Sydney Girls High School

MATHEMATICS EXTENSION 1

Year 11

Task 2 (Half-yearly)

April 2015

Time Allowed: 60 minutes + 5 minutes reading time

Topics: Basic Arithmetic & Algebra; Inequalities; Real Functions; Trigonometric ratios

Total: 64 marks

Instructions:

- There are FOUR (4) questions which are of equal value.
- Attempt all questions.
- Show all necessary working. Marks will be deducted for badly arranged work or incomplete working.
- Start each question on a new page.
- Write on one side of the paper only.
- Diagrams are NOT to scale.
- Board-approved calculators may be used.
- Write your student number at the top of each question and clearly number each question.

Student's Name:_____ **Teacher's Name:**_____

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Question 1: (16 Marks)

a) If $f(x)=3x^2-5x+4$ and $g(x)=2x+10$ find $f(1)-g(-1)$. 2

b) Find the exact value of $\sin\theta$ if $\tan\theta=\frac{3}{7}$ and $\cos\theta<0$. 3

c) Find the integers a and b such that $\frac{2}{2-\sqrt{3}}=a+\sqrt{b}$. 2

d) Express $\frac{x+1}{x^2-x}-\frac{x-1}{x^2+x}$ as a fraction in its lowest terms. 3

e) Solve $\frac{2}{x+3}\leq 5$ 3

f) Find the range of the function $y=8x-x^2$ 1

g) Expand and simplify $(2x-y)^4$ 2

Question 2: (16 Marks)

a) Find the value of m if $\sec 27^o = \text{cosec}(m+20)^\circ$ 1

b) Determine if the function $f(x) = x^3 - x^2$ is odd, even or neither. 2
Show working to justify your answer.

c) Sketch the following showing all relevant features.

i) $2x - y = 1$ 1

ii) $y = 3 - x^2$ 1

iii) $y = (2x-1)^3$ 1

iv) $y = -2^x$ 1

v) $y = |x^2 - 1|$ 1

vi) $y = \frac{2}{x^2} + 1$ 2

d) On the same diagram shade the following region for which $y \le x^2$, $y \ge 0$ and 3
$y \le \sqrt{9 - x^2}$ are simultaneously true.

e) Find the value of x and y if $2^{2x+y} = 128$ and $\frac{4^x}{16} = 8^{x+y}$ 3

Question 3: (16 Marks)

a) A function is defined by the rule $f(x)\begin{cases}1 & if \quad x \le -2 \\ -1 & if \quad -2 < x < 0 \\ x & if \quad x \ge 0\end{cases}$

i) Sketch the curve 2

ii) Find $f(a^2)$ 1

b) i) Show that $\frac{x+4}{x+3} = 1 + \frac{1}{x+3}$. 1

ii) Find the domain and range of the function 2

$$y = \frac{x+4}{x+3}$$

iii) Sketch the graph of $y = \frac{x+4}{x+3}$ 2

c) Solve the following equations where $0 \le \theta \le 360^o$

i) $\cos\theta = 0.7$ 2

ii) $\sin 2\theta = -\frac{\sqrt{3}}{2}$ 2

d) State the domain of the function $h(x) = \sqrt{2+x} - \sqrt{3-x}$ 2

e) On the same graph sketch the following for $0 \le x \le 360^\circ$ 2

i) $y = \sin x$

ii) $y = \text{cosec}\, x$

Question 4: (16 Marks)

a) From a ship travelling north, a tower on shore is observed to be on a bearing of 37^{o}. Later when the ship is 2 nautical miles from the tower, the bearing of the tower from the ship is 127^{o}.

i) Draw a diagram marking on it all the information given. 1

ii) Find, correct to 3 significant figures, the distance of the tower from the initial point of observation. 2

b) Prove $\dfrac{\tan x}{\sec x - 1} = \dfrac{\sec x + 1}{\tan x}$. 2

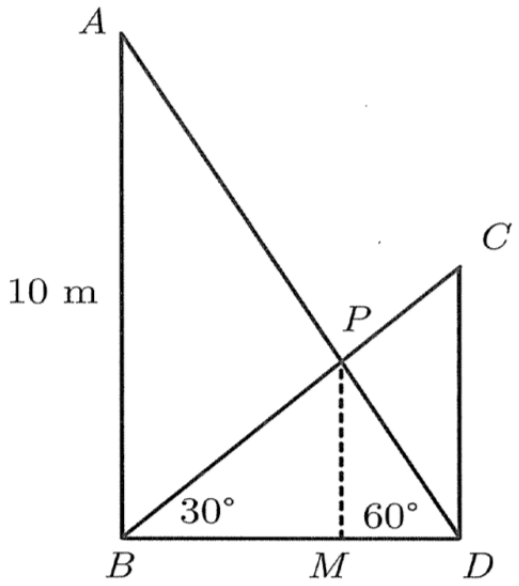
c) i) Sketch the curves $y = |x - 3|$ and $y = 5 - |x|$ on the same diagram. 2

ii) Hence or otherwise solve, $|x - 3| + |x| < 5$ 1

d) Solve $6\tan\theta - 5\operatorname{cosec}\theta = 0$ for $0 \le \theta \le 360^{o}$. 4

Question 4(continued): (16 Marks)

e) AB is a vertical pole height 10m. CD is a shorter vertical pole of unknown height. The angle of elevation from the base of AB to the top of CD is given as 30° and the angle of elevation from the base of CD to the top AB is given as 60°. Another vertical pole PM is placed at the point of intersection of the angles of elevation.



i) Copy the diagram onto your exam paper

ii) Find the **exact** distance the poles AB and CD are apart. 1

iii) Find the **exact** height of CD 1

iv) Find the **exact** height of PM 2

THE END

2015 YR EXT TASK 1

Q1

a) $f(x) = 3x^2 - 5x + 4$

$g(x) = 2x + 10$

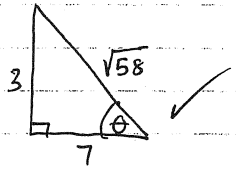
Find $f(1) - g(-1)$ ✓

$= 3(1)^2 - 5(1) + 4 - (2(-1) + 10)$

$= -6$ ✓

b) $\tan\theta > 0$, $\cos\theta < 0$ ∴ 3rd quadrant ✓ $\quad \pi < \theta < \frac{3\pi}{2}$

$\tan\theta = \frac{3}{7}$



✓

$\therefore \sin\theta = -\frac{3}{\sqrt{58}}$ ✓

c) $\frac{2}{2-\sqrt{3}} = a + \sqrt{b}$

$\frac{2(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})}$ ✓ $= \frac{4 + 2\sqrt{3}}{1} = a + \sqrt{b}$

$\therefore a = 4$

$4 \times 3 = b$

$\therefore \begin{cases} a = 4 \\ b = 12 \end{cases}$ ✓

d) $\frac{x+1}{x(x-1)} - \frac{x-1}{x(x+1)}$

$\frac{(x+1)^2 - (x-1)^2}{x(x-1)(x+1)}$ ✓ $= \frac{x^2 + 2x + 1 - x^2 + 2x - 1}{x(x-1)(x+1)}$ ✓

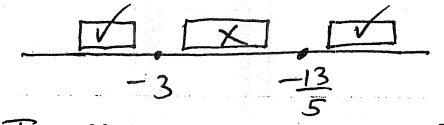
$= \frac{4}{(x-1)(x+1)}$ ✓

e) $\frac{2}{x+3} \leq 5$

Critical points : $x = -3$

$\frac{2}{x+3} = 5 \therefore 2 = 5x + 15$

$x = -\frac{13}{5}$



✓

- Try $x = -4$: $-2 \leq 5$ True
- Try $x = -2.8$: $10 \leq 5$ False ✓
- Try $x = -1$: $1 \leq 5$ False

Final solutions

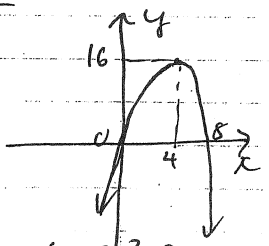
$x < -3$ OR $x \geq -\frac{13}{5}$ ✓

f) $y = 8x - x^2 = -x(x-8)$

vertex : $x = \frac{-b}{2a} = \frac{-8}{2(-1)} = 4$

$y = 8(4) - (4)^2$

$y = 16$



Range : $y \leq 16$ ✓

g) $(2x - y)^4 = (2x)^4 - 4(2x)^3(y) + 6(2x)^2y^2 - 4(2x)y^3 + y^4$ ✓

$= 16x^4 - 32x^3y + 24x^2y^2 - 8xy^3 + y^4$ ✓

QUESTION 2

(a) $90° - 27° = 63°$ 1

$m + 20° = 63°$

$m = 43°$

(b) $f(-x) = (-x)^3 - (-x)^2$

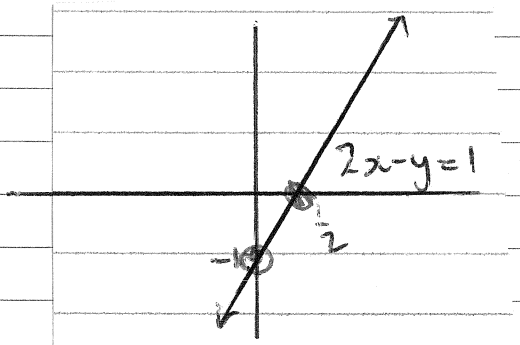
$= -x^3 - x^2$

$\neq f(x)$ or $-f(x)$. 2

NEITHER

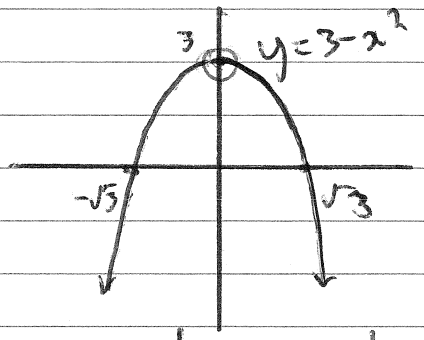
(c)

(i)



1

(ii)



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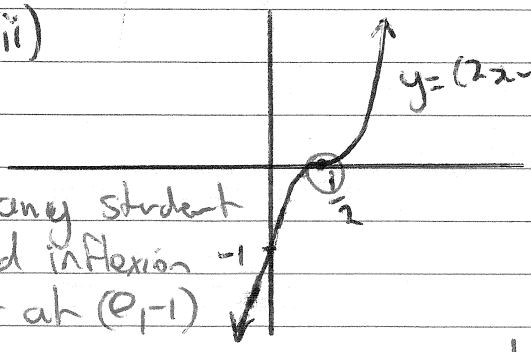
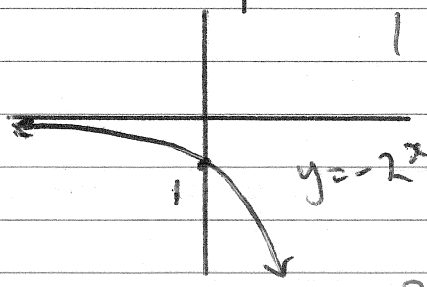
(iii)



Many student had inflexion -1 pt at $(0,-1)$

1

(iv)



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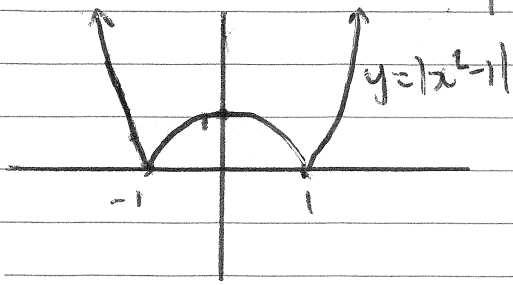
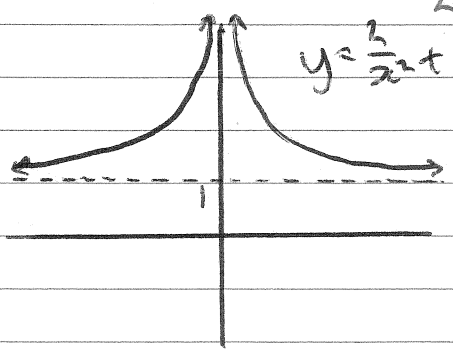
(v)



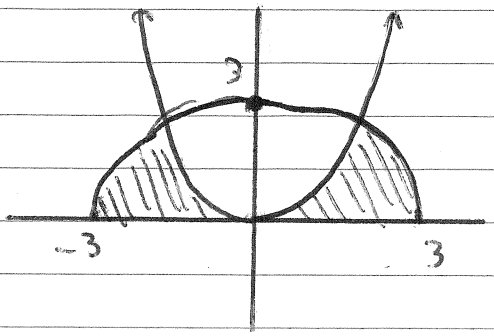
1

(vi)



2

(d)



3

Most student shaded above the parabola $y=x^2$.

(e) $2^{2x+y} = 128$

$2^{2x+y} = 2^7$

$2x+y=7$ (A)

Sub (B) into (A)

$-6y-8+y=7$

$-5y-8=7$

$-5y=15$

$y=-3$

Sub into (B)

$x=9-4$

$x=5$

$x=5,\ y=-3$

$\frac{4^x}{16} = 8^{x+y}$

$\frac{2^{2x}}{2^4} = 2^{3x+3y}$

$2^{2x-4} = 2^{3x+3y}$

$2x-4 = 3x+3y$

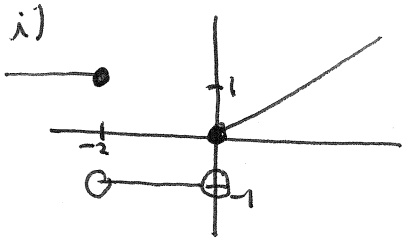
$-x = 3y+4$ (B)

$x = -3y-4$

3

Many student simplified $\frac{2^{2x}}{2^4}$ as $2^{\frac{2x}{4}}$.

3a) i)



Open and closed circles must be clearly shown

ii) $f(a^2) = a^2$

b) i) $\frac{x+4}{x+3} = \frac{x+3+1}{x+3}$

$= \frac{x+3}{x+3} + \frac{1}{x+3}$

$= 1 + \frac{1}{x+3}$ Q.E.D.

ii) domain : $x \neq -3$

range : $y \neq 1$

c) i) $\theta = 46°$ or $360° - 46°$

$= 46°$ or $314°$

ii) $2\theta = 180° + 60°, 360° - 60°,$

$540° + 60°, 720° - 60°$

$= 240°, 300°, 600°, 660°$

$\theta = 120°, 150°, 300°, 330°$

[If $0 \leq \theta \leq 360°$ then $0° \leq 2\theta \leq 720°$]

d) $2 + x \geq 0$ and $3 - x \geq 0$

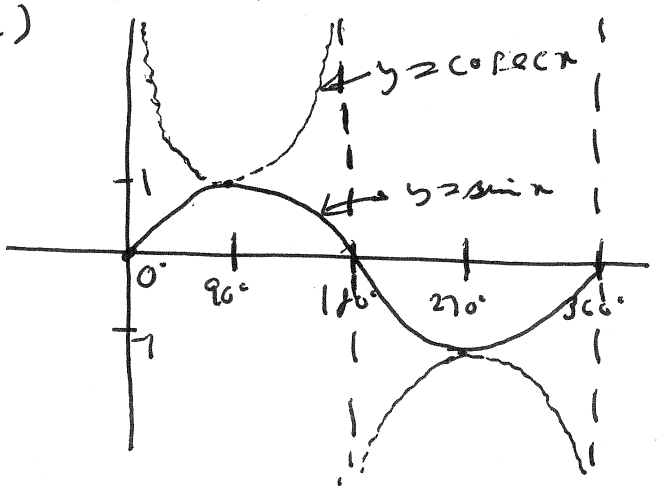
$x \geq -2$ $\quad -x \geq -3$

$x \leq 3$

$\therefore -2 \leq x \leq 3$

Some forgot to combine the solutions.

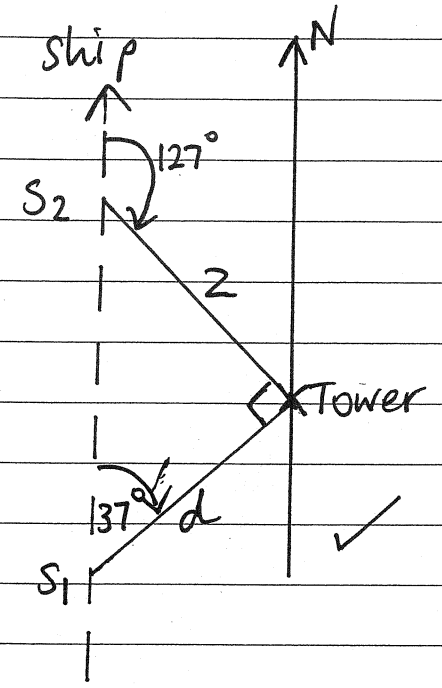
e)



Any inflexions must be drawn for $y = \text{cosec } x$.

Question 4 (16 Marks)

i) (1)



ii) (2)

$\tan 37° = \frac{2}{d}$ ✓

$\therefore d = \frac{2}{\tan 37°}$

$d \doteq 2.65 nm$ ✓

(3 sig figs)

iii) (2)

$$LHS = \frac{\tan x}{\sec x - 1}$$

$$= \frac{\tan x}{(\sec x - 1)} \times \frac{(\sec x + 1)}{(\sec x + 1)}$$

$$= \frac{\tan x(\sec x + 1)}{\sec^2 x - 1}$$ ✓

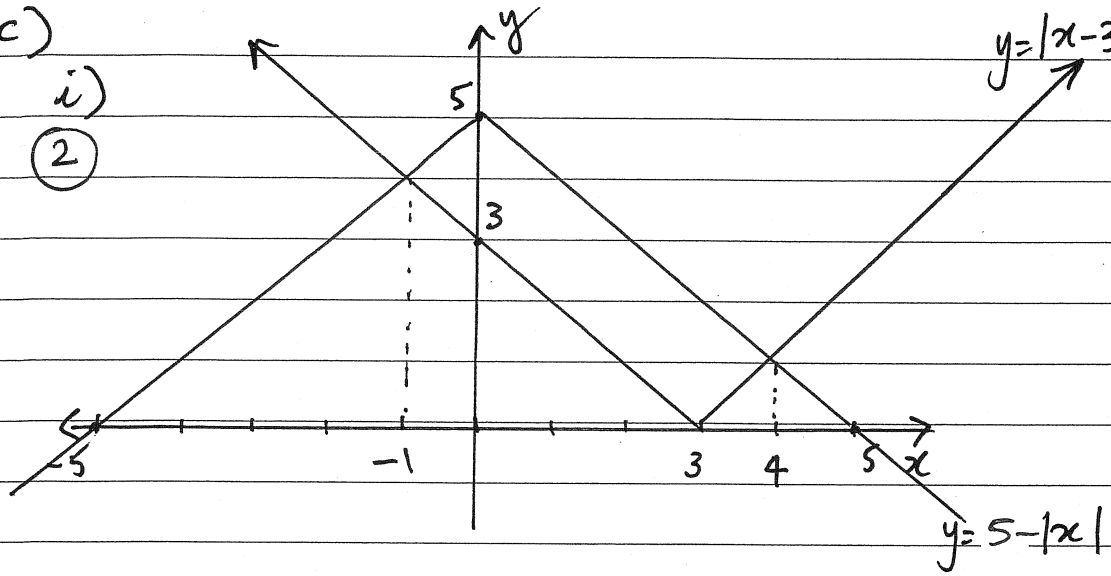
$$= \frac{\tan x(\sec x + 1)}{\tan^2 x}$$ ✓

$$= \frac{(\sec x + 1)}{\tan x}$$

$$= RHS$$

c)

i) (2)



scale intercepts ✓

shape ✓

ii) (1) $|x-3| + |x| < 5$

$|x-3| < 5 - |x|$

$\therefore \quad -1 < x < 4$ (from graph). ✓

d) (4) $6\tan\theta - 5\operatorname{cosec}\theta = 0$, $0 \le \theta \le 360°$.

$6\frac{\sin\theta}{\cos\theta} - 5.\frac{1}{\sin\theta} = 0$ ✓ $(\sin\theta \neq 0, \cos\theta \neq 0)$

$6\sin^2\theta - 5\cos\theta = 0$

$6(1-\cos^2\theta) - 5\cos\theta = 0$ ✓

$\therefore \quad 6\cos^2\theta + 5\cos\theta - 6 = 0$

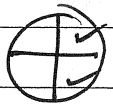
$6\cos^2\theta + 9\cos\theta - 4\cos\theta - 6 = 0$

$3\cos\theta(2\cos\theta+3) - 2(2\cos\theta+3) = 0$

$(2\cos\theta+3)(3\cos\theta-2) = 0$

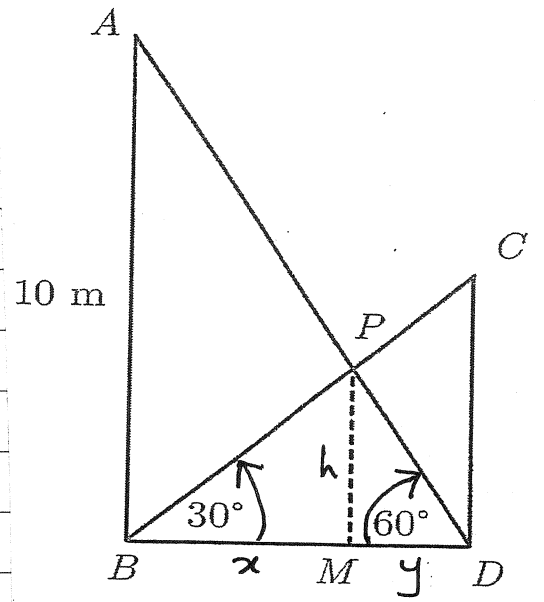
$\therefore \quad 2\cos\theta+3 = 0 \quad$ or $\quad 3\cos\theta - 2 = 0$

$\cos\theta = \frac{-3}{2} \qquad \cos\theta = \frac{2}{3}$



$\theta =$ No solutions ✓ $\qquad \theta = 48°11', 311°49'$ ✓

e) i)



ii) Find BD:

(1) In ΔABD: $\tan 60° = \frac{AB}{BD}$

$\therefore BD = \frac{AB}{\tan 60°}$

$\therefore BD = \frac{10}{\sqrt{3}} = \frac{10\sqrt{3}}{3}$ m. ✓

iii) Find CD:

(1) In ΔCDB: $\tan 30° = \frac{CD}{BD}$

$\frac{1}{\sqrt{3}} = CD \times \frac{\sqrt{3}}{10}$

$\therefore CD = \frac{1}{\sqrt{3}} \times \frac{10}{\sqrt{3}} = \frac{10}{3}$ m ✓

iv) Find PM. Let $BM = x$, $MD = y$ and $PM = h$.

(2) In ΔPMB: $\tan 30° = \frac{h}{x}$ In ΔPMD: $\tan 60° = \frac{h}{y}$

$\therefore x = \frac{h}{\tan 30}$ $\therefore y = \frac{h}{\tan 60°}$

Now: $x + y = \frac{h}{\tan 30} + \frac{h}{\tan 60°}$

$BD = \sqrt{3}h + \frac{h}{\sqrt{3}}$

from ii) $\frac{10}{\sqrt{3}} = \left(\frac{3+1}{\sqrt{3}}\right)h$

$\therefore h = \frac{10}{\sqrt{3}} \times \frac{\sqrt{3}}{4} = \frac{10}{4}$

$\therefore h = 2.5$ metres.