Sydney Girls High School

MATHEMATICS EXTENSION 1

Half Yearly

Year 11
2016

60 minutes + 5 minutes reading time

Topics: Basic Arithmetic and Algebra, Functions, Trigonometry.

Total: 60 marks

Instructions:

- There are FOUR (4) questions which are of equal value
- Attempt all questions
- Show all necessary working. Marks may be deducted for badly arranged work or incomplete working.
- Start each question on a new page.
- Write on one side of the paper only.
- Diagrams are NOT to scale.
- Board approved calculators may be used.

Name:..... **Teacher:** .....

Question 1 (Start a new page) **Marks**

(a) Find a and b if $\frac{2}{5-\sqrt{3}}-\frac{4}{6-2\sqrt{3}}=a+b\sqrt{3}$. 3

(b) Solve $x^2-6x+8>0$ and graph your solution on a number line. 3

(c) Factorise $x^4+x^2-y^2-y^4$ fully. 2

(d) If $\sin\theta=\frac{7}{10}$ find the exact value of $\tan\theta$ if θ is obtuse. 2

(e) Find the bearing of B from A if the bearing from B to A is 213°. 1

(f) A ladder 2m long reaches 1.5m up a vertical wall. Find the angle the ladder makes with the ground correct to the nearest degree. 2

(g) $x=\frac{3}{2}$ is a solution of $ax^2+5x-12=0$. Find the other solution. 2

Question 2 (Start a new page) **Marks**

(a) Evaluate $4^{3^{2^1}}$. 1

(b) Expand and simplify $(2x-3y)^4$. 2

(c) Solve $\frac{2}{x-1} \geq 3$. 3

(d) A ship sails on a bearing of 136° for a distance of 256km. How far South is the ship of the starting point? 2

(e) Draw the graph of $y = \cot x$ for $0^\circ \leq x \leq 360^\circ$. 2

(f) Find the exact value of

(i) $\sin 45^\circ$ 1

(ii) $\cos 240^\circ$ 1

(iii) cosec 300° 1

(g) Simplify

(i) $\sec\theta \cot\theta$ 1

(ii) $1+\sin(-\theta)\cos(90^\circ-\theta)$ 1

Question 3 (Start a new page) **Marks**

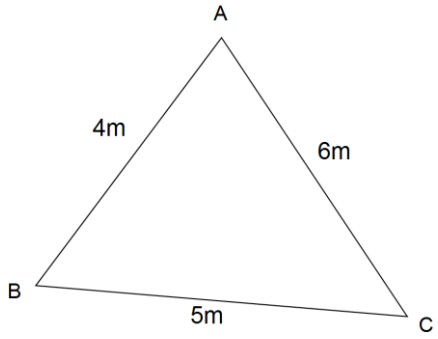
(a) Solve $\frac{x+1}{x^2+5x+6} + \frac{x-1}{x^2-9} = 0.$ 3

(b) For $y = \frac{x-4}{x+1}$

(i) Find the equations of the asymptotes. 2

(ii) Find the intercepts and sketch the curve. 2

(c)



Find the area of the above triangle correct to 2 significant figures. 3

(d) On a number plane sketch $y = 4 - |x+3|$ showing all important features. 3

(e) Find the domain and range of $y = \frac{4}{\sqrt{1-x}}$. 2

Question 4 (Start a new page) **Marks**

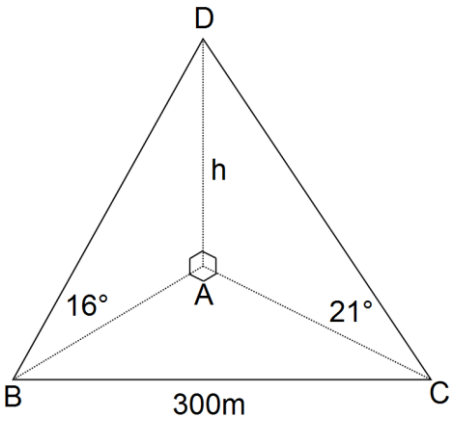
(a) (i) Graph $y=(3-x)(x-1)(x+2)$ showing the intercepts on both axes. 2

(ii) Solve $(3-x)(x-1)(x+2)\le 0$. 2

(b) Solve $\sin x-2\sin x\cos x+2\cos x=1$ for $0^\circ \le x\le 360^\circ$. 3

(c) Sketch a possible graph of $y=f(x)$ if $f(x)-f(-x)=0$ and the graph passes through the points (0,3) and (-2,0). 2

(d)



The above diagram shows a triangular pyramid with a right angled triangle as its base.

(i) Find an expression for AB in terms of h. 1

(ii) Find h correct to 2 decimal places. 2

(e) Prove that $\dfrac{1+\cot x}{cosecx}-\dfrac{\sin x}{\tan x\cot x}=\cos x$. 3

The End

Q1

a) $\frac{2}{5-\sqrt{3}} - \frac{4}{6-2\sqrt{3}} = a + b\sqrt{3}$

LHS: $\frac{2(5+\sqrt{3})}{25-3} - \frac{4(6+2\sqrt{3})}{36-4\times3}$

$\frac{10+2\sqrt{3}}{22} - \frac{(24+8\sqrt{3})}{24}$ ✓

$\frac{5+\sqrt{3}}{11} - \frac{(12+4\sqrt{3})}{12}$

$\frac{12(5+\sqrt{3}) - 11(12+4\sqrt{3})}{132}$

$\frac{60+12\sqrt{3}-132-44\sqrt{3}}{132}$

$\frac{-72-32\sqrt{3}}{132} = \frac{-72}{132} - \frac{32}{132}\sqrt{3}$

Thus $a = \frac{-72}{132}$ ✓, $b = \frac{-32}{132}$ ✓ $= \frac{-8}{33}$

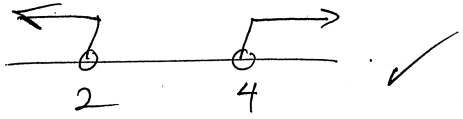
$a = -6/11$

A number of students could not rationalise the denominators correctly

b) $x^2 - 6x + 8 > 0$

$(x-4)(x-2) > 0$ ✓

$\begin{bmatrix} x < 2 \\ x > 4 \end{bmatrix}$ ✓



✓

Some students forgot to sketch the solutions on a number line as the question requested.

c) Factorise: $x^4 + x^2 - y^4 - y^2$

$= x^4 - y^4 + x^2 - y^2$

$= (x^2)^2 - (y^2)^2 + x^2 - y^2$

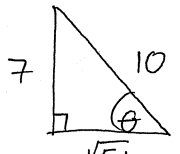
$= (x^2+y^2)(x^2-y^2) + (x^2-y^2)$ ✓

$= (x^2-y^2)(x^2+y^2+1)$

$= (x+y)(x-y)(x^2+y^2+1)$ ✓

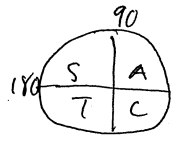
Some students did not factorise fully.

d) $\sin\theta = \frac{7}{10}$



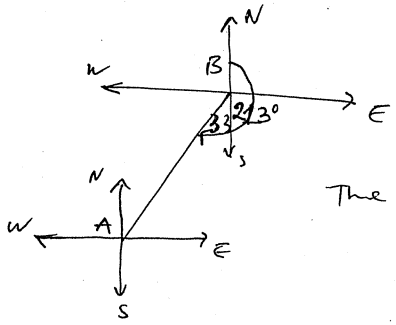
✓

$\tan\theta = -\frac{7}{\sqrt{51}}$ ✓



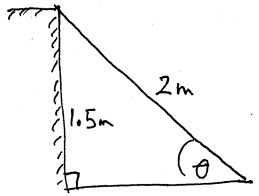
$\tan\theta < 0$ for $90° < \theta < 180°$

e)



The Bearing of B from A is 033° ✓

f)



$\sin\theta = \frac{1.5}{2}$ ✓ $\Rightarrow \theta = 49°$ ✓ (nearest degrees)

g) $ax^2 + 5x - 12 = 0$. If $x = \frac{3}{2}$

$a \times \frac{9}{4} + 5 \times \frac{3}{2} - 12 = 0$

$a = \frac{2}{3}$ ✓ OR ~~$\frac{x^2}{2} + 5x - 12 = 0$~~

$2x^2 + 5x - 12 = 0$

$(2x-3)(x+4) = 0$

$x = 3/2$ ✓

$x = -4$

Question 2

(a) $4^{3^2} = 4^9 = 262\ 144$

(a) was incorrectly evaluated by many students. A simple way to do this would have involved using your calculator.

(b) $(2x-3y)^4 = (2x)^4 - 4(2x)^3(3y)^1 + 6(2x)^2(3y)^2 - 4(2x)(3y)^3 + (3y)^4$

$= 16x^4 - 96x^3y + 216x^2y^2 - 216xy^3 + 81y^4$

This could have also been expanded using $(2x-3y)^2(2x-3y)^2 = (4x^2 - 12xy + 9y^2)(4x^2 - 12xy + 9y^2)$

$= \ldots\ldots$

Students need to be careful with simplifying terms and be careful with the + or - sign.

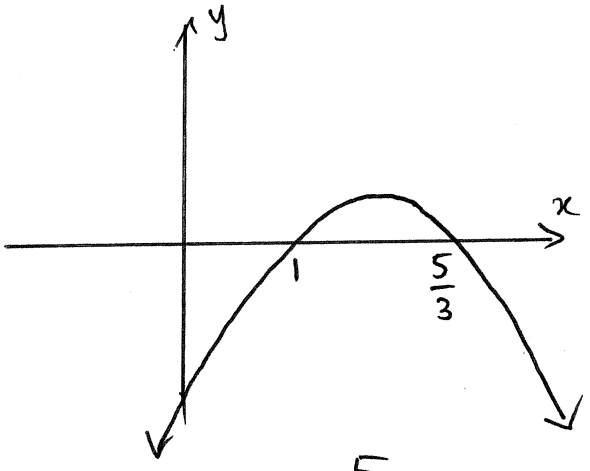
(c) $\frac{2}{x-1} \times (x-1)^2 \geq 3(x-1)^2$

$(x-1)[2 - 3(x-1)] \geq 0$

$(x-1)(2 - 3x + 3) \geq 0$

$(x-1)(5 - 3x) \geq 0$

$\therefore \quad 1 \leq x \leq \frac{5}{3}$

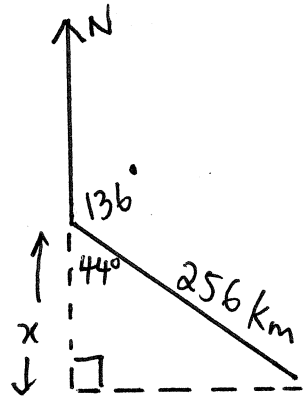


Since $x \neq 1$, the solution is $1 < x \leq \frac{5}{3}$.

Other methods were used to solve the inequation, including consideration of critical points. Common errors included excluding $x = 1$ from the solution set.

Question 2 (continued)

(d)



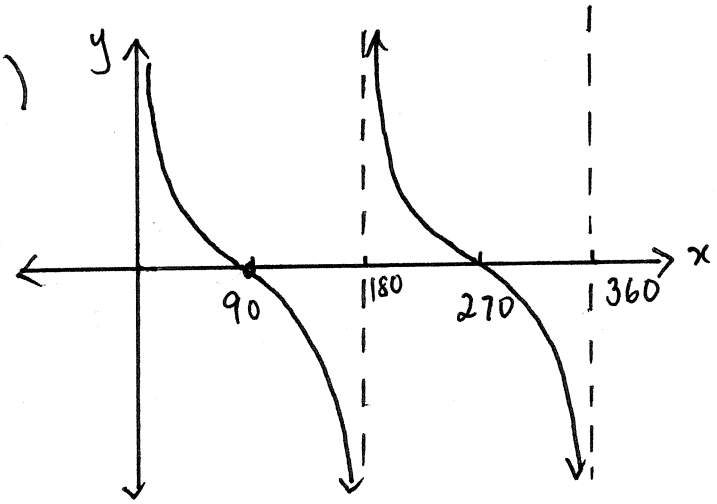
$\cos 44 = \frac{x}{256}$

$x = 256 \cos 44$

$\therefore x \doteq 184.2$ km

Well done by vast majority.

(e)



Many "attempts" were poorly done.
Asymptotes must be clearly marked.
The graph should be clearly shown to be approaching them.

(f) (i) $\sin 45 = \frac{1}{\sqrt{2}}$

(ii) $\cos 240 = \cos(180+60)$

$= -\cos 60$

$= -\frac{1}{2}$

S	A
T	C

(iii) $\text{cosec } 300 = \frac{1}{\sin 300}$

$= \frac{1}{-\sin 60}$

$= -\frac{2}{\sqrt{3}}$ since $\sin 60 = \frac{\sqrt{3}}{2}$

Mostly done well.
However, many students did not recognise the − sign in (ii) and (iii)

(g) (i) $\sec\theta \cot\theta = \frac{1}{\cos\theta} \times \frac{\cos\theta}{\sin\theta} = \frac{1}{\sin\theta}$ or $\text{cosec}\,\theta$

(ii) $1 + \sin(-\theta)\cos(90-\theta) = 1 - \sin\theta \times \sin\theta$

$= 1 - \sin^2\theta$

$= \cos^2\theta$

This was one of the weaker parts of this question. Some students need to ensure they read the solution.

3 (a) $\frac{x+1}{(x+2)(x+3)} + \frac{x-1}{(x+3)(x-3)} = 0$

$(x+1)(x-3) + (x-1)(x+2) = 0$

$x^2 - 2x - 3 + x^2 + x - 2 = 0$

$2x^2 - x - 5 = 0$

$x = \frac{1 \pm \sqrt{1^2 - 4 \times 2 \times -5}}{2 \times 2}$

$= \frac{1 \pm \sqrt{41}}{4}$

(b) (i) $y = \frac{x+1}{x+1} - \frac{5}{x+1}$

$= 1 - \frac{5}{x+1}$

$\therefore x = -1, y = 1$

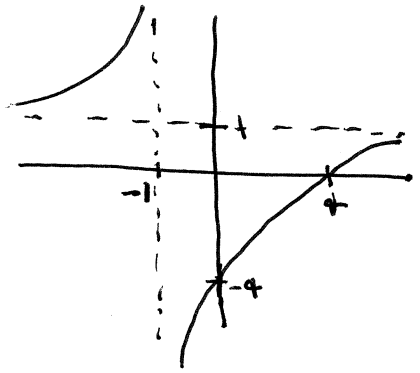
The equations of the asymptotes are not $x \neq -1$ and $y \neq 1$

(ii) $x = 0, y = \frac{-4}{1}$

$= -4$

$y = 0, x - 4 = 0$

$x = 4$



Students needed to show all intercepts on their graphs.

(c) $\cos \hat{B} = \frac{5^2 + 4^2 - 6^2}{2 \times 5 \times 4}$

$\hat{B} = 92° 1' 16.9811$

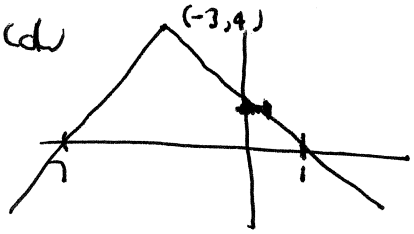
$A = \frac{1}{2} \times 5 \times 4 \times \sin(92° \ldots)$

$= 9.9215076$

$\doteq 9.9$

One should not round off in the middle of a calculation

(d)



Many students left out some of the intercepts

(e) domain $1 - x > 0$

$-x > -1$

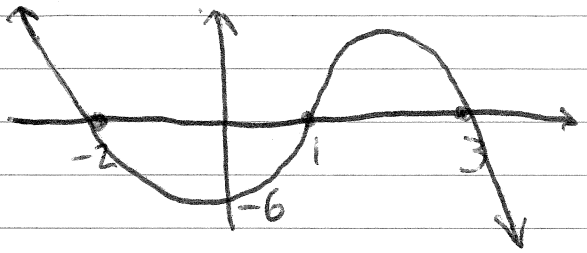
$x < 1$

range $y > 0$

2016 Ext 1 Yr 11 Task 1

Q4.

(a) (i)



When $x=0$

$y=(3)(-1)(2)$.

$=-6$.

(ii) $-2 \leq x \leq 1$, $x \neq 3$.

(b) $\sin x - 2\sin x \cos x + 2\cos x - 1 = 0$.

$-1(1-\sin x) + 2\cos x(1-\sin x) = 0$.

$(1-\sin x)(2\cos x - 1) = 0$.

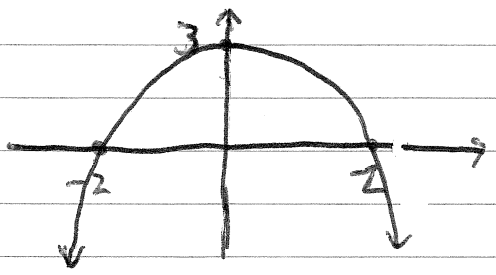
Factorising by pairs is an important technique

$\sin x = 1$ or $\cos x = \frac{1}{2}$.

$x = 90°$ $x = 60°, 300°$

$\therefore x = 60°, 90°, 300°$

(c) If $f(x) - f(-x) = 0$ then $f(x) = f(-x)$, that is $f(x)$ is even.



y-intercept 3.
Goes through $(-2,0)$ $(2,0)$ and symmetrical about y-axis.

Many students didn't recognise this as an even function.

(d)(i) $\tan 16° = \frac{h}{AB}$

$AB = \frac{h}{\tan 16°}$

(ii) $AC = \frac{h}{\tan 21°}$

$AC^2 + AB^2 = 300^2$

A common mistake was to square 300 and get 900.

$\frac{h^2}{\tan^2 21°} + \frac{h^2}{\tan^2 16°} = 300^2$

$h^2\left(\frac{1}{\tan^2 21°} + \frac{1}{\tan^2 16}\right) = 300^2$

$$h = \frac{300}{\sqrt{\frac{1}{\tan^2 21°} + \frac{1}{\tan^2 16°}}}$$

$\doteq 68.92\text{m}$

(e) $$LHS = \frac{1 + \frac{\cos x}{\sin x}}{\frac{1}{\sin x}} \times \frac{\sin x}{\sin x} - \frac{\sin x}{\frac{\sin x}{\cos x}} \times \frac{\cos x}{\sin x}$$

$$= \frac{\sin x + \cos x}{1} - \frac{\sin x}{1}$$

$= \cos x$

$= RHS.$

Some students confused $\sin x + \cos x$ with $\sin^2 x + \cos^2 x$