

Sydney Girls High School



YEAR 12

MATHEMATICS EXTENSION 1

HSC Assessment Task 3

2022

Reading time : 5 minutes

Writing time : 60 minutes

Total marks : 50

Topics: Mathematical Induction; Further Calculus; Motion, Forces and Projectiles

General Instructions:

- There are four (4) questions which are of approximately equal value.
- Attempt all questions.
- Show all necessary working. Marks may be deducted for badly arranged work or incomplete working.
- Start each question on a new page. Write on one side of the paper only.
- Diagrams are NOT to scale.
- Board-approved calculators may be used.
- Write using a black or blue pen.
- White-out or correction tape is NOT to be used.
- A correct answer without working may be awarded a maximum of 1 mark.
- A Mathematics reference sheet is provided.

Student Name:_____ **Teacher Name:**_____

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Question 1 (13 marks)

(a) Differentiate: $y = 4 \sin^{-1} x$ 1

(b) Find: $\int 2x \sec^2(x^2) dx$ 1

(c) Find: $\int \frac{dx}{16+9x^2}$ 2

(d) Use the substitution $u = x^2 + 1$ to evaluate: 3

$$\int_0^1 x^3(x^2 + 1)^4 dx$$

(e) A ball is projected horizontally from the top of a 15 m high cliff with a velocity of 20 m/s. Assume $g = 10 \text{ m/s}^2$.

(i) Prove the following expressions for x and y , in terms of t :

$$x = 20t \text{ and } y = -5t^2 + 15 \quad 2$$

Hence, find:

(ii) the time taken to reach the water. (Answer to 2 decimal places). 1

(iii) the horizontal distance travelled. (Answer to 1 decimal place). 1

(iv) the acute angle the ball makes with the horizontal when it hits the water. (Answer to the nearest degree). 2

Question 2 (12 marks)

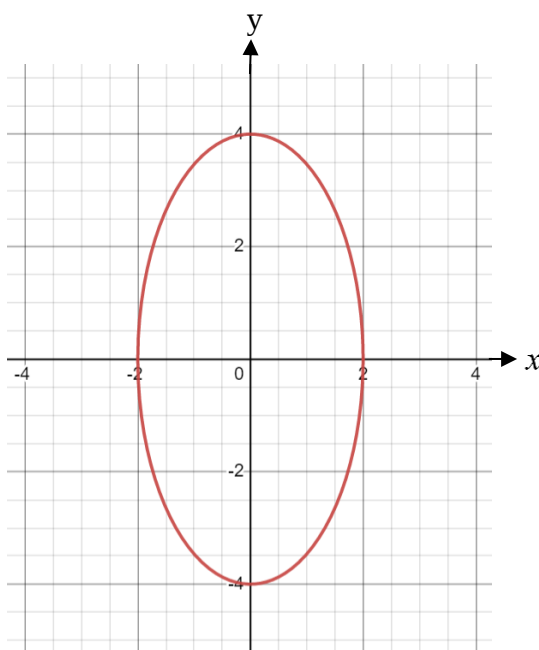
- (a) Find the derivative of: $y = \cos^{-1} \sqrt{1-x}$.

Express your answer without negative indices.

2

- (b) The diagram below shows an ellipse which has the equation $4x^2 + y^2 = 16$. Find the exact volume of the solid formed when the area inside the ellipse is rotated about the x -axis.

3



- (c) Prove by mathematical induction that for all integers $n \geq 1$:

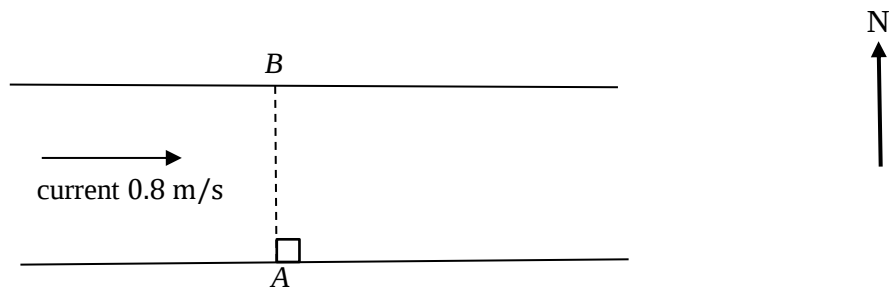
$9^n + 3$ is divisible by 6

3

Question 2 continued on next page

Question 2 continued

- (d) In still water, Bella can swim at 3 m/s . Bella is at point A on the edge of a canal, and considers point B directly opposite. A current is flowing from the left at a constant speed of 0.8 m/s .



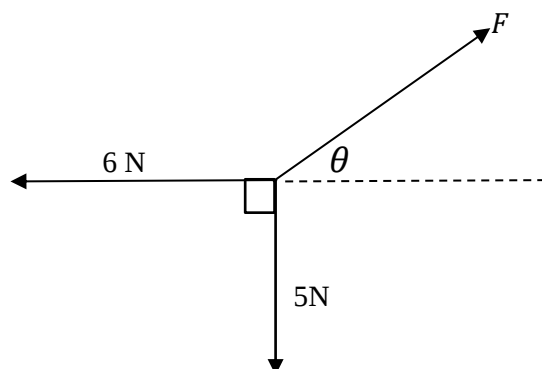
- (i) If Bella dives in straight towards B , and swims without allowing for the current, what will her actual speed and direction be? Answer to 2 decimal places. 2
- (ii) Bella wants to swim directly across the canal to point B . At what angle should Bella aim to swim in order that the current will correct her direction and what will Bella's actual speed be? 2

Question 3 (12 marks)

- (a) Find the equation of the tangent to the curve $y = e^{\tan^{-1} x}$ at the point where the curve cuts the y -axis.

3

- (b) The diagram shows three forces which act in the same plane and are in equilibrium.



- (i) Find the magnitude of the force F . 1
- (ii) Find the value of θ , correct to the nearest degree. 2

- (c) Prove by mathematical induction that for all positive integer values of n :

$$\frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \frac{1}{5 \times 7} + \dots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$$

3

- (d) The parabola $y = 4x - x^2$ and the line $y = 2x$ intersect at the points $(0,0)$ and $(2,4)$. The region between the curves is rotated about the y -axis to form a solid of revolution. Calculate the exact volume of this solid.

3

Question 4 (13 marks)

- (a) Find: 2

$$\int \cos 3x (\tan 3x - \sin 3x) dx$$

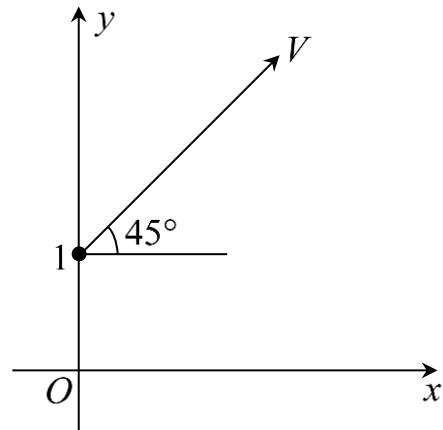
- (b) Using the substitution $x = 2 \sin \theta - 1$, find: 3

$$\int \sqrt{1-x} \sqrt{x+3} dx$$

- (c) A volleyball player serves a ball with initial speed V metres per second at an angle of projection of 45° to the horizontal.

The ball is 1 metre above the ground at the moment of projection.

For the following, assume that $g = 10 \text{ m/s}^2$.



- (i) State in component form $\underline{a}(t)$, $\underline{v}(0)$ and $\underline{r}(0)$. 1

- (ii) Hence, show the vector displacement of the ball is given by: 2

$$\underline{r}(t) = \left(\frac{Vt}{\sqrt{2}} \right) \underline{i} + \left(\frac{Vt}{\sqrt{2}} - 5t^2 + 1 \right) \underline{j}$$

- (iii) At the moment of projection, the horizontal distance of the ball from the net is 9 metres. The ball just clears the net, which is 2 metres high. Calculate the amount of time it takes for the volleyball to reach the net. Give your answer in seconds, correct to 2 decimal places. 2

- (iv) Determine the time the volleyball takes to travel from its maximum height to the ground. Give your answer in seconds, correct to 2 decimal places. 3

End of paper

2022 Yr 12 Ext. 1 Assessment Task 3

Question 1

(a) $y = 4 \sin^{-1} x$

$$\frac{dy}{dx} = \frac{4}{\sqrt{1-x^2}}$$

1

(b) $\int 2x \sec^2(x^2) dx$

$$= \tan(x^2) + C$$

1

(c) $\int \frac{1}{4^2 + (3x)^2} dx$

$$= \frac{1}{3} \int \frac{3}{4^2 + (3x)^2} dx$$

$$= \frac{1}{3} \times \frac{1}{4} \tan^{-1} \frac{3x}{4} + C$$

$$= \frac{1}{12} \tan^{-1} \frac{3x}{4} + C$$

1/2

Comments:

* General comment
→ write large and clearly enough for solutions to be clear

* Most common mistakes was in part (c).

* For (c)(i) - Proving the expressions: need to show how the expressions were derived.

(d) $u = x^2 + 1 \quad x=1$

$$\frac{du}{dx} = 2x \quad u=2$$

$$du = 2x dx \quad x=0 \quad u=1$$

$$\int_0^1 x^3 (x^2 + 1)^4 dx$$

$$= \frac{1}{2} \int 2x \cdot x^2 (u)^4 dx$$

$$= \frac{1}{2} \int_1^2 (u-1) u^4 du$$

$$= \frac{1}{2} \int_1^2 (u^5 - u^4) du$$

$$= \frac{1}{2} \left[\frac{u^6}{6} - \frac{u^5}{5} \right]_1^2$$

$$= \frac{1}{2} \left(\left(\frac{32}{3} - \frac{32}{5} \right) - \left(\frac{1}{6} - \frac{1}{5} \right) \right)$$

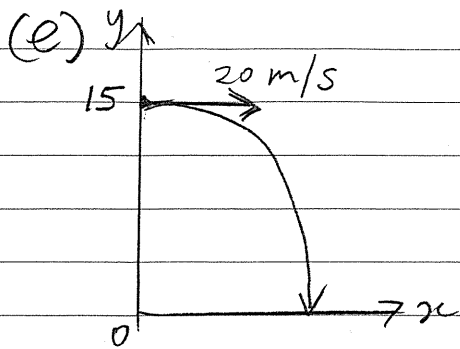
$$= \frac{1}{2} \left(\frac{64}{15} + \frac{1}{30} \right)$$

$$= \frac{1}{2} \times \frac{43}{10}$$

$$= \frac{43}{20}$$

1/3

Q1 cont.



$$\begin{aligned} \text{(i)} \quad \ddot{x} &= 0 & \ddot{y} &= -10 \\ \dot{x} &= C_1 & \dot{y} &= -10t + C_2 \\ \text{When } t=0, \dot{x} &= 20, \dot{y} &= 0 \\ 20 &= C_1 & 0 &= C_2 \\ \dot{x} &= 20 & \dot{y} &= -10t \\ x &= 20t + C_3 & y &= -5t^2 + C_4 \end{aligned}$$

$$\begin{aligned} \text{When } t=0, x &= 0, y = 15 \\ 0 &= C_3 & 15 &= C_4 \\ x &= 20t & y &= -5t^2 + 15 \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad t &=? \text{ when } y=0 \\ 0 &= -5t^2 + 15 \end{aligned}$$

$$5t^2 = 15$$

$$t^2 = 3$$

$$t = \sqrt{3} \quad (t \geq 0)$$

$$t \doteq 1.73 \quad (2 \text{ d.p.})$$

$\therefore 1.73$ secs to reach water

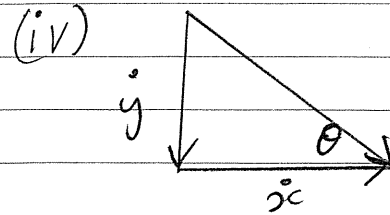
$$\text{(iii)} \quad x = ? \text{ when } t = \sqrt{3}$$

$$x = 20 \times \sqrt{3}$$

$$= 20\sqrt{3}$$

$$\doteq 34.6 \quad (1 \text{ d.p.})$$

$$\therefore \text{Horiz. dist.} = 34.6 \text{ m}$$



$$\text{When } t = \sqrt{3},$$

$$\dot{x} = 20 \quad \dot{y} = -10\sqrt{3}$$

$$\tan \theta = \frac{10\sqrt{3}}{20}$$

$$= \frac{\sqrt{3}}{2}$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\doteq 40^\circ 54'$$

$\therefore$ Ball hits water at an angle of 41° to the horizontal.

Question 2 (12 marks)

- (a) Find the derivative of: $y = \cos^{-1} \sqrt{1-x}$.
Express your answer without negative indices.

2

Reference sheet:

$$\text{If } y = \cos^{-1} f(x) \text{ then } \frac{dy}{dx} = \frac{-f'(x)}{\sqrt{1-[f(x)]^2}}$$

$$y = \cos^{-1} \sqrt{1-x}$$

$$\frac{dy}{dx} = \frac{-f'(x)}{\sqrt{1-[f(x)]^2}}$$

$$= \frac{-\left(\frac{-1}{2\sqrt{1-x}}\right)}{\sqrt{1-(\sqrt{1-x})^2}}$$

$$= -\left(\frac{-1}{2\sqrt{1-x}}\right) \times \frac{1}{\sqrt{1-(1-x)}}$$

$$= \frac{1}{2\sqrt{1-x}} \times \frac{1}{\sqrt{x}} \quad \checkmark$$

$$= \frac{1}{2\sqrt{x(1-x)}}$$

$$\therefore \frac{dy}{dx} = \frac{1}{2\sqrt{x-x^2}}$$

$$f(x) = (1-x)^{1/2}$$

$$f'(x) = \frac{1}{2} (1-x)^{-\frac{1}{2}} \times (-1)$$

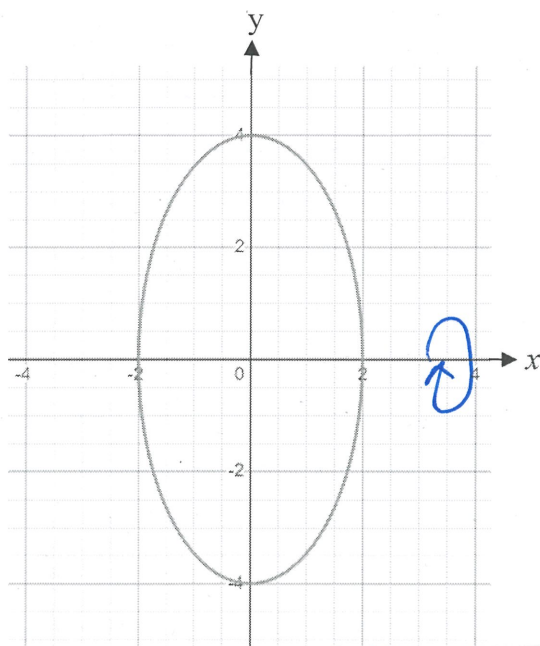
$$\therefore f'(x) = \frac{-1}{2\sqrt{1-x}} \quad \checkmark$$

* Most students found $f'(x)$ correctly but when they substituted it into $\frac{dy}{dx}$ the negative sign was missed resulting in $\frac{1}{2}$.

* Students should refer to NESA reference sheet.

- (b) The diagram below shows an ellipse which has the equation $4x^2 + y^2 = 16$. Find the exact volume of the solid formed when the area inside the ellipse is rotated about the x -axis.

3



$$4x^2 + y^2 = 16$$

$$y^2 = 16 - 4x^2$$

$$V = \pi \int y^2 dx$$

$$= \pi \int_{-2}^2 (16 - 4x^2) dx \quad \checkmark \text{ } y^2 \text{ substitution}$$

$$= 2\pi \int_0^2 (16 - 4x^2) dx$$

$$= 2\pi \left[16x - \frac{4x^3}{3} \right]_0^2 \quad \checkmark \text{ integration/limits}$$

$$= 2\pi \left[\left(16(2) - \frac{4(2)^3}{3} \right) - (0) \right]$$

$$\therefore V = \underline{\underline{\frac{128\pi}{3} \text{ units}^3}} \quad \checkmark \text{ correct answer}$$

* Most students did well in this question.

There was a little confusion about substituting " y^2 " straight after rearranging the formula for some students.

(c) Prove by mathematical induction that for all integers $n \geq 1$:

$9^n + 3$ is divisible by 6

3

Let $S(n) = 9^n + 3$, be divisible by 6, $n \geq 1$.

For $n=1$: $S(1) = 9^1 + 3$

$$= 12$$

$$= 6 \times 2 \text{ (divisible by 6)}$$

$\therefore S(n)$ is true for $n=1$. ✓

Assume $S(n)$ is true for $n=k$, integer k , $k \geq 1$.

That is: $\frac{S(k)}{6} = M$ (integer M)

$$S(k) = 6M$$

$$\Rightarrow 9^k + 3 = 6M$$

$$9^k = 6M - 3$$

Prove that $S(k+1)$ is true. ✓

That is show $S(k+1) = 9^{k+1} + 3$, is divisible by 6.

Proof: $9^{k+1} + 3$

$$= 9 \cdot 9^k + 3$$

$$= 9 \cdot (6M - 3) + 3 \text{ (by assumption)}$$

$$= 54M - 27 + 3$$

$$= 54M - 24$$

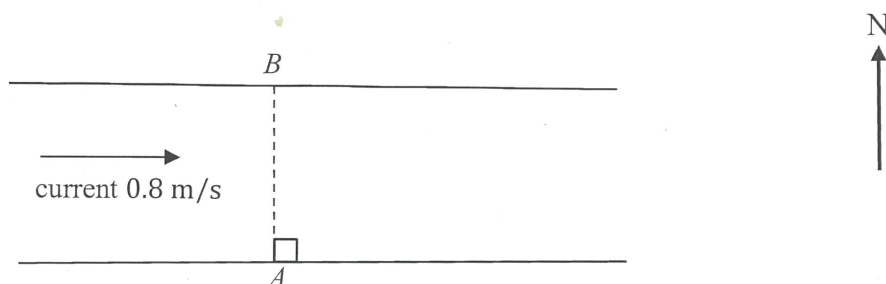
$$= 6(9M - 4), \text{ which is divisible by 6.}$$

$\therefore S(k+1)$ is true when it is true for $S(k)$ and $S(1)$ is true.

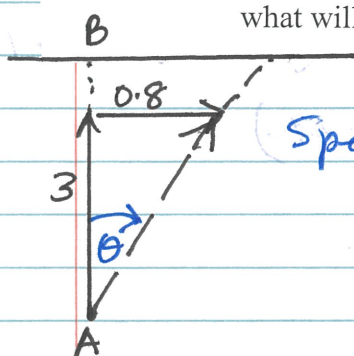
Hence, proven true by the principle of mathematical induction for all integers $n \geq 1$. ✓

* The majority of students completed the induction proof successfully. ✓

- (d) In still water, Bella can swim at 3 m/s. Bella is at point A on the edge of a canal, and considers point B directly opposite. A current is flowing from the left at a constant speed of 0.8 m/s.



- (i) If Bella dives in straight towards B, and swims without allowing for the current, what will her actual speed and direction be? Answer to 2 decimal places. 2



$$\begin{aligned} \text{Speed} &= \sqrt{3^2 + 0.8^2} \\ &= \sqrt{9.64} \\ &\doteq 3.10 \text{ m/s} \end{aligned}$$

$$\tan \theta = \frac{0.8}{3}$$

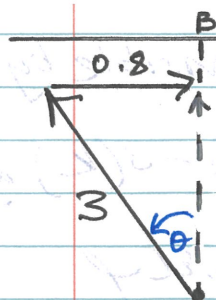
$$\theta = \tan^{-1}\left(\frac{0.8}{3}\right)$$

$$\theta \doteq 14^{\circ}56' \text{ (or } 14.93^{\circ}\text{)}$$

$\therefore$ Bella swims at 3.10 m/s in the direction $N 14^{\circ}56' E$ or $014^{\circ}56' T$

* Many students did not give a 3-digit bearing for direction and were not penalised for their answer. This should be noted in future.

- (ii) Bella wants to swim directly across the canal to point B. At what angle should Bella aim to swim in order that the current will correct her direction and what will Bella's actual speed be? 2



$$\begin{aligned} \text{Speed} &= \sqrt{3^2 - 0.8^2} \\ &= \sqrt{8.36} \\ &\doteq 2.89 \text{ m/s} \end{aligned}$$

$$\sin \theta = \frac{0.8}{3}$$

$$\theta = \sin^{-1}\left(\frac{0.8}{3}\right)$$

$$\theta = 15^{\circ}28' \text{ (or } 15.47^{\circ}\text{)}$$

$\therefore$ Bella's actual speed is 2.89 m/s in the direction of $N 15^{\circ}28' W$ or $344^{\circ}32' T$ or $344.5^{\circ} T$

* many students did not label their diagram clearly and so did not use the correct trig. ratio to find direction of the swim. This question was nearly identical to the SGHS booklet!

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3a)

$$y' = \frac{1}{1+x^2} e^{\tan^{-1}x}$$

at $x=0$ $y=1$

$$m = \frac{1}{1+0} e^0 = 1$$

$$y-1 = 1(x-0)$$

$$y = x+1$$

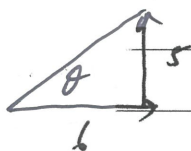
* students differentiated this function well however some d.dn't substitute the x -value to get the gradient $\therefore$ they lost mark.

b) i) $\underline{F} = -5\underline{j} + 6\underline{i}$

$$|F| = \sqrt{25+36} = \sqrt{61}$$

* this question was done well

ii)



$$\tan \theta = \frac{5}{6}$$

$$\theta = 40^\circ$$

c) Prove true for $n=1$

LHS $\frac{1}{3}$

$$RHS = \frac{1}{2(1)+1}$$

$$= \frac{1}{3}$$

$\therefore$ true for $n=1$

Assume true for $n=k$

$$\frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \frac{1}{5 \times 7} + \dots + \frac{1}{(2k-1)(2k+1)} = \frac{k}{2k+1}$$

Prove true for $n=k+1$

$$\begin{aligned} \text{LHS} &= \frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \dots + \frac{1}{(2(k+1)-1)(2(k+1)+1)} & \text{RHS} &= \frac{k+1}{2(k+1)+1} \\ &= \frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \dots + \frac{1}{(2k-1)(2k+1)} + \frac{1}{(2k+1)(2k+3)} & &= \frac{k+1}{2k+3} \end{aligned}$$

$$= \frac{k}{2k+1} + \frac{1}{(2k+1)(2k+3)}$$

$$= \frac{k(2k+3) + 1}{(2k+1)(2k+3)}$$

$$= \frac{2k^2 + 3k + 1}{(2k+1)(2k+3)}$$

$$= \frac{2k^2 + 2k + k + 1}{(2k+1)(2k+3)}$$

$$= \frac{2k(k+1) + (k+1)}{(2k+1)(2k+3)}$$

$$= \frac{\cancel{(2k+1)}(k+1)}{\cancel{(2k+1)}(2k+3)}$$

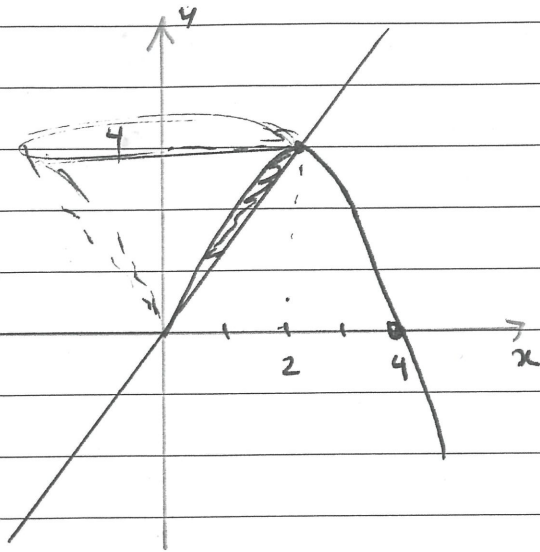
$$= \frac{k+1}{2k+3}$$

$\therefore$ RHS

This question was done well

$\therefore$ by mathematical induction true for all n .

d)



$$y = 2x \rightarrow x = \frac{y}{2}$$

$$y = x^2 - 4x$$

$$4 - y = x^2 - 4x + 4$$

$$(x - 2)^2 = 4 - y$$

$$x - 2 = \pm \sqrt{4 - y}$$

$$x = 2 \pm \sqrt{4 - y}$$

$$\therefore x = 2 - \sqrt{4 - y}$$

$$V = \pi \int_0^4 \left(\left(\frac{y}{2} \right)^2 - (2 - \sqrt{4 - y})^2 \right) dy$$

$$= \pi \int_0^4 \left(\frac{y^2}{4} - (4 - 4\sqrt{4 - y} + 4 - y) \right) dy$$

$$= \pi \int_0^4 \left(\frac{y^2}{4} - 8 + 4\sqrt{4 - y} + y \right) dy$$

$$= \pi \left[\frac{y^3}{12} - 8y + \frac{4(4 - y)^{\frac{3}{2}}}{\frac{3}{2} \times -1} + \frac{y^2}{2} \right]_0^4$$

$$= \pi \left[\frac{64}{12} - 32 + \frac{16}{2} - \left(-\frac{8}{3} (4)^{\frac{3}{2}} \right) \right]$$

$$= \pi \left[\frac{64}{12} - 32 + 8 + \frac{64}{3} \right]$$

$$= \frac{8\pi}{3} u^3$$

* This question was done poorly as many students didn't complete the square to make x the subject, the volume was around the y -axis.

Question 4

(a) Find:

2

$$\int \cos 3x (\tan 3x - \sin 3x) dx$$

$$I = \int \cos 3x \left(\frac{\sin 3x}{\cos 3x} - \sin 3x \right) dx$$

$$= \int (\sin 3x - \sin 3x \cos 3x) dx$$

$$\sin 6x = 2 \sin 3x \cos 3x$$

$$= \int \left(\underbrace{\sin 3x}_{\text{OR}} - \frac{1}{2} \sin 6x \right) dx \quad (1)$$

$$\therefore I = \underbrace{-\frac{1}{3} \cos 3x + \frac{1}{12} \cos 6x + C}_{(1)}$$

Generally well done but still many students didn't use $\tan 3x$ as $\frac{\sin 3x}{\cos 3x}$ or didn't recognise the opportunity to use the double angle formula to replace $\sin 3x \cos 3x$ with $\frac{1}{2} \sin 6x$.

Question 4 (continued)

(b) Using the substitution $x = 2 \sin \theta - 1$, find:

3

$$\int \sqrt{1-x} \sqrt{x+3} dx$$

$$x = 2 \sin \theta - 1$$

$$dx = 2 \cos \theta d\theta$$

$$I = \int \sqrt{1 - 2 \sin \theta + 1} \sqrt{2 \sin \theta - 1 + 3} \times 2 \cos \theta d\theta \quad (1)$$

$$= \int \sqrt{2 - 2 \sin \theta} \sqrt{2 + 2 \sin \theta} \times 2 \cos \theta d\theta$$

$$= 2 \int \sqrt{4 - 4 \sin^2 \theta} \cos \theta d\theta$$

$$= 2 \int 2 \sqrt{1 - \sin^2 \theta} \cos \theta d\theta$$

$$= 2 \int 2 \cos^2 \theta d\theta$$

$$= 2 \int (\cos 2\theta + 1) d\theta$$

$$= 2 \left(\frac{\sin 2\theta}{2} + \theta \right) + C \quad (1)$$

$$= 2 \sin \theta \cos \theta + 2\theta + C$$

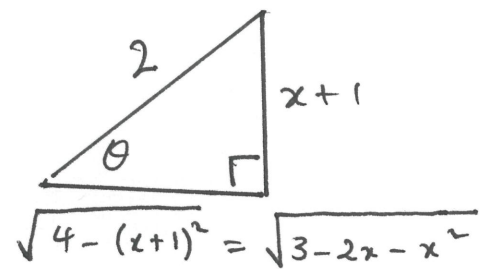
$$= 2x \frac{x+1}{2} \times \frac{\sqrt{3-2x-x^2}}{2} + 2 \sin^{-1} \left(\frac{x+1}{2} \right) + C$$

$$\therefore I = \frac{(x+1) \sqrt{3-2x-x^2}}{2} + 2 \sin^{-1} \left(\frac{x+1}{2} \right) + C \quad (1)$$

Key Concerns:

- Students mixing x and θ in the integration.
- Students not returning to the original variable x .
- Careless errors, especially with misplaced - signs.

$$\sin \theta = \frac{x+1}{2}$$

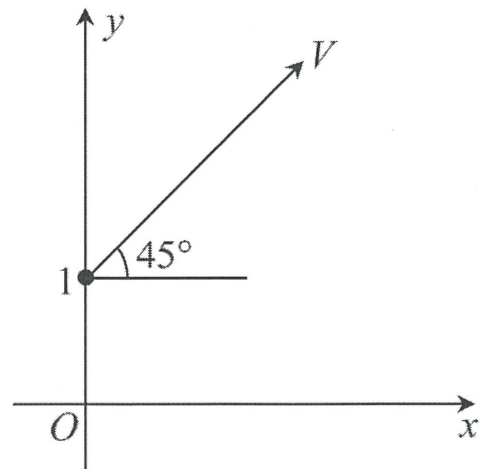


Question 4 (continued)

- (c) A volleyball player serves a ball with initial speed V metres per second at an angle of projection of 45° to the horizontal.

The ball is 1 metre above the ground at the moment of projection.

For the following, assume that $g = 10 \text{ m/s}^2$.



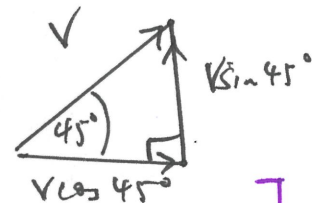
- (i) State in component form $\underline{a}(t)$, $\underline{v}(0)$ and $\underline{r}(0)$.

1

- (ii) Hence, show the vector displacement of the ball is given by:

2

$$\underline{r}(t) = \left(\frac{Vt}{\sqrt{2}} \right) \underline{i} + \left(\frac{Vt}{\sqrt{2}} - 5t^2 + 1 \right) \underline{j}$$



$$(i) \quad \underline{a}(t) = 0 \underline{i} - 10 \underline{j} \quad \underline{v}(0) =$$

Key concern: Students not answering the question and not recognising $\underline{v}(0)$ means

when $t=0$

$$\underline{r}(0) = 0 \underline{i} + \underline{j}$$

$$\frac{V}{\sqrt{2}} \underline{i} + \frac{V}{\sqrt{2}} \underline{j}$$

$$(ii) \quad \underline{a}(t) = 0 \underline{i} - 10 \underline{j}$$

$$\underline{v}(t) = C_1 \underline{i} + (C_2 - 10t) \underline{j}$$

Since $\underline{v}(0) = \frac{V}{\sqrt{2}} \underline{i} + \frac{V}{\sqrt{2}} \underline{j}$ $C_1 = \frac{V}{\sqrt{2}}$, $C_2 = \frac{V}{\sqrt{2}}$

$$\underline{v}(t) = \frac{V}{\sqrt{2}} \underline{i} + \left(\frac{V}{\sqrt{2}} - 10t \right) \underline{j}$$

$$\underline{r}(t) = \left(\frac{Vt}{\sqrt{2}} + C_3 \right) \underline{i} + \left(\frac{Vt}{\sqrt{2}} - 5t^2 + C_4 \right) \underline{j}$$

Since $\underline{r}(0) = 0 \underline{i} + \underline{j}$ $C_3 = 0$, $C_4 = 1$

$$\underline{r}(t) = \left(\frac{Vt}{\sqrt{2}} \right) \underline{i} + \left(\frac{Vt}{\sqrt{2}} - 5t^2 + 1 \right) \underline{j}$$

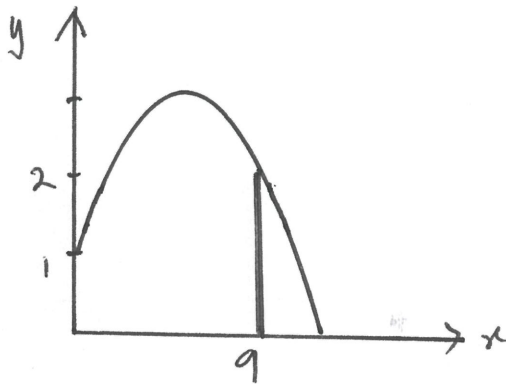
Key concerns:

- Many students did not provide a complete solution and had half of their solution in (i).
- Students should be showing the constants and how their values have been determined.
- Students should be working in component form.

Question 4 (c) (continued)

- (iii) At the moment of projection, the horizontal distance of the ball from the net is 9 metres. The ball just clears the net, which is 2 metres high. Calculate the amount of time it takes for the volleyball to reach the net. Give your answer in seconds, correct to 2 decimal places. 2
- (iv) Determine the time the volleyball takes to travel from its maximum height to the ground. Give your answer in seconds, correct to 2 decimal places. 3

(iii)



Let $t = T$ for time to reach the net.

Notes:

- Well attempted by some students but many didn't make much progress.

- Many solutions were less efficient than the one shown.

$$\therefore 9 - 5T^2 = 1$$

$$5T^2 = 8$$

$$T = \sqrt{\frac{8}{5}} \quad (T > 0) \quad \therefore \text{Approx. 1.26 seconds.}$$

$$\left. \begin{array}{l} \frac{VT}{\sqrt{2}} = 9 \quad (1) \\ \frac{VT}{\sqrt{2}} - 5T^2 + 1 = 2 \quad (2) \end{array} \right\} \text{for both}$$

(iv)

From (1)

$$V = \frac{9\sqrt{2}}{T} = 9\sqrt{2} \times \sqrt{\frac{5}{8}}$$

$$V = \frac{9\sqrt{5}}{2} \Rightarrow \boxed{A} \doteq 10.06$$

Max. height when $\dot{y} = 0$, i.e. $\frac{V}{\sqrt{2}} - 10t = 0$

$$t = \frac{9\sqrt{5}}{20\sqrt{2}} \quad (1)$$

Time to hit ground when $y=0$, i.e. $\frac{Vt}{\sqrt{2}} - 5t^2 + 1 = 0$

$$5t^2 - \frac{Vt}{\sqrt{2}} - 1 = 0$$

$$t = \frac{-B \pm \sqrt{B^2 + 20}}{2 \times 5} \doteq 1.55 \quad (1)$$

$\boxed{B}$

PTO

Q 4 (c) (iv) continued

$$\text{Time difference} = 1.55 - \frac{9\sqrt{5}}{20\sqrt{2}}$$

$$\approx \underline{0.84 \text{ seconds}} \quad (1)$$

Notes:

- Quality of responses varied greatly.
- A number of students mistakenly assumed that the net was the highest point on the volleyball's path.
- Students should be careful of solving simultaneously with t when the value of t in one equation is different to the value of t in another equation.