SYDNEY BOYS HIGH SCHOOL

NESA Number:

Name:

2021

YEAR 11
TERM 2
COHORT TASK 1

Maths Class:
11MX

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 90 minutes
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided with this paper
- Marks may **NOT** be awarded for messy or badly arranged work
- **In Questions 11 – 33, show ALL relevant mathematical reasoning and/or calculations**

Total Marks: 76

Section I – 10 marks (pages 2 – 5)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 66 marks (pages 6 – 32)

- Attempt Questions 11 – 33
- Allow about 1 hour 15 minutes for this section

Examiner:

RW

NOT TO BE REMOVED FROM EXAMINATION ROOM

Section I

10 marks

Attempt all questions.
Allow about 15 minutes for this section.
Use the multiple choice answer sheet for Questions 1 – 10.

Question 1

Which of the following is 1.0987 corrected to 3 significant figures?

A. 1.1
B. 1.10
C. 1.09
D. 1.099

Question 2

Which of the following is equal to $\left(3\sqrt{7}+7\sqrt{3}\right)^2$?

A. $6\left(35+7\sqrt{21}\right)$
B. $6\left(35+7\sqrt{7}\right)$
C. $6\left(35+7\sqrt{3}\right)$
D. 210

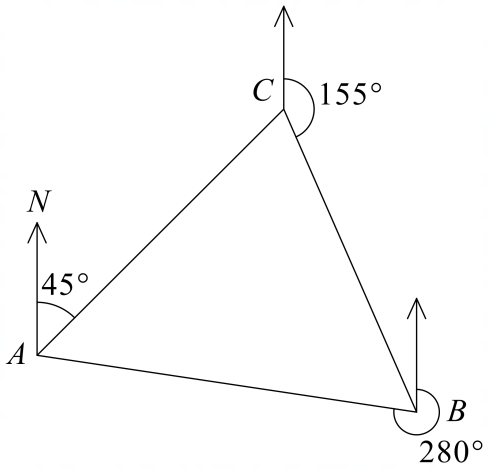
Question 3

Which of the following is the solution set to the number line below?

A. $[-6,-2]$
B. $(-6,-2)$
C. $[-6,-2)$
D. $(-6,-2]$

Question 4

Which of the following is true given the diagram below?



A. The bearing of A from C is $N135°W$.
B. The bearing of B from A is $E10°S$.
C. The bearing of C from B is $335°T$.
D. The bearing of A from C is $45°T$.

Question 5

Which of the following is true regarding the equation of a negative definite parabola?

A. $\Delta > 0, a > 0$
B. $\Delta < 0, a > 0$
C. $\Delta > 0, a < 0$
D. $\Delta < 0, a < 0$

Question 6

Which of the following is true regarding the graph of $y = f\left(\frac{x}{4}\right)$?

A. It is a vertical dilation of $y = f(x)$ by a factor of 4.
B. It is a vertical dilation of $y = f(x)$ by a factor of $\frac{1}{4}$.
C. It is a horizontal dilation of $y = f(x)$ by a factor of 4.
D. It is a horizontal dilation of $y = f(x)$ by a factor of $\frac{1}{4}$.

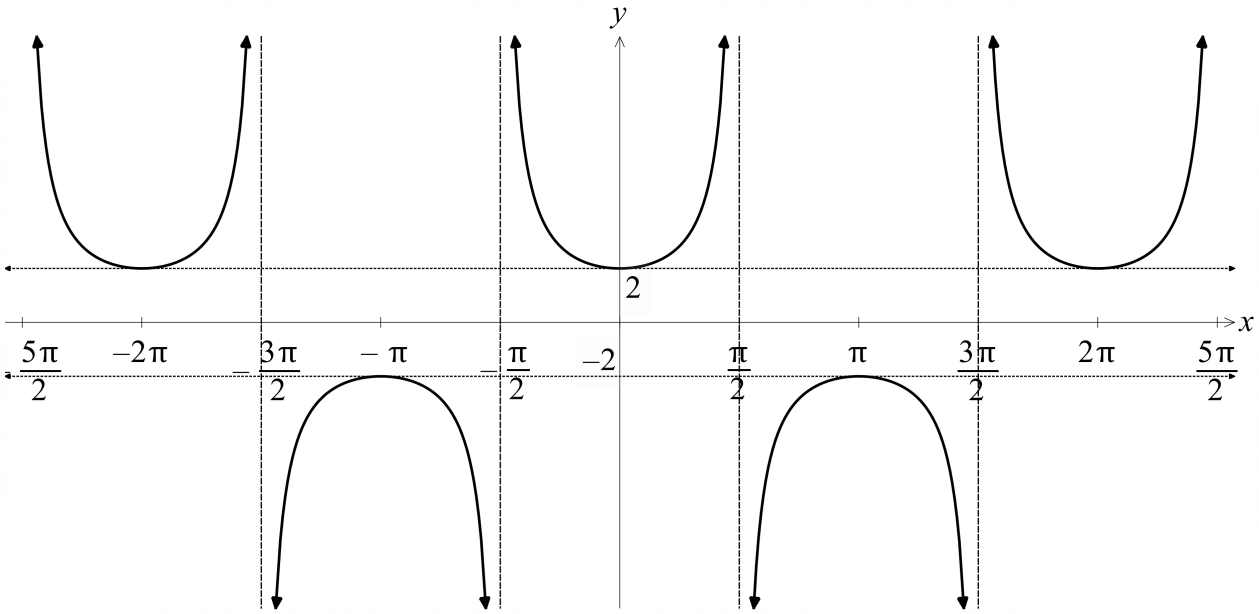
Question 7

An investment earns interest at a rate of 0.75% p.a., compounded monthly, over 5 years.
The future value of the investment is $100 000.
Which of the following is the present value of the investment, rounded to the nearest dollar?

A. $63 870
B. $96 321
C. $96 333
D. $99 688

Question 8

Which of the following is the equation of the graph below?



A. $y = 2\sec x$

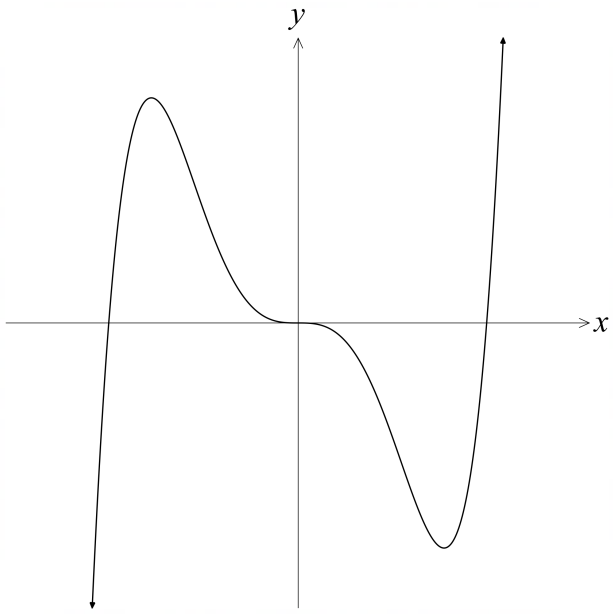
B. $y = \frac{1}{2}\sec x$

C. $y = \sec(2x)$

D. $y = \sec\left(\frac{x}{2}\right)$

Question 9

Which of the following is **false** regarding the graph below?



A. There are 3 intercepts.
B. There are 3 stationary points.
C. There are 3 turning points.
D. There are 3 inflection points.

Question 10

Below are two student responses when asked how they would sketch the graph of $y = f(2-x)$:

Student A: "Translate $y = f(x)$ to the right by 2 units, then reflect the result about the line $x = 2$."
Student B: "Reflect $y = f(x)$ about the line $x = 1$."

Which student is correct?

A. A only.
B. B only.
C. Both A and B.
D. Neither A nor B.

NESA Number										Q11-19	

YEAR 11 Mathematics Extension 1

TERM 2 COHORT TASK #1

Part A

Section II

Part A 15 marks

Attempt all questions.
Answer each question in the space provided.
A blank page is provided at the end of each part to allow rewriting of a question.
Your response should include relevant mathematical reasoning and/or calculations.

Question 11

Expand $(1-x)^3$ completely. **1**

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Question 12

Evaluate $\sum_{n=1}^{3} n^n$. **1**

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Question 13

Write $\log_5 7$ correct to 3 significant figures. **1**

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Question 14

Express $36°$ in radians as a multiple of π. **1**

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Question 15

State whether $f(x) = \sqrt[3]{x}$ is odd, even or neither. **1**

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Question 16

Differentiate the following with respect to x:

A. $y = 4x^3 + x^2 + 1$ **1**

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B. $y = (2x+1)^3$ **1**

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C. $y = \dfrac{x^2}{x+1}$ **2**

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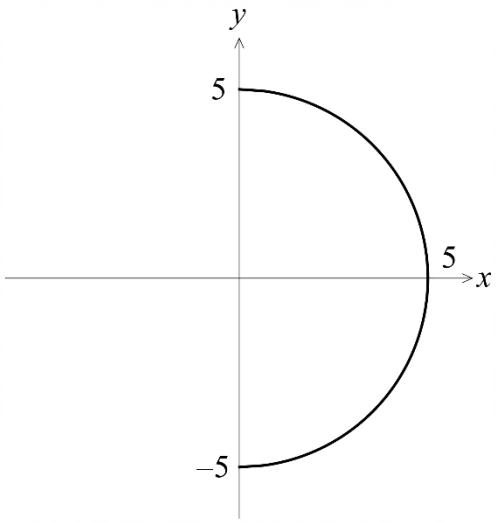
Question 17

State the domain and range of the graph below. **2**



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Question 18

Given that $\log_{10} x = y$, express $\log_{10} \frac{1}{\sqrt{x}}$ in terms of y. **1**

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Question 19

Given an arithmetic sequence with terms $T_5 = 17$ and $T_{12} = 52$, evaluate T_{36}. **3**

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End of Part A

Use this page to rewrite any questions from Part A.

NESA Number										Q20-23	

YEAR 11 Mathematics Extension 1

TERM 2 COHORT TASK #1

Part B

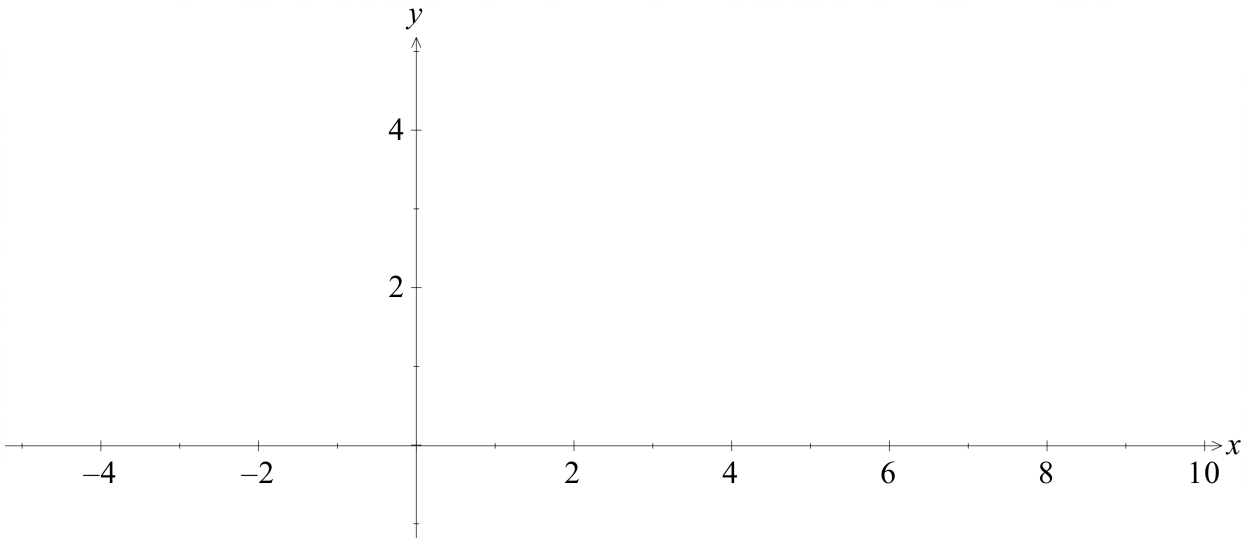
Section II

Part B 13 marks

Attempt all questions.
Answer each question in the space provided.
A blank page is provided at the end of each part to allow rewriting of a question.
Your response should include relevant mathematical reasoning and/or calculations.

Question 20

A. Sketch $y = \left| \frac{2-x}{2} \right|$ on the number plane below. **2**



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B. Hence, solve $\left| \frac{x}{2} - 1 \right| \geq 3$. **2**

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Question 21

Find the equation of the normal to the curve $y = (x-1)\left(\frac{1}{x}+1\right)$ passing through $P(-2, 3)$. **3**

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Question 22

The mass M of a radioactive substance after t hours is given by $M = 250e^{-0.1t}$. **2**
Find the half life of the substance, correct to nearest hour.
Note: The half life is the time needed for a radioactive substance to reduce to half of its initial value.

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Question 23

RW runs a business that sells x cups of bubble milk tea daily.
The revenue generated as a result is given by the function $R(x) = -0.01x^2 + 7.1x$.
The cost of running the business is given by the function $C(x) = 2.5x + 408$.

A. Find the values of x such that the business breaks even. **2**

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B. Deduce the maximum daily profit that can be generated. **2**

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End of Part B

Use this page to rewrite any questions from Part B.

NESA Number										Q24-27	

YEAR 11 Mathematics Extension 1

TERM 2 COHORT TASK #1

Part C

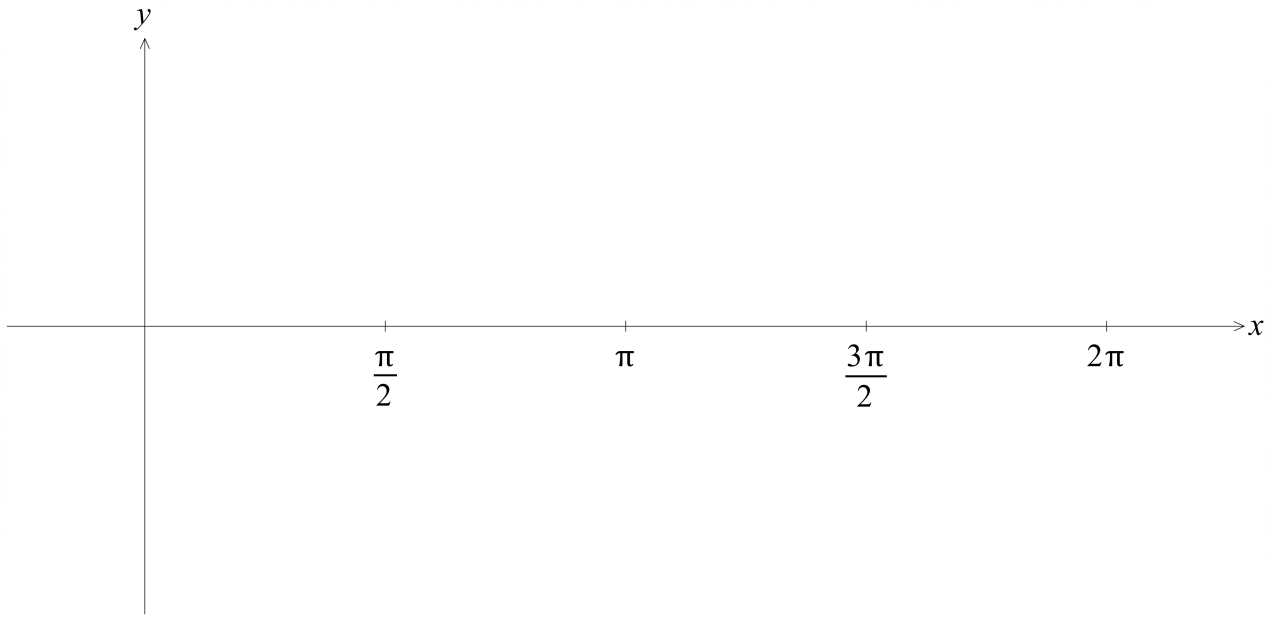
Section II

Part C 13 marks

Attempt all questions.
Answer each question in the space provided.
A blank page is provided at the end of each part to allow rewriting of a question.
Your response should include relevant mathematical reasoning and/or calculations.

Question 24

A. Sketch $y = \cot x$ for $0 \le x \le 2\pi$ on the number plane below. **2**



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B. Hence or otherwise, show that $\tan x \times \tan\left(x + \frac{\pi}{2}\right) = -1$ for all $x \ne \frac{k\pi}{2}$, $k \in \mathbb{Z}$. **2**

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Question 25

Differentiate $f(x) = 2x^2 + 3x$ using $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$. **2**

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Question 26

Prove that $\frac{1 + \cot\theta}{\operatorname{cosec}\theta} - \frac{\sec\theta}{\tan\theta + \cot\theta} = \cos\theta$. **2**

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Question 27

A couple take out a mortgage of \$$P$ at a reducible interest rate of 2.4% p.a.
The couple plans to pay off the mortgage over n months with a monthly repayment of \$$M$.

A. Let $\$A_n$ be the amount owing after n repayments. **2**

Show that $A_3 = 1.002^3 P - M\left(1 + 1.002 + 1.002^2\right)$.

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Question 27 continues over the page

B. Hence, show that $A_n = 1.002^n P - M\left(\frac{1.002^n - 1}{0.002}\right)$. **1**

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Suppose that the couple borrowed \$750 000.

C. Show that the couple will never pay off the mortgage with a monthly repayment of \$1 000. **1**

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D. Find the minimum monthly repayment needed to eventually pay off the mortgage. **1**

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End of Part C

Use this page to rewrite any questions from Part C.

NESA Number										Q28-30	

YEAR 11 Mathematics Extension 1

TERM 2 COHORT TASK #1

Part D

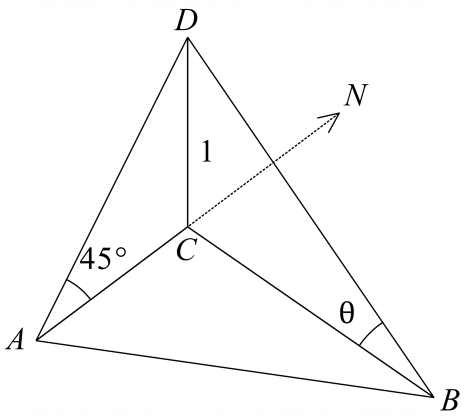
Section II

Part D 13 marks

Attempt all questions.
Answer each question in the space provided.
A blank page is provided at the end of each part to allow rewriting of a question.
Your response should include relevant mathematical reasoning and/or calculations.

Question 28

In the diagram below, CD is a vertical pole of height 1 metre with a base C on horizontal ground.
The angle of elevation of D from a point A south of C is $45°$.
The angle of elevation of D from a point B east of C is $\theta°$.



A. Show that $BC = \cot\theta$ and hence show that $AB = \operatorname{cosec}\theta$. **3**

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Question 28 continues over the page

B. Show that $CM = \frac{1}{2}\operatorname{cosec}\theta$, where M is the midpoint of AB. **2**

Question 29

St. Jorge bank provides the following annuity tables to all of its customers:

Future value of an annuity with instalments of \$1

Periods n	*Interest rate per period*				
	1%	2%	3%	4%	5%
5	5.1010	5.2040	5.3091	5.4163	5.5256
10	10.4622	10.9497	11.4639	12.0061	12.5779
15	16.0969	17.2934	18.5989	20.0236	21.5786
20	22.0190	24.2974	26.8704	29.7781	33.0660

Present value of an annuity with instalments of \$1

Periods n	*Interest rate per period*				
	1%	2%	3%	4%	5%
5	4.8534	4.7135	4.5797	4.4518	4.3295
10	9.4713	8.9826	8.5302	8.1109	7.7217
15	13.8651	12.8493	11.9379	11.1184	10.3797
20	18.0456	16.3514	14.8775	13.5903	12.4622

AW, a loyal St. Jorge customer, pays \$5 000 every year into his superannuation scheme, which pays 4% per annum, compounded annually.

A. Find the total value of AW's fund at the end of 20 years. **1**
Round your answer to the nearest dollar.

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B. JM, a new St. Jorge customer, wins the \$75 000 lottery jackpot. **2**
She decides to invest it all at 4% per annum for 20 years, compounded annually.
To match JM's payout after 20 years, AW increases his annual payment by some amount x.
Find x, rounding your answer to the nearest dollar.

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Question 30

An integer z consists of the digit 4, followed by n lots of the digit 4, followed by n lots of the digit 8, followed by the digit 9, i.e. $z = 4\overbrace{444...4}^{n}\overbrace{888...8}^{n}9$ for some integer $n \geq 0$.

A. Write z as a basic numeral for $n = 3$. **1**

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B. By writing in expanded notation, show that $z = \left(\frac{1+\left(2\times 10^{n+1}\right)}{3}\right)^2$ for all integers $n \geq 0$. **3**

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C. Hence, write $\left(\frac{1+\left(2\times 10^{n+1}\right)}{3\times 10^{n-1}}\right)^2$ as a basic numeral for $n = 6$ without rounding. **1**

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End of Part D

Use this page to rewrite any questions from Part D.

NESA Number										Q31-33	

YEAR 11 Mathematics Extension 1

TERM 2 COHORT TASK #1

Part E

Section II

Part E 12 marks

Attempt all questions.
Answer each question in the space provided.
A blank page is provided at the end of each part to allow rewriting of a question.
Your response should include relevant mathematical reasoning and/or calculations.

Question 31

A. Factorise $e^{3x} + e^{3y}$. **1**

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B. Given that $e^x + e^y = 3$ and $e^{3x} + e^{3y} = 10$, find the exact value of $x + y$. **2**

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Question 32

Solve $|2x - 1| + 8 = |3 - 6x|$. **2**

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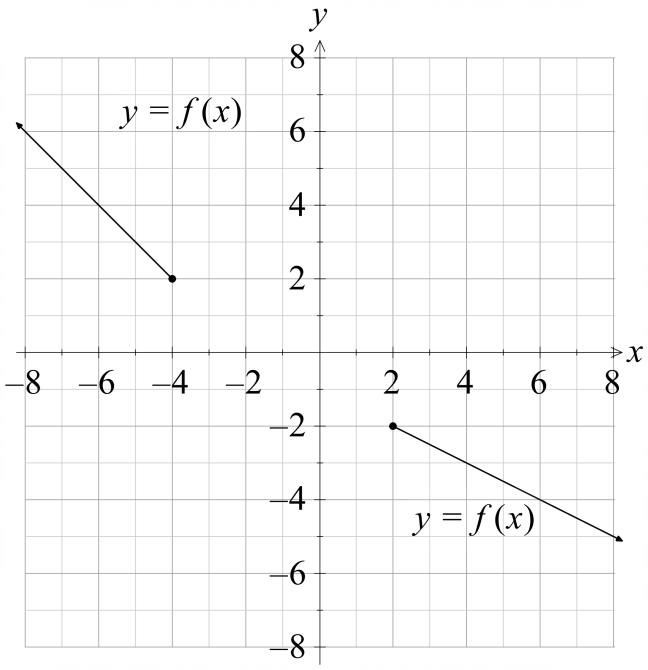
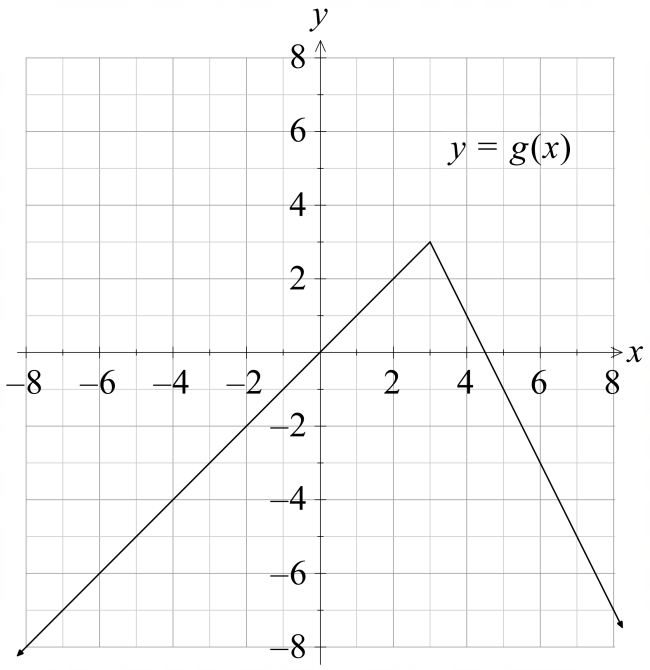
Question 33

The graphs of $y = f(x)$ and $y = g(x)$ are drawn **to scale** on separate number planes below.





A. Given that $A(x) = f(x) + g(x)$, express $A'(x)$ as a piecewise (piecemeal) function. **3**

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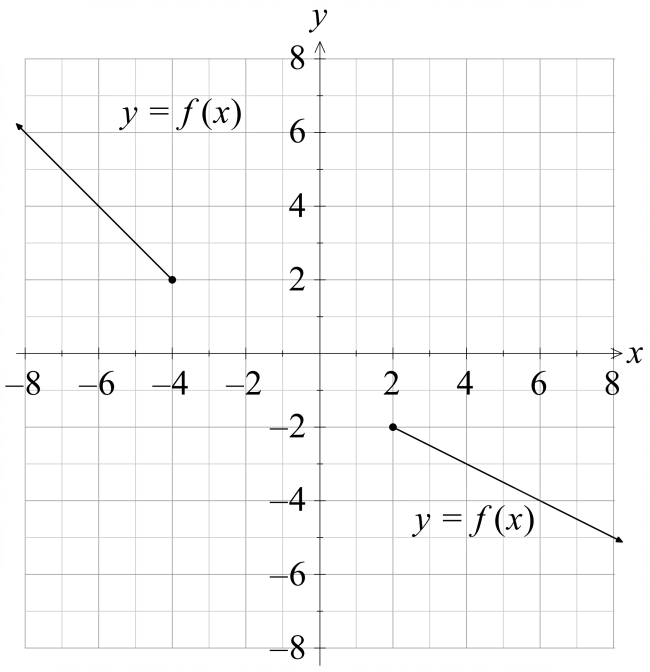
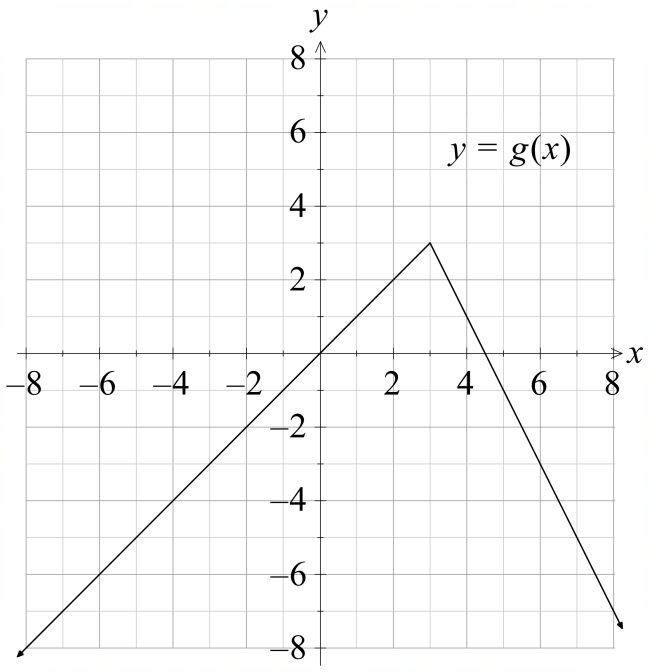
Question 33 continues over the page





B. Given that $B(x) = f(x)g(x)$, find the domain of $B'(x)$. **1**

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Question 33 continues over the page

C. Given that $C(x) = \big(f \circ g\big)(x)$, find $C'(-6)$. **2**

D. Given that $D(x) = \big(g \circ f\big)(x)$, show that $D'(-5)$ is undefined. **1**

End of paper

Use this page to rewrite any questions from Part E.

SYDNEY BOYS HIGH SCHOOL

2021

YEAR 11
TERM 2
COHORT TASK 1

Mathematics Extension

Sample Solutions

NOTE: **Some of you may be disappointed with your mark.**
This process of checking your mark is NOT the opportunity to improve your marks.
Improvement will come through further revision and practice, as well as reading the solutions and comments.

Before putting in an appeal re marking, first consider that the mark is not linked to the amount of ink you have used.

Just because you have shown 'working' does not justify that your solution is worth any marks.

Quick MC Answers

1. B
2. A
3. D
4. C
5. D
6. C
7. B
8. A
9. C*
10. C

2021 Y11 HY Ext Multiple choice solutions

Mean (out of 10): 6.03

1. $1.09\dot{8}7 \approx 1.10$ (B)

A	2
B	160
C	6
D	7

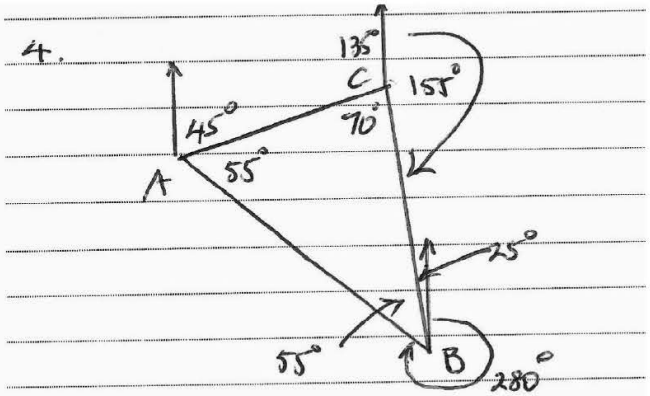
2. $(3\sqrt{7}+7\sqrt{3})^2 = 9\times7+2\times21\sqrt{21}+49\times3$
$=63+147+42\sqrt{21}$
$=210+42\sqrt{21}$
$=6(35+7\sqrt{21})$ (A)

A	173
B	0
C	2
D	0

3. $-6 < x \leq -2$
$\Rightarrow (-6, -2]$ (D)

A	1
B	1
C	7
D	166

4.



A A from C N 135° W
non-standard format X

B B from A E 10° S
non-standard format X

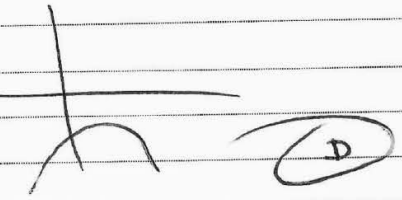
C C from B 135° T ✓

D A from C 225° T X

(C)

A	25
B	37
C	104
D	9

5. Negative definite parabola
$\Rightarrow a < 0$ and $\Delta < 0$



(D)

A	3
B	10
C	85
D	77

6. $y = f\left(\frac{x}{4}\right)$

$\therefore$ Horizontal dilation factor of $\frac{1}{\left(\frac{1}{4}\right)} = 4$ (C)

A	12
B	19
C	115
D	29

7. 0.75% pa = 0.0625% per month

Amount after 1 month $= 1 \times 1.000625^{1}$

2 months $= 1 \times 1.000625^{2}$

60 months $= 1 \times 1.000625^{60}$

$= 100\,000$

$\therefore A = 96320.57$

$\approx \$96321$ (B)

A	9
B	152
C	9
D	5

8. $y = \sec x$ has discontinuities at $x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \ldots$

$\Rightarrow$ A or B

$y = \sec 2x$ has discontinuities at $2x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{4}, \ldots$

$\therefore x = \pm\frac{\pi}{4}, \pm\frac{3\pi}{4}, \ldots$

$y = \sec\left(\frac{x}{2}\right)$ has discontinuities at $\frac{x}{2} = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \ldots$

$\therefore x = \pm\pi, \pm 3\pi, \ldots$

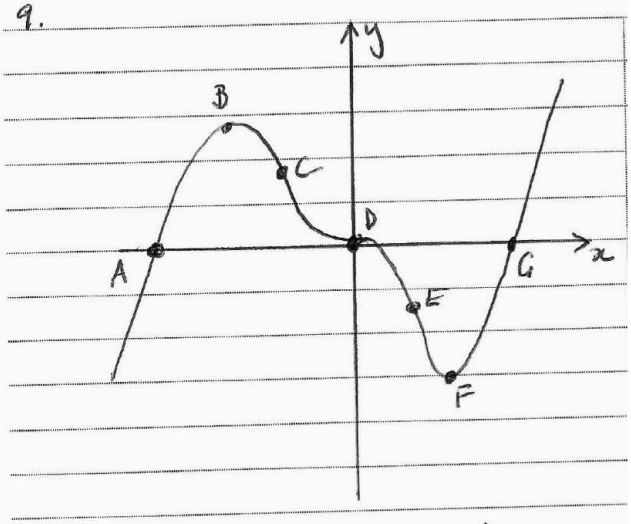
$y = \operatorname{cosec} x$ has turning point at $(0, 1), (\pi, -1), \ldots$

$\therefore$ For turning points at $(0, 2), (\pi, -2), \ldots$

$y = 2\operatorname{cosec} x$ (A)

A	142
B	12
C	14
D	7

9.



A There are 3 intercepts A, D and G (D is both an an x intercept and a y intercept) ✓

B There are 3 stationary points B, D and F ✓

C. There are 2 turning points B and F ✓ ✗

D There are 3 inflection points C, D and E ✓

C

NOTE: THIS ANSWER WAS NOT INCLUDED IN RESULTS AS IT SHOULD NOT HAVE BEEN EXAMINED.

A	4
B	10
C	66
D	95

10.

	Translation 2 units to right	Reflection in $x=2$
x	$\Rightarrow$ x	$\Rightarrow$ x
-1	1	3
0	2	2
1	3	1
2	4	0
3	5	-1
	$X = x - 2$	$X = 2 - x$

Reflection in $x=1$

X	x
-1	3
0	2
1	1
2	0
3	-1

$X = 2 - x$

Both A and B are correct

C

A	23
B	19
C	34
D	98

Section II

Part A 15 marks

Attempt all questions.
Answer each question in the space provided.
A blank page is provided at the end of each part to allow rewriting of a question.
Your response should include relevant mathematical reasoning and/or calculations.

Question 11

Expand $(1-x)^3$ completely. **1**

$(1-x)(1-2x+x^2) \Rightarrow 1-2x+x^2-x+2x^2-x^3$

Which simplifies to: $1-3x+3x^2-x^3$

NOTE: This caught out a lot of pupils who used the factorisation of 1^3-x^3. Some successfully used binomial expansion (not taught at school as of yet) but didn't get the signs right. Those people should have considered using $(1+(-x))^3$.

Question 12

Evaluate $\sum_{n=1}^{3} n^n$. **1**

$1^1+2^2+3^3=32$

NOTE: Many pupils wrote the correct sum but then used 3^2 when evaluating and hence wrote down the wrong evaluation.

Question 13

Write $\log_5 7$ correct to 3 significant figures. **1**

$\log_5 7=\frac{\ln 7}{\ln 5}$ OR $\frac{\log_{10} 7}{\log_{10} 5}$ which are both approximately 1.21 (to 3 significant figures)

NOTE: This was done very well by almost all pupils.

Question 14

Express $36°$ in radians as a multiple of π. **1**

Let α be the angle we need in radians. $\alpha=\frac{\theta}{180°}\times\pi$, where θ is in degrees

$\alpha=\frac{36°}{180°}\times\pi$, which gives a value of $\frac{\pi}{5}$ for α

NOTE: This was done very well by almost all pupils.

Question 15

State whether $f(x)=\sqrt[3]{x}$ is odd, even or neither. **1**

$f(-x)=\sqrt[3]{-x}$ which equals $-\sqrt[3]{x}$, then $f(-x)=-f(x)$ and the function is therefore odd.

NOTE: This was generally done well by almost all pupils, however a handful failed to do what the question asked (which was to state which type of function this is).

Question 16

Differentiate the following with respect to x:

A. $y=4x^3+x^2+1$ **1**

For each term use the fact that for $a\times x^n$, $n\neq 0$ that $\frac{dy}{dx}=n\times a\times x^{n-1}$ and for all constant terms $\frac{dy}{dx}=0$.

$\frac{dy}{dx}=12x^2+2x$

NOTE: This was done very well by almost all pupils.

B. $y=(2x+1)^3$ **1**

Using the Chain Rule let $y=u^3$, where $u=2x+1$. Then $\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}$.

$\frac{dy}{du}=3u^2$ and $\frac{du}{dx}=2$. Then $\frac{dy}{dx}=3\times u^2\times 2$ and we substitute $u=2x+1$ in order to get our equation in x.

$\frac{dy}{dx}=6(2x+1)^2$. NOTE: This was done very well by almost all pupils.

C. $y=\frac{x^2}{x+1}$ **2**

Let $y=\frac{u}{v}$ and use Leibnitz rule: $\frac{dy}{dx}=u'\times v^{-1}+-v^{-2}\times v'\times u$ which simplifies to $\frac{u'v-v'u}{v^2}$.

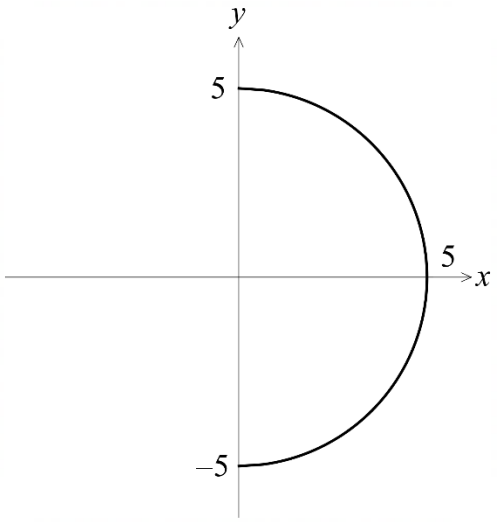
$u'=2x$ and $v'=1$

Hence $\frac{dy}{dx}=\frac{2x\times(x+1)-1\times x^2}{(x+1)^2}$ OR $\frac{x(x+2)}{(x+1)^2}$

NOTE: A handful of pupils used the Chain Rule to do this, which is the incorrect method. A handful got the signs wrong in the numerator. Pupils should have double checked their formula (those who used the simplified version of Leibnitz Rule for rational functions – 'the quotient rule').

Question 17

State the domain and range of the graph below. **2**



Domain is $[0,5]$ and Range is $[-5,5]$

NOTE: This was generally done well. A handful of students stated that the Range was in the variable x – which is obviously incorrect. A very small number decided to put 2 solutions for each required answer, assuming they were different versions of the same solutions. Sometimes they were not equivalent and one is not allowed to put 2 different answers!

Question 18

Given that $\log_{10} x = y$, express $\log_{10} \frac{1}{\sqrt{x}}$ in terms of y. **1**

$\log_{10} \frac{1}{\sqrt{x}} = -\frac{1}{2}\log_{10} x$ which is $-\frac{\log_{10} x}{2}$. Since $y = \log_{10} x$, this is equal to $-\frac{y}{2}$.

NOTE: Many pupils failed to understand the question and started with $y = \log_{10} \frac{1}{\sqrt{x}}$. There was often a lot of working for this question which was not given marks since the working didn't appear to be leading to the correct solution.

Question 19

Given an arithmetic sequence with terms $T_5 = 17$ and $T_{12} = 52$, evaluate T_{36}. **3**

$T_n = a + (n-1)d$, hence $T_{12} = T_5 + 7 \times d$ since $T_5 = a + 4 \times d$

Therefore $d = \frac{52-17}{7}$ which is 5.

$T_{36} = T_{12} + 24 \times d$ which is $52 + 24 \times 5 = 172$

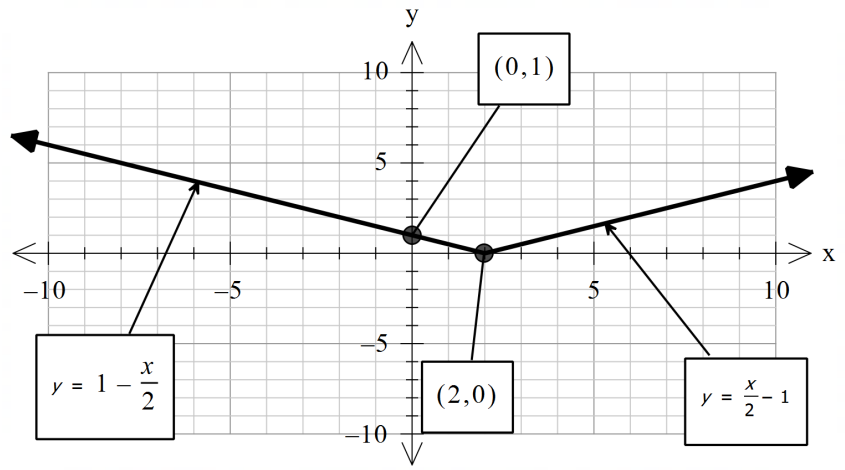
NOTE: This was generally well done by most pupils. All but 2 pupils worked out the first term after working out a value for the arithmetic difference. As you can see from the solutions above, there was NO NEED to evaluate this. Everyone who did this did more work than was necessary.

End of Part A

Solutions to Year 11 2021 Half Yearly: Section B

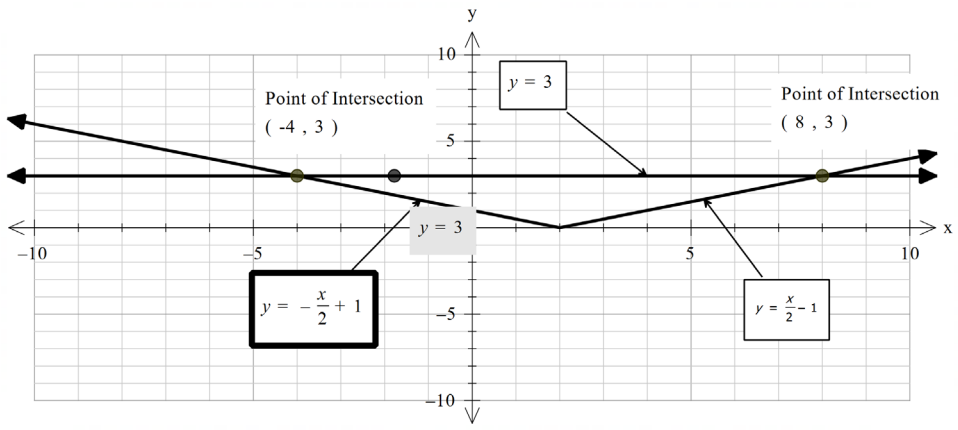
Question 20

A.



Full 2 marks for a correct sketch with y-intercept shown.

B.



The curves $y = 3$ and $y = |\frac{x}{2} - 1|$ intersect at $x = 8$ and $x = -4$.

The region where $|\frac{x}{2} - 1| \geq 3$ is occurs where $x \geq 8$ or $x \leq -4$.

For full marks in part B, students had to demonstrate that they had used the graph to find the solution.

Question 21

$$y = 1 + x - \frac{1}{x} - 1$$
$$= x - \frac{1}{x}$$
$$= x - x^{-1}$$
$$y' = 1 + x^{-2}$$
$$= 1 + \frac{1}{x^2}$$

$$\text{For } x = -2: \qquad y' = 1 + \frac{1}{4} = \frac{5}{4}$$

$$y' = 1 + \frac{1}{4} = \frac{5}{4}$$

$$m_{\text{normal}} = -\frac{4}{5}$$

$$y - 3 = -\frac{4}{5}(x + 2)$$

The Common mistake was differentiating incorrectly, this commonly occurred when students attempted to use the product rule to differentiate.

Question 22

$$125 = 250e^{-0.1t}$$
$$\therefore e^{-0.1t} = 0.5$$
$$\therefore -0.1t = 0.5$$
$$\therefore t = \frac{\log_e(0.5)}{-0.1}$$
$$\therefore t = 7 \text{ hours (to nearest hour)}$$

This question was generally well done. The most common error was subbing in an incorrect value for M in the first step, eg M=0.5 or setting M to $0.5M$. Most students had good understanding of how to solve log equations.

Question 23

A.

At break even $R(x) = C(x)$

$\therefore -0.01x^2 + 7.1x = 2.5x + 408$

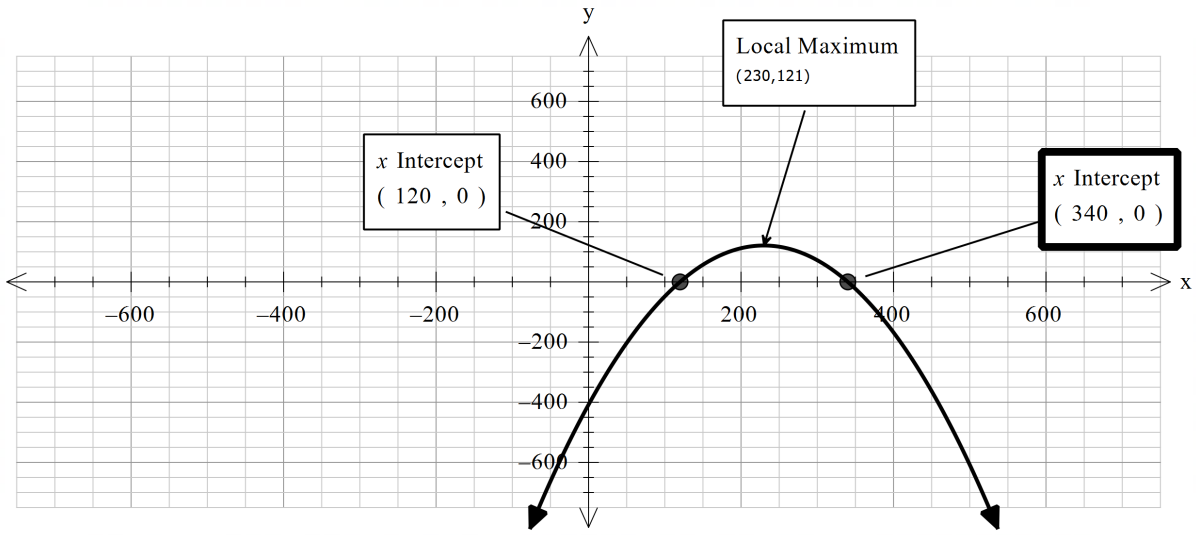
$\therefore -0.01x^2 + 4.6x - 408 = 0$

$\therefore x^2 + 460x - 40800 = 0$

$$\therefore x = \frac{-460 \pm \sqrt{460^2 - 4 \times -40800}}{2}$$

$\therefore x = 120$ or 340

B.



The maximum profit occurs at the midpoint of the two x-intercepts, which is

$$x = \frac{340 + 120}{2} = 230$$

At $x = 230$: Profit $= -0.01 \times 230^2 + 4.6 \times 230 - 408 = 121$ units of currency

Note that the vertex corresponds to a maximum value, as the curve is concave down $a < 0$.

It was required that students either explain why the vertex is a *maximum* value or show it graphically.

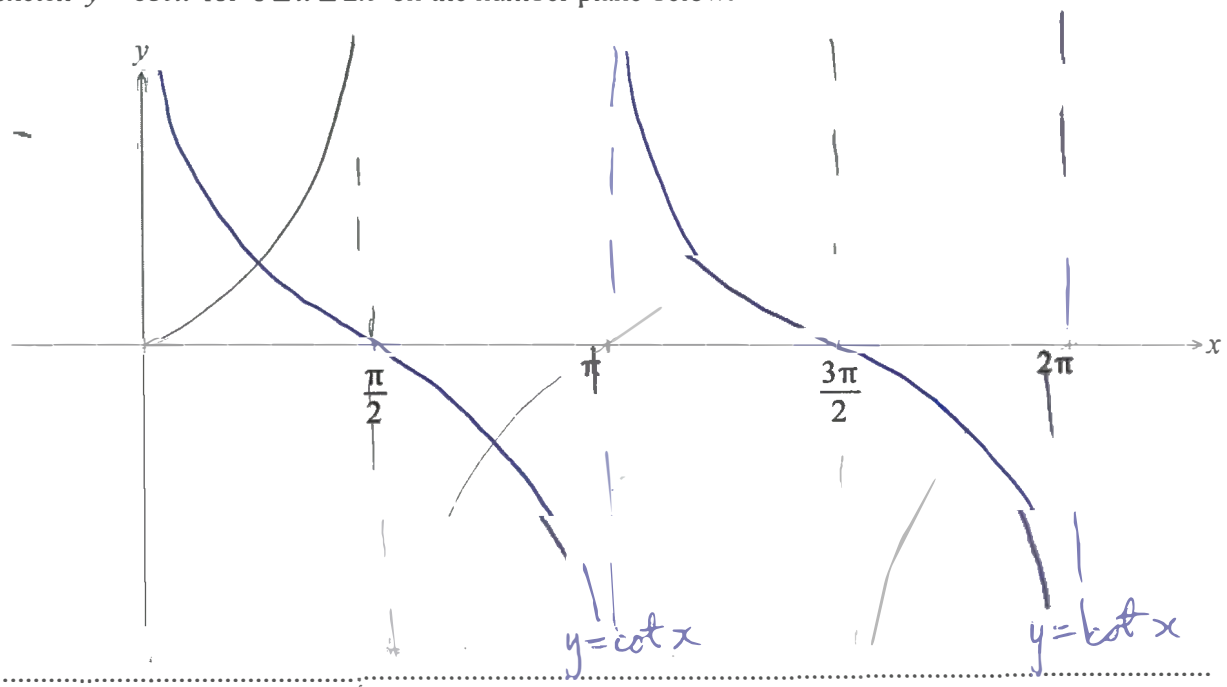
Section II

Part C **13 marks**

Attempt all questions.
Answer each question in the space provided.
A blank page is provided at the end of each part to allow rewriting of a question.
Your response should include relevant mathematical reasoning and/or calculations.

Question 24

A. Sketch $y = \cot x$ for $0 \le x \le 2\pi$ on the number plane below. **2**



Results were binary. Students either knew the graph or made a poor guess. Only a few seemed to try to derive it from the graph of tanx.

B. Hence or otherwise, show that $\tan x \times \tan\left(x + \frac{\pi}{2}\right) = -1$ for all $x \neq \frac{k\pi}{2}$, $k \in \mathbb{Z}$. **2**

Graph of $y = \tan x$ added above.

$\tan\left(x + \frac{\pi}{2}\right) = -\cot x$ by symmetry

Then $\tan x \times \tan\left(x + \frac{\pi}{2}\right)$

$= \tan x \times -\cot x$

$= \tan x \times \frac{-1}{\tan x}$

$= -1$ for $x \neq \frac{k\pi}{2}$, $k \in \mathbb{Z}$

- Again some students knew the trig identity and could just state it and get the marks. No students were able to deduce the identity from the graph.

Question 25

Differentiate $f(x)=2x^2+3x$ using $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$. **2**

$$f'(x)=\lim_{h\to 0}\frac{2(x+h)^2+3(x+h)-(2x^2+3x)}{h}$$

$$=\lim_{h\to 0}\frac{2x^2+4xh+2h^2+3x+3h-2x^2-3x}{h}$$

$$=\lim_{h\to 0}\frac{h(4x+2h+3)}{h}$$

$$=\lim_{h\to 0}(4x+2h+3)$$

$$\Rightarrow f'(x)=4x+3$$

Most did this well but half a mark was deducted if students didn't preface each line until the last line with the limit as h approaches 0.

Question 26

Prove that $\frac{1+\cot\theta}{\operatorname{cosec}\theta}-\frac{\sec\theta}{\tan\theta+\cot\theta}=\cos\theta$. **2**

$$LHS=\left(1+\frac{\cos\theta}{\sin\theta}\right)\sin\theta-\frac{1}{\frac{\cos\theta}{\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}}}$$

$$=\sin\theta+\cos\theta-\left(\frac{1}{\cos\theta}\div\frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}\right)$$

$$=\sin\theta+\cos\theta-\frac{1}{\cos\theta}\times\frac{\cos\theta\sin\theta}{1}$$

$$=\sin\theta+\cos\theta-\sin\theta$$

$$=\cos$$

$$=RHS$$

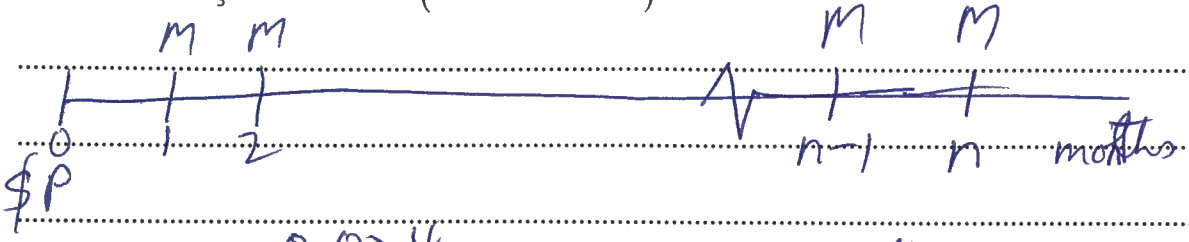
Some students did not simplify to functions of only sin and cos and then did not get the result. There were many kinds of errors eg applying Pythagorean identity when term was not squared and poor manipulation of fractions.

Question 27

A couple take out a mortgage of $\$P$ at a reducible interest rate of 2.4% p.a.
The couple plans to pay off the mortgage over n months with a monthly repayment of $\$M$.

A. Let $\$A_n$ be the amount owing after n repayments. **2**

Show that $A_3 = 1.002^3 P - M\left(1+1.002+1.002^2\right)$.



$r = \frac{0.024}{12} = 0.002$ per month

$A_1 = P(1.002) - M$

$A_2 = A_1(1.002) - M$

$= [P(1.002) - M]1.002 - M$

$= P(1.002)^2 - 1.002M - M$

$A_3 = A_2(1.002) - M$

$= [P(1.002)^2 - 1.002M - M]1.002 - M$

$= P(1.002)^3 - 1.002^2M - 1.002M - M$

$= P(1.002)^3 - M[1 + 1.002 + 1.002^2]$

Students were awarded full marks if they developed each term, A1, A2 and A3 from the previous term.

Question 27 continues over the page

B. Hence, show that $A_n = 1.002^n P - M\left(\frac{1.002^n - 1}{0.002}\right)$. 1

Then $A_n = P(1.002)^n - M[1 + 1.002 + 1.002^2 + \ldots + 1.002^{n-1}]$

GP $a = 1$, $r = 1.002$, n terms $\quad S_n = \frac{a(r^n - 1)}{r - 1}$

$\Rightarrow A_n = P(1.002)^n - M\left(\frac{1.002^n - 1}{0.002}\right)$

Students needed to have the top line and identify the GP and the parts of the GP and then apply the formula for the sum of a GP.

Suppose that the couple borrowed \$750 000.

C. Show that the couple will never pay off the mortgage with a monthly repayment of \$1 000. 1

Let $P = \$750000$, $M = 1000$

$A_1 = 1.002 \times 750000 - 1000$

$= \$750500$ ✓

$\therefore A_n$ keeps increasing & loan will never be paid.

Variable results. Many saw the above method. Some proved that the general result using \$1000 reduced to the log of a negative number. This was given full marks.

D. Find the minimum monthly repayment needed to eventually pay off the mortgage. 1

Let $A_1 = 750000$ (ie. assume payment = interest)

$\Rightarrow 750000 = 1.002 \times 750000 - M$

$750000 = 751500 - M$

$\Rightarrow M = \$1500$ ✓

$\therefore$ Min monthly payment, $M > \$1500$ to pay off loan

End of Part C

If students did part C the way presented above then they generally got this result as well.

Part D

Q28

A. In ΔBCD

$$\tan\theta = \frac{CD}{BC}$$

$$= \frac{1}{BC}$$

$$\therefore \cot\theta = BC$$

Similarly, in ΔACD

$$\cot 45 = AC$$

$$AC = 1$$

In ΔABC, $\angle ACB = 90°$, since B is due east of C, and A is due south

$$\therefore AB^2 = AC^2 + BC^2, \text{ by Pythagoras' Theorem}$$

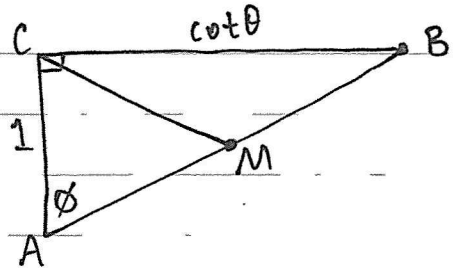
$$= 1 + \cot^2\theta$$

$$= \operatorname{cosec}^2\theta$$

$$\therefore AB = \operatorname{cosec}\theta, \text{ taking positive only, as } AB > 0, \text{ and } \operatorname{cosec}\theta > 0 \text{ if } \theta > 0$$

- Overall, this question was done well, but the marking was lenient due to the general lack of clarity in solutions.
- Students were not penalised for not labelling which triangle they derive a result from, nor for failing to state that $\angle ACB = 90°$ or their use of Pythag.
- Some students unnecessarily used Cosine rule on ΔABC.
  Don't bring a gun to a knife fight.
- Some students are not doing enough to 'show' the required result.
  For example, many assumed $\angle BCBA = \theta$ (which it is), but that makes the 'proof' trivial if an explanation is not given.

B. Method 1



Let $\angle CAB = \phi$

In $\triangle ABC$,

$$\cos\phi = \frac{AC}{AB}$$

$$= \frac{1}{\operatorname{cosec}\theta}$$

$$= \sin\theta$$

In $\triangle ACM$, using the Cosine Rule,

$$CM^2 = AC^2 + AM^2 - 2 \times AC \times AM \times \cos\angle CAM$$

$$= 1 + \left(\tfrac{1}{2}\operatorname{cosec}\theta\right)^2 - 2 \times 1 \times \tfrac{1}{2}\operatorname{cosec}\theta \times \sin\theta$$

$$= 1 + \tfrac{1}{4}\operatorname{cosec}^2\theta - 1$$

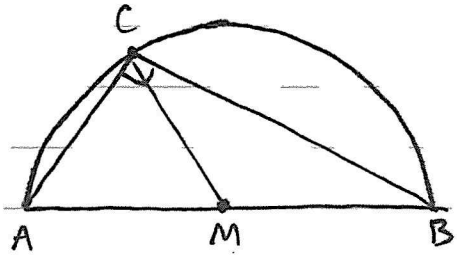
$$= \tfrac{1}{4}\operatorname{cosec}^2\theta$$

$$\therefore CM = \tfrac{1}{2}\operatorname{cosec}\theta$$

* $AM = \frac{1}{2}AB$

$= \frac{1}{2}\operatorname{cosec}\theta$

Method 2



As $\angle ACB = 90°$, AB is a diameter of a circle passing through A, B and C.

M, midpoint of AB, is the centre of the circle.

$$CM = AM \text{ (equal radii)}$$

$$= \tfrac{1}{2}AB$$

$$= \tfrac{1}{2}\operatorname{cosec}\theta$$

- This question was done poorly. Many students assumed $\phi = 45°$, or left it blank. Note that Method 1 could be adapted to use $\angle ABC$ instead.
- Some students successfully showed $\triangle ABC \equiv \triangle DBC$ in proving $CM = \frac{1}{2}\operatorname{cosec}\theta$

Q29

A. Future value multiplier = 29·7781

total value = 29·7781 × $5000

= $148,890·50

≈ $148,891

- Done well overall. Some students unnecessarily derived the result by summing a geometric series. Consider the mark allocation of the question.

B. Method 1

JM's future value = $75,000 × $(1+0.04)^{20}$

= $164,334·24

Difference in future values = $164,334·24 − $148,890·50

= $15 443·74

Present value installment = $15 443·74 ÷ 29·7781

= $518·63

≈ $519

Method 2

Present value multiplier = 13·5903

Present value installment = $75 000 ÷ 13·5903

= $5,518.64

∴ Required increase = $518·64 ≈ $519

- Many students got this wrong, getting values that were astronomically high. Consider the 'reasonableness' of your answer.

Q30

A. $z = 44,448,889$

- Most students got this correct. The most common error was answering 4,448,889.

B. $z = 4\ \underbrace{44\ldots4}_{n\text{-times}}\ \underbrace{88\ldots8}_{n\text{-times}}\ 9$

$= 4\times10^{2n+1} + 4\times10^{2n} + \ldots + 4\times10^{n+1} + 8\times10^{n} + 8\times10^{n-1} + \ldots + 8\times10^{1} + 9$

$=$ series 1 + series 2 + 9

$= \dfrac{4\times10^{n+1}(10^{n+1}-1)}{9} + \dfrac{8\times10^{1}(10^{n}-1)}{9} + 9$

$= \dfrac{4\times10^{2n+2} - 4\times10^{n+1} + 8\times10^{n+1} - 80 + 81}{9}$

$= \dfrac{4\times10^{2n+2} + 4\times10^{n+1} + 1}{9}$

$= \dfrac{(2\times10^{n+1})^2 + 2\times1\times(2\times10^{n+1}) + 1}{3^2}$

$= \left(\dfrac{1 + 2\times10^{n+1}}{3}\right)^2$

- Very poorly done. Most students did not express z in expanded notation as asked
- Of those who did, many used incorrect powers of 10.
  eg $4\times10^{2n} + \ldots \times 4\times10^{n}$,
- When expressing series, some were too vague. eg. $4\times10^{2n} + \ldots + 8\times10^{n} + \ldots$
  You should explicitly state at least the first two, and last one term in a series.

C. $z = 4,444.44488888889$

- Students who answered $z = 4,444.444889$, which is what a calculator displays, received $\frac{1}{2}$ mark only.

Part E Solutions

Question 31

A. Factorise $e^{3x}+e^{3y}$. **1**

$$e^{3x}+e^{3y}=\left(e^{x}\right)^{3}+\left(e^{y}\right)^{3}$$

$$=\left(e^{x}+e^{y}\right)\left(e^{2x}-e^{x}e^{y}+e^{2y}\right)$$

$$A^3+B^3=\left(A+B\right)\left(A^2-AB+B^2\right)$$

$$=\left(e^{x}+e^{y}\right)\left(e^{2x}-e^{x+y}+e^{2y}\right)$$

Comment:

The poor performance with this question seems to indicate that students are relying too much on the Reference sheet rather than their memory.

B. Given that $e^{x}+e^{y}=3$ and $e^{3x}+e^{3y}=10$, find the exact value of $x+y$. **2**

$$e^{3x}+e^{3y}=\left(e^{x}+e^{y}\right)\left(e^{2x}-e^{x+y}+e^{2y}\right)\Rightarrow 10=3\left(e^{2x}-e^{x+y}+e^{2y}\right)$$

$$e^{2x}-e^{x+y}+e^{2y}=\frac{10}{3} \qquad -(1)$$

Now $\left(e^{x}+e^{y}\right)^{2}=e^{2x}+2e^{x+y}+e^{2y}$

$$\therefore e^{2x}+2e^{x+y}+e^{2y}=3^{2}=9 \qquad -(2)$$

$(2)-(1)$: $\quad 3e^{x+y}=\frac{17}{3}$

$$\therefore e^{x+y}=\frac{17}{9}$$

$$\therefore x+y=\ln\left(\frac{17}{9}\right)$$

Comment:

This was well done by students who were able to do Part A.

'Ecf' was not awarded in part B where students significantly oversimplified things or wrote things like $e^{3}=\frac{10}{3}$.

Question 32

Solve $\lvert 2x-1\rvert+8=\lvert 3-6x\rvert$.	**2**

First step: $\lvert 3-6x\rvert=\lvert 6x-3\rvert$

$\therefore \lvert 2x-1\rvert+8=3\lvert 2x-1\rvert$

$\therefore 2\lvert 2x-1\rvert=8\Rightarrow\lvert 2x-1\rvert=4$

Now

$\lvert 2x-1\rvert=4\Rightarrow 2x-1=\pm 4$

$\therefore 2x-1=-4$ or $2x-1=4$

$\therefore x=-1.5$ or 2.5

Alternative Solution

$\lvert 2x-1\rvert+8=\lvert 6x-3\rvert$

$x<\frac{1}{2}$: $-(2x-1)+8=-(6x-3)$

$\therefore -2x+1+8=-6x+3$

$\therefore 4x=-6$

$\therefore x=-1.5$

$x\geq\frac{1}{2}$: $\therefore 2x-1+8=6x-3$

$\therefore -4x=-10$

$\therefore x=2.5$

Comment

This question was badly done by most students.
Not much care was taken with starting the problem, let alone verifying the solutions.

Most students scored 0.5 per correct solution as little evidence of understanding was present.
Those who showed understanding scored better, e.g. checking validity of solutions etc

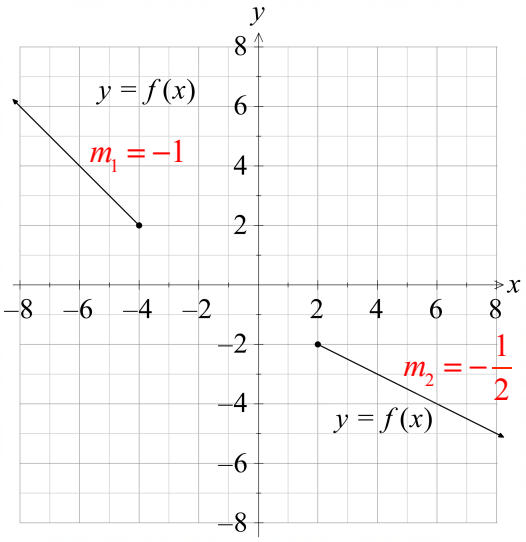
Many students thought 'squaring both sides' was a method to use. This method was not appropriate here.

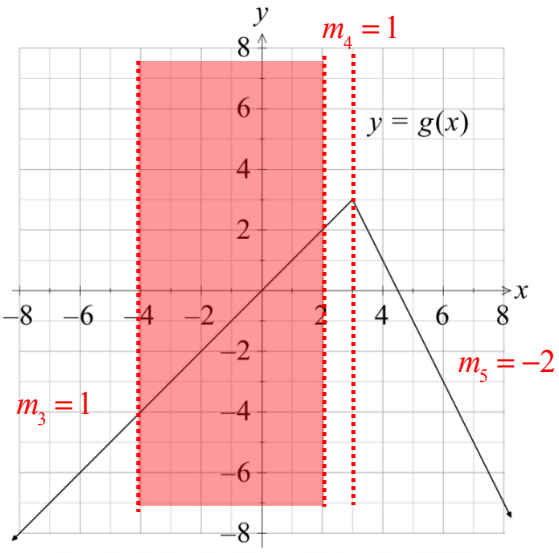
For students doing cases (like the alternative solution) they need to be careful:

$$\lvert x+a\rvert=\begin{cases}x+a, & x\geq -a\\ -(x+a), & x<-a\end{cases}\quad\textbf{NOT}\quad\lvert x+a\rvert=\begin{cases}x+a, & x\geq 0\\ -(x+a), & x<0\end{cases}$$

Question 33

The graphs of $y = f(x)$ and $y = g(x)$ are drawn **to scale** on separate number planes below.





A. Given that $A(x) = f(x) + g(x)$, express $A'(x)$ as a piecewise (piecemeal) function. **3**

$$f'(x) = \begin{cases} m_1 = -1, & x < -4 \\ m_2 = -\dfrac{1}{2}, & x > 2 \end{cases}$$

$$g'(x) = \begin{cases} m_{3/4} = 1, & x < 3 \\ m_5 = -2, & x > 3 \end{cases}$$

$$A'(x) = f'(x) + g'(x) = \begin{cases} m_1 + m_3 = 0, & x < -4 \\ m_2 + m_4 = \dfrac{1}{2}, & 2 < x < 3 \\ m_2 + m_5 = -\dfrac{5}{2}, & x > 3 \end{cases}$$

Comment

Students were not penalised if their answer was not expressed in strict piecewise form.

Students were penalised for things like including $x = 3$ for two different branches.
A function is not differentiable at an end point or at 'sharp' points, solutions had to reflect this.
The function $f(x)$ is not defined for $-4 < x < 2$, so $A(x)$ and consequently $A'(x)$ cannot be defined for $-4 < x < 2$.

A longer alternative solution is over the page.
This method is not recommended due to the time it takes to write it all out.

A. (Continued)

Alternative solution

$$f(x)=\begin{cases}-x-2, & x\le -4\\ -\frac{1}{2}x-1, & x\ge 2\end{cases} \qquad g(x)=\begin{cases}x, & x<3\\ -2x+9, & x\ge 3\end{cases}$$

$$A(x)=f(x)+g(x)=\begin{cases}-x-2+x=-2, & x\le -4\\ -\frac{1}{2}x-1+x=\frac{1}{2}x-1, & 2<x<3\\ -\frac{1}{2}x-1+9-2x=-\frac{5}{2}x+8, & x\ge 3\end{cases}$$

$$\therefore A'(x)=f'(x)+g'(x)=\begin{cases}0, & x<-4\\ \frac{1}{2}, & 2<x<3\\ -\frac{5}{2}, & x>3\end{cases}$$

B. Given that $B(x)=f(x)g(x)$, find the domain of $B'(x)$. **1**

$B'(x)=f(x)\times g'(x)+f'(x)\times g(x)$

The domain for $f'(x)$ is the same as $f(x)$ except for the "=".
Similarly, for $g(x)$.

With the addition and product of functions the domain is the intersection of the domains of the relevant functions. So using part **A**, the domain of $B'(x)$ is:

$(-\infty,-4)\cup(2,3)\cup(3,\infty)$ OR $x<-4,\ 2<x<3,$ or $x>3$.

Comment

For those students who had the time to do Part A, Part B should have been a quick question.
It is worth 1 mark and students should have taken this into account before they tried to work out a rule for $B(x)$, let alone $B'(x)$.

C. Given that $C(x) = (f \circ g)(x)$, find $C'(-6)$. **2**

$$C'(x) = f'(g(x)) \times g'(x)$$

$$\begin{aligned} C'(-6) &= f'(g(-6)) \times g'(-6) \\ &= f'(-6) \times 1 && [g(-6) = -6;\ g'(-6) = 1 \text{ from graph}] \\ &= -1 \times 1 && [f'(-6) = -1 \text{ from graph}] \\ &= -1 \end{aligned}$$

Comment

Students could have used a substitution/chain rule to help them:

$C(x) = f(g(x))$: Let $u = g(x)$: $C(x) = f(u)$

$$\begin{aligned} C'(x) &= \frac{d}{du}(f(u)) \times \frac{du}{dx} \\ &= f'(u) \times g'(x) \\ &= f'(g(x)) \times g'(x) \end{aligned}$$

D. Given that $D(x) = (g \circ f)(x)$, show that $D'(-5)$ is undefined. **1**

$$D'(x) = g'(f(x)) \times f'(x)$$

$$\begin{aligned} D'(-5) &= g'(f(-5)) \times f'(-5) && [f(-5) = 3;\ f'(-5) = -1 \text{ from graph}] \\ &= g'(3) \times (-1) \end{aligned}$$

Now $g'(-3)$ is undefined as $g(x)$ is not differentiable at $x = 3$ because $(3, g(3))$ is a 'sharp' point.

$\therefore D'(-5)$ is undefined.

Comment

Many didn't get here, but for those who did it might have been better if they had put the time into earlier questions.

End of solutions