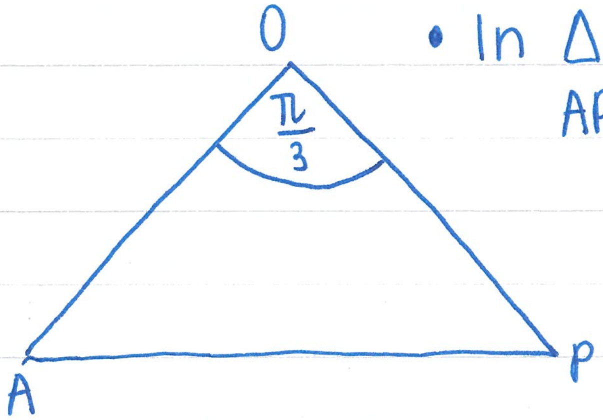


$$\sin\alpha = \frac{h}{PT}$$

$$PT = \frac{h}{\sin\alpha}$$

$$\tan\alpha = \frac{h}{OP}$$

$$OP = \frac{h}{\tan\alpha}$$



• In ΔAOP:

$$AP^2 = OP^2 + AO^2 - 2(AO)(OP)\cos\left(\frac{\pi}{3}\right)$$

$$= \frac{h^2}{\tan^2\alpha} + h^2 - \frac{2h^2}{\tan\alpha} \cdot \cos\left(\frac{\pi}{3}\right)$$

$$= h^2\left[1 + \cot^2\alpha - \cot\alpha\right]$$

In ΔTAP:

$$AP^2 = (\sqrt{2}h)^2 + \left(\frac{h}{\sin\alpha}\right)^2 - 2(\sqrt{2}h)\left(\frac{h}{\sin\alpha}\right)\cos\theta$$

$$= h^2\left[2 + \operatorname{cosec}^2\alpha - \frac{2\sqrt{2}\cos\theta}{\sin\alpha}\right]$$

$$h^2\left[1 + \cot^2\alpha - \cot\alpha\right] = h^2\left[2 + \operatorname{cosec}^2\alpha - \frac{2\sqrt{2}\cos\theta}{\sin\alpha}\right]$$

$$-\frac{\cos\alpha}{\sin\alpha} = 2 - \frac{2\sqrt{2}\cos\theta}{\sin\alpha}$$

$$\frac{2\sqrt{2}\cos\theta}{\sin\alpha} = 2 + \frac{\cos\alpha}{\sin\alpha}$$

$$\therefore \quad \cos\theta = \frac{1}{\sqrt{2}}\sin\alpha + \frac{1}{2\sqrt{2}}\cos\alpha.$$

Comments:

This Question was very poorly done.

23. (a) $f(-3+h) = (-3+h)^2 + 2(-3+h) + 3$

$$= 9 - 6h + h^2 - 6 + 2h + 3$$

$$= h^2 - 4h + 6$$

$$f(-3-h) = (-3-h)^2 + 2(-3-h) + 3$$

$$= 9 + 6h + h^2 - 6 - 2h + 3$$

$$= h^2 + 4h + 6$$

$$\therefore \quad f'(-3) = \lim_{h\to 0} \frac{f(-3+h) - f(-3-h)}{2h}$$

$$= \lim_{h\to 0} \frac{h^2 - 4h + 6 - (h^2 + 4h + 6)}{2h}$$

$$= \lim_{h\to 0} \frac{-8h}{2h}$$

$$= -4.$$

(b) gradient normal $\Rightarrow \dfrac{-1}{\text{gradient tangent}}$

$\therefore \quad m = \dfrac{-1}{-4}$ at $(-3, 6)$

$$= \frac{1}{4}$$

$$\therefore \quad y - 6 = \frac{1}{4}(x - (-3))$$

$$4y - 24 = x + 3$$

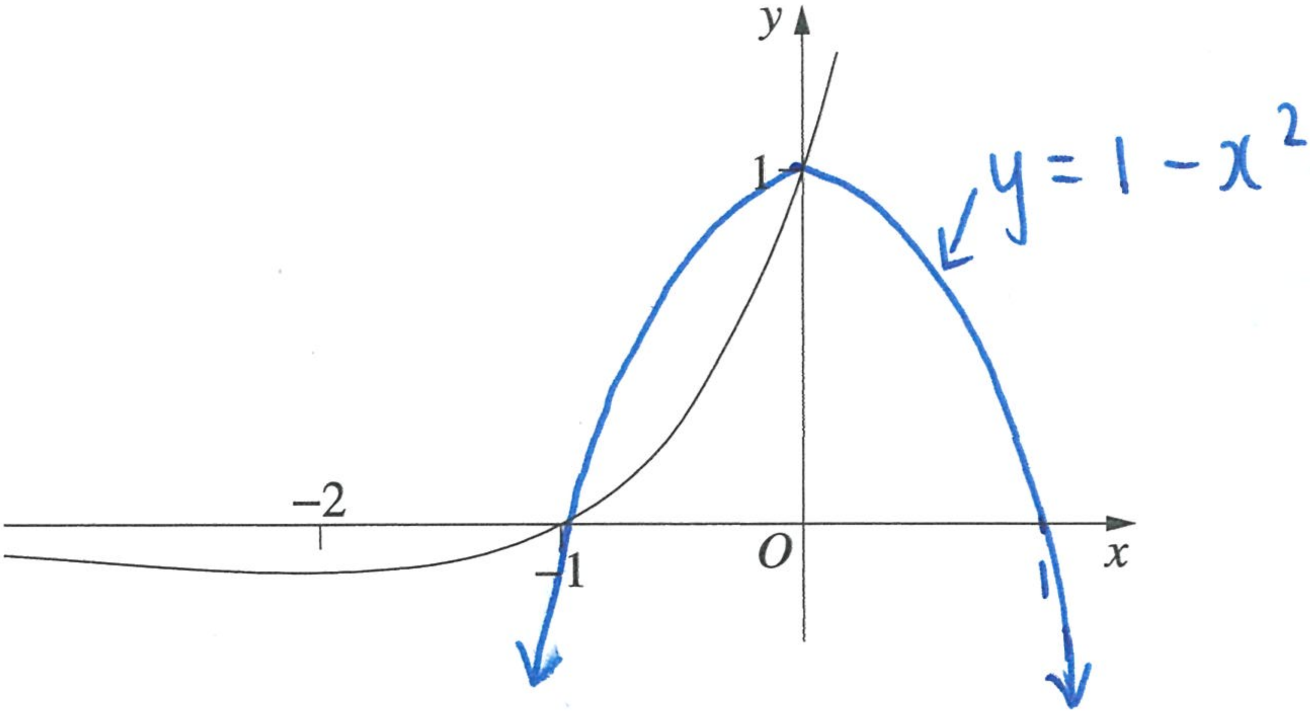
$$\therefore \quad x - 4y + 27 = 0$$

Comments:

This Question was very poorly done.

Most students didn't use the formula for finding the derivative.

As well as found the equation of the tangent not the normal to the equation.



$\therefore$ 2 solutions
($x = 0$ and $x = -1$)

Comments:

Most students did well with this question.

YEAR 11 Mathematics Extension

TERM 2 Cohort Task #1

Part D
Sample Solutions

Section II

Part D 9 marks
Attempt Questions 25–27

Answer each question in the spaces provided. A blank page is provided at the end of each question to allow rewriting of a part.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 25 (3 marks)

The population of a country grew exponentially between 1920 and 2020. **3**
This population can be modelled by the equation $P(t) = 92e^{kt}$, where $P(t)$ is the population of the country in millions, t is the time in years after 1920 and k is a positive constant.
The population of the country in 1970 was 184 million.

Assuming that this model continues to be valid after 2020, estimate the population of the country in 2030 to the nearest million.

$P(50) = 92e^{k(50)} = 184$

$e^{50k} = 2$

$50k = \log_e 2$

$k = \dfrac{\log_e 2}{50}$

≈ 0.014

$P(110) = 92e^{0.014(110)}$

$= 422.7209946$

≈ 423 million (to nearest million)

COMMENTS:

- $P(t)$ is in millions. The population in 1920 is 92 million, in 1970 it's 184 million which means the population doubles every 50 years. The population in 2020 is 368 million (2×184).
- When calculating $P(110)$ the value for k still on the calculator should be used, not 0.014.

Question 26 (3 marks)

Let $x = \tan\phi + \cot\phi$ and $y = \sin\phi + \cos\phi$.

By finding xy^2, or otherwise, find an equation linking x and y without ϕ. **3**

$$x = \tan\phi + \cot\phi$$

$$= \frac{\sin\phi}{\cos\phi} + \frac{\cos\phi}{\sin\phi}$$

$$= \frac{\sin^2\phi + \cos^2\phi}{\sin\phi\cos\phi}$$

$$= \frac{1}{\sin\phi\cos\phi}$$

$$y^2 = (\sin\phi + \cos\phi)^2$$

$$= \sin^2\phi + 2\sin\phi\cos\phi + \cos^2\phi$$

$$= 1 + 2\sin\phi\cos\phi$$

$$= 1 + \frac{2}{\left(\frac{1}{\sin\phi\cos\phi}\right)}$$

$$= 1 + \frac{2}{x}$$

$$\therefore y^2 = 1 + \frac{2}{x}$$

OR $xy^2 = x + 2$

COMMENTS:

- Most students could find an expression for xy^2 but failed to notice that x could be used for the $\sin\phi\cos\phi$ term.

Question 27 (3 marks)

Markham, the Mayor of Gaynforde, wants to redevelop a block of land to assist with social distancing for his family. **3**
Markham will need to borrow \$350 000 to make this happen.
Interest on this loan will be charged at the rate of 4.9% per annum, compounding fortnightly.
After three years of equal fortnightly repayments, the balance of Markham's loan will be \$262 332.33

It can be shown that the amount, $\$A_n$, owing after the nth repayment is given by

$$A_n = 350\,000R^n - M\left(1 + R + R^2 + ... + R^{n-1}\right),$$ (Do NOT show this)

where R is related to the interest rate and $\$M$ is the fortnightly repayment.

What is the value of each fortnightly repayment that Markham will make?
Round your answer to the nearest cent. [Take 1 year = 26 fortnights]

$$A_n = 350\,000R^n - M(1+R+R^2+...+R^{n-1})$$

$$A_n = 350\,000R^n - M\frac{(R^n-1)}{R-1}$$

$$A_n(R-1) = 350\,000R^n(R-1) - M(R^n-1)$$

$$M(R^n-1) = 350\,000R^n(R-1) - A_n(R-1)$$

$$M = \frac{(350\,000R^n - A_n)(R-1)}{R^n-1}$$

$r = 0.049$ per annum

$= \frac{0.049}{26}$ per fortnight

$= \frac{49}{26000}$

$R = \frac{26049}{26000}$

$n = 3 \times 26$

$= 78$

$A_{78} = 262332.33$

$$M = \frac{\left(350000\left(\frac{26049}{26000}\right)^{78} - 262332.33\right)\left(\frac{49}{26000}\right)}{\left(\frac{26049}{26000}\right)^{78} - 1}$$

Use this space to re-write any questions for Part D.

$M = \$1704.03$

COMMENTS:

- Just like in Q25, it is important not to round too early. Particularly when expressions are getting raised to powers.

End of Part D

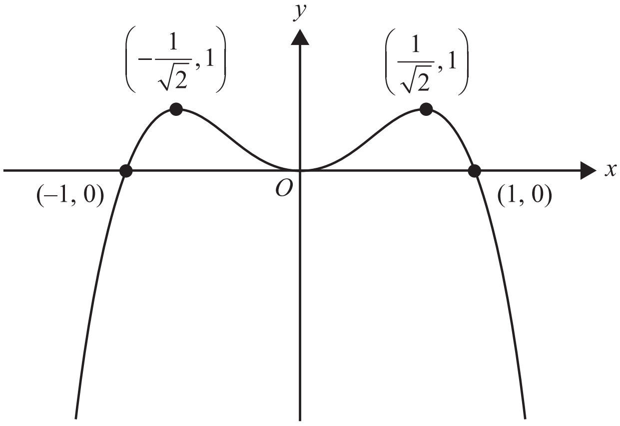
YEAR 11 Mathematics Extension

TERM 2 Cohort Task #1

Part E
Sample Solutions

Question 28 (4 marks)

The function g is a polynomial function of degree 4.
Part of the graph of g is shown below. The graph of g touches the x-axis at the origin.



(a) What is the rule for g? **1**

$$g(x) = -k(x+1)(x-1)x^2$$
$$= -kx^2(x^2-1)$$

$$g\left(\frac{1}{\sqrt{2}}\right) = 1 \Rightarrow 1 = -k\left(\frac{1}{\sqrt{2}}\right)^2\left[\left(\frac{1}{\sqrt{2}}\right)^2 - 1\right]$$

$$\therefore k = 4$$

$$\therefore g(x) = 4x^2(1-x^2)$$

Comment:

Unfortunately, the language of this question stumped most students.

For those who did understand what the question required many ignored the fact that $g\left(\frac{1}{\sqrt{2}}\right) = 1$.

Question 28 (continued)

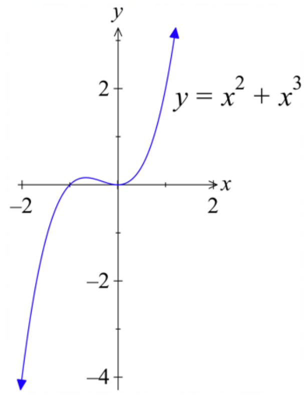
(b) Let the function G be defined such that $G(x) = \log_e(g(x)) - \log_e(x^3 + x^2)$. **3**

What is the domain and range of G?

$$G(x) = \log_e\left(4x^2\left(1-x^2\right)\right) - \log_e\left(x^2(x+1)\right) \qquad -(*)$$

$$= \log_e\left(\frac{4x^2\left(1-x^2\right)}{x^2(x+1)}\right)$$

$$= \log_e\left(4(1-x)\right) \qquad -(**)$$



Domain: Need to check original equation i.e. (*)

Using the graph of $g(x)$ then $g(x) > 0$ for $-1 < x < 1, x \neq 0$

Note: $x^2 + x^3 > 0$ for $x > -1, x \neq 0$

D: $-1 < x < 1, x \neq 0$ or $(-1, 1) \setminus \{0\}$

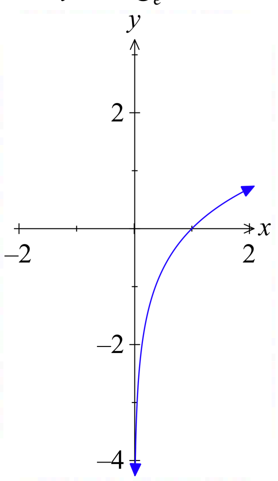
Range: As $x \to 1^-$, $G(x) \to -\infty$ [using the graph given]

As $x \to -1^+$, $G(x) \to \ln 8$ [using (**)]

As $x \to 0$, $G(x) \to \ln 4$ [using (**)]

R: $-\infty < y < \ln 8,\ y \neq \ln 4$ or $(-\infty, \ln 8) \setminus \{\ln 4\}$

$y = \log_e x$



Comment:

Many students were able to determine the domain, but to get the range students needed to use the original graph and the equation (**). This meant that they needed the "rule" from part (a).

Question 29 (7 marks)

(a) By completing the square, find in terms of θ, the minimum value as x varies of **2**

$$9x^2 - 12x\cos\theta + 4$$

$$9x^2 - 12x\cos\theta + 4 = 9\left(x^2 - \frac{12}{9}x\cos\theta\right) + 4$$

$$= 9\left[x^2 - \underbrace{\frac{4}{3}\cos\theta}_{b} x + \left(\underbrace{\frac{2}{3}\cos\theta}_{\frac{b}{2}}\right)^2\right] + 4 - \underbrace{4\cos^2\theta}_{\left(\frac{b}{2}\right)^2}$$

$$= 9\left(x - \frac{2}{3}\cos\theta\right)^2 + 4\sin^2\theta$$

$$\geq 4\sin^2\theta$$

$\therefore$ minimum value $= 4\sin^2\theta$

(b) Similarly, find the maximum value as x varies of $12x^2\sin\theta - 9x^4$. **2**

$$12x^2\sin\theta - 9x^4 = -9\left(x^4 - \frac{12}{9}x^2\sin\theta\right)$$

$$= -9\left[x^4 - \frac{4}{3}x^2\sin\theta + \left(\frac{2}{3}\sin\theta\right)^2\right] + 4\sin^2\theta$$

$$= 4\sin^2\theta - 9\left(x^2 - \frac{2}{3}\sin\theta\right)^2$$

$$\leq 4\sin^2\theta$$

$\therefore$ maximum value $= 4\sin^2\theta$

Comment for (a) and (b)

- First up, students were provided with an expression NOT an equation. Most students started with equating the given expressions to zero. As a result most could not get the required result.
- For parts (a) and (b) $\cos\theta$ and $\sin\theta$ are constants.
- For those who did manage to complete the square they were either unable or forgot to answer the question i.e. $y = k(x-p)^2 + q$ has a minimum value of q and $y = -k(x-p)^2 + q$ has a maximum value of q.

Question 29 (continued)

(c) Hence determine the values of x and θ, for $0<\theta<\pi$, that satisfy the equation **3**

$$9x^4+\left(9-12\sin\theta\right)x^2-12x\cos\theta+4=0.$$

$$9x^4+\left(9-12\sin\theta\right)x^2-12x\cos\theta+4=0 \Leftrightarrow 9x^2-12x\cos\theta+4=12x^2\sin\theta-9x^4 \qquad -(\boldsymbol{+})$$

Where do the graphs of $y=9x^2-12x\cos\theta+4$ and $y=12x^2\sin\theta-9x^4$ intersect?
From parts (a) and (b) they would only intersect when $y=4\sin^2\theta$.

For $y=9x^2-12x\cos\theta+4$ this occurs at $\left(\frac{2}{3}\cos\theta, 4\sin^2\theta\right)$,

For $y=12x^2\sin\theta-9x^4$ this occurs at $\left(\pm\sqrt{\frac{2}{3}\sin\theta}, 4\sin^2\theta\right)$

For this intersection to occur these points must be coincidental:

$$\frac{2}{3}\cos\theta=\pm\sqrt{\frac{2}{3}\sin\theta} \Rightarrow \frac{4}{9}\cos^2\theta=\frac{2}{3}\sin\theta$$

$$\therefore 2\left(1-\sin^2\theta\right)=3\sin\theta$$

$$\therefore 2\sin^2\theta+3\sin\theta-2=0$$

$$\therefore \left(2\sin\theta-1\right)\left(\sin\theta+2\right)=0$$

$$\therefore 2\sin\theta-1=0 \Rightarrow \sin\theta=\frac{1}{2}$$

$$\therefore \theta=\frac{\pi}{6} \text{ or } \frac{5\pi}{6}$$

$$\therefore x=\frac{2}{3}\cos\frac{\pi}{6} \text{ or } \frac{2}{3}\cos\frac{5\pi}{6}$$

$$=\frac{2}{3}\frac{\sqrt{3}}{2} \text{ or } -\frac{2}{3}\frac{\sqrt{3}}{2}$$

$$=\frac{\sqrt{3}}{3} \text{ or } -\frac{\sqrt{3}}{3}$$

Comment:

A few students were able to see the connection to parts (a) and (b) i.e. equation (**+**)

End of solutions