Sydney Girls High School

YEAR 12 MATHEMATICS EXTENSION 1

2024 HSC Task 3

Reading time: 5 minutes

Writing time: 75 minutes

Total marks: 55

Section I – 5 marks

- Attempt Questions 1-5
- Allow about 5-10 minutes for this section

Section II – 50 marks

- Attempt Question 6-9
- Allow about 65-70 minutes for this section

Topics: Mathematical Induction, Further Calculus and Motion, Forces and Projectiles

General Instructions:

- There are nine (9) questions which are not all of equal value.
- Attempt all questions.
- Show all necessary working. Marks may be deducted for badly arranged work or incomplete working.
- Diagrams are NOT to scale, unless indicated otherwise.
- NESA approved calculators may be used.
- Write using a black or blue pen.
- White-out or correction tape is NOT to be used.
- A correct answer without working will be awarded a maximum of 1 mark.

Student Name:________________________ **Teacher:**____________________________

Section I

5 marks

Attempt Questions 1-5

Allow about 5-10 minutes for this section

Use the multiple-choice answer sheet for Questions 1-5.

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1) Which of the following is an expression for $\int \frac{x}{\sqrt{x^2-2}}\,dx$.

Use the substitution $u = x^2 - 2$.

(A) $\sqrt{x^2-2}+C$

(B) $\left(\sqrt{x^2-2}\right)^3+C$

(C) $2\sqrt{x^2-2}+C$

(D) $2\left(\sqrt{x^2-2}\right)^3+C$

2) Which of the following represents $f\,'(x)$ if $f(x)=\cos^{-1}3x + x\cos^{-1}3x$?

(A) $\cos^{-1}3x-\frac{x+1}{\sqrt{9-x^2}}$

(B) $\cos^{-1}3x-\frac{3(x+1)}{\sqrt{1-9x^2}}$

(C) $\cos^{-1}3x-\frac{3(x-1)}{\sqrt{1-9x^2}}$

(D) $\cos^{-1}3x-\frac{x-1}{\sqrt{9-x^2}}$

3) A duck waddles 2.5 m east and 6m north. What are the magnitude and direction of the duck's displacement with respect to its original position?

(A) 3.5 m at 19° north of east
(B) 6.3 m at 67° north of east
(C) 6.5 m at 72° north of east
(D) 6.5 m at 67° north of east

4) Which expression is equal to $\int \sin^2 3x\,dx$?

(A) $\frac{1}{2}\left(x-\frac{1}{3}\sin 3x\right)+C$
(B) $\frac{1}{2}\left(x+\frac{1}{3}\sin 3x\right)+C$
(C) $\frac{1}{2}\left(x+\frac{1}{6}\sin 6x\right)+C$
(D) $\frac{1}{2}\left(x-\frac{1}{6}\sin 6x\right)+C$

5) Which of the following expressions is equivalent to $\int \frac{1}{\sqrt{1-4x^2}}\,dx$?

(A) $\sin^{-1}\left(\frac{x}{2}\right)+C$

(B) $\sin^{-1}(2x)+C$

(C) $\frac{1}{2}\sin^{-1}(2x)+C$

(D) $\frac{1}{2}\sin^{-1}\left(\frac{x}{2}\right)+C$

Section II

50 marks

Attempt Questions 6-9

Allow about 65-70 minutes for this section

Answer each question on the writing paper supplied. Start each question on a NEW page. Extra writing paper are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (12 Marks)

a) Use the substitution $u = e^x$ to find: $\int \frac{dx}{e^x + 9e^{-x}}$. (2)

b) Find the area between the curve $y = \cos^2 2x$ and the x-axis from $x = 0$ and $x = \frac{\pi}{6}$. (2)

c) A weather station releases a balloon that rises at a constant 15m/s relative to the air, but there is a wind blowing at 6.5m/s towards the west. (2)

What are the magnitude and direction of the velocity of the balloon?

d) Use the substitution $u = \tan^{-1} x$ to evaluate the following. Leave your answer in exact form. (3)

$$\int_0^1 \frac{\tan^{-1} x}{1+x^2} dx$$

e) Use mathematical induction to show that for all positive integers $n \geq 2$: (3)

$$2\times1 + 3\times2 + 4\times3 + \cdots + n(n-1) = \frac{n(n^2-1)}{3}.$$

Question 7 (13 Marks)

a) i) Find: $\dfrac{d}{dx}\cos^{-1}\left(\dfrac{x-10}{10}\right)$. (2)

ii) Hence, evaluate: $\displaystyle\int_5^{10}\frac{1}{\sqrt{20x-x^2}}dx$. (2)

b) Evaluate $\displaystyle\int_0^1 \frac{x\,dx}{\sqrt{1+x}}$ using the substitution $u^2 = 1+x$. (2)

c) Use mathematical induction to prove that for all positive integers n:
(3)

$$1\times1!+2\times2!+3\times3!+\cdots+n\times n! = (n+1)!-1$$

d) Ella was playing fetch with her dog at the park. She threw the tennis ball with a speed of 15m/s and initial angle of $45°$ from the horizontal.
Assume that $g = 10\text{ms}^{-2}$.

i) Find the velocity vector for the tennis ball. (1)

ii) Find the position vector for the tennis ball. (1)

iii) Determine the maximum height reached by the tennis ball. (1)

iv) How far did the ball land from Ella's position? (1)

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Question 8 (12 Marks)

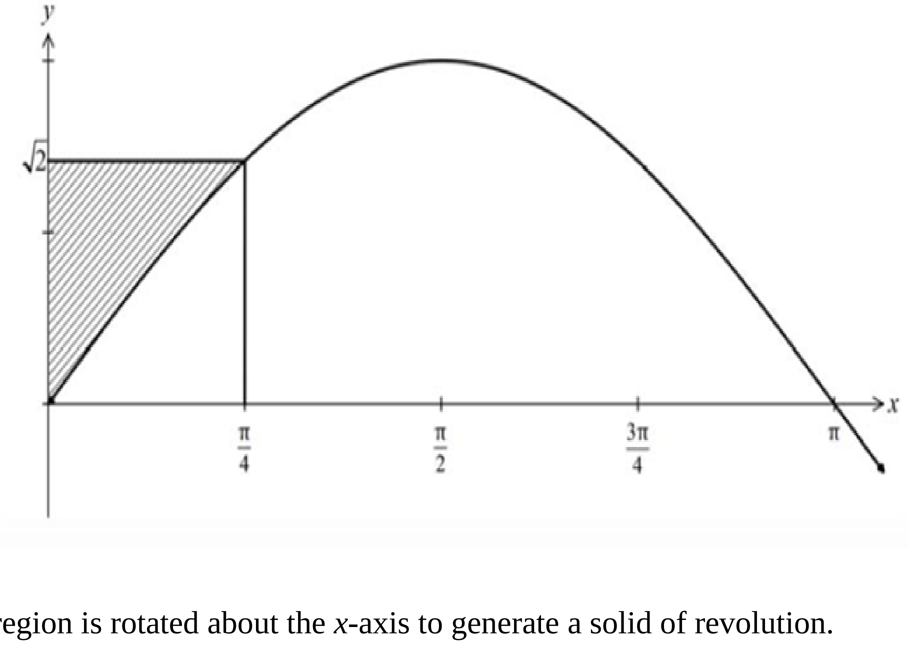
a) Differentiate : $\dfrac{\cos^{-1} x}{x}$. (2)

b) Prove by mathematical induction that $2^n + (-1)^{n+1}$ is divisible by 3 for all integers $n \geq 1$. (3)

c) i) Find $\dfrac{d}{dx}\left(x\cos^{-1} x\right)$. (2)

ii) Hence or otherwise find $\int \cos^{-1} x\, dx$? (2)

d) The diagram below shows the shaded region bounded by the curve $y = \sin 2x$ and the y-axis for $0 \leq y \leq \sqrt{2}$.



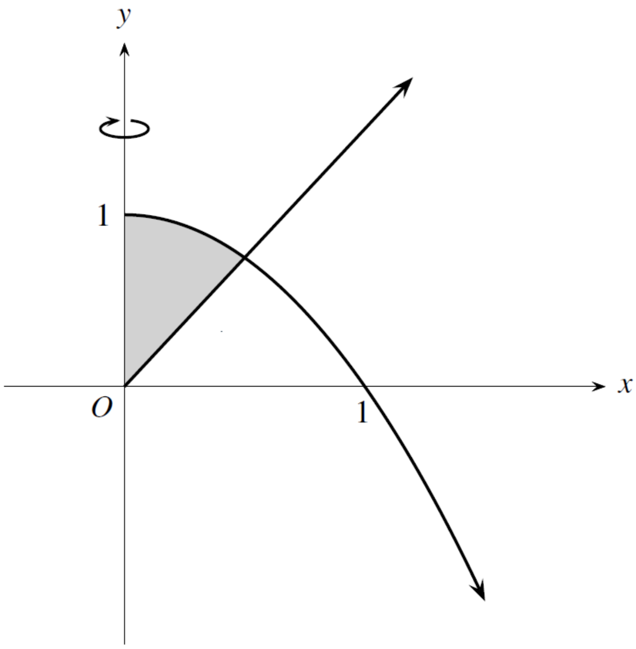
The region is rotated about the x-axis to generate a solid of revolution. (3)

Find the exact volume of the solid.

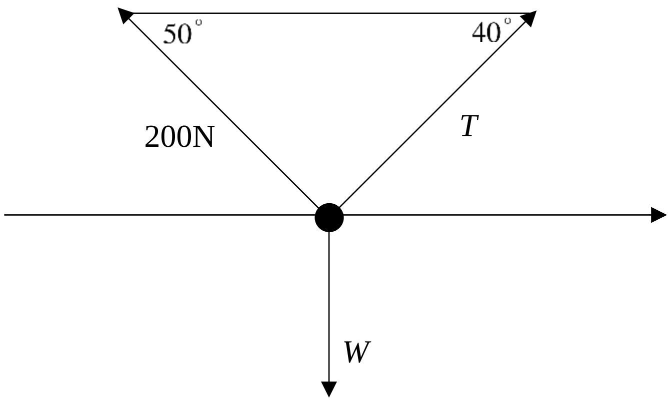
Question 9 (13 Marks)

a) In the diagram, the shaded region is bounded by the parabola $y = -x^2 + 1$, the y-axis and the line $y = \frac{3x}{2}$. (2)

Find the volume of the solid formed when the region is rotated about the y-axis.



b) An object is hanging from two ropes. One rope has a tension of 200 N and makes an angle of $50°$ with the ceiling. The other rope makes an angle of $40°$ with ceiling.

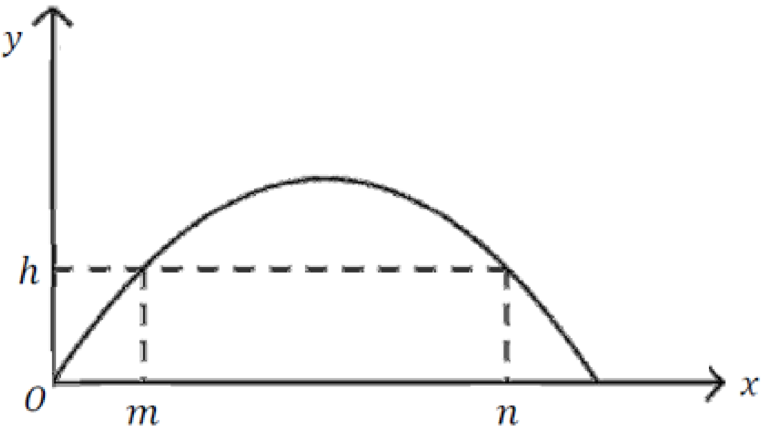


i) Calculate the tension T in the second rope. (2)

ii) Calculate the weight of the object. (2)

c) An object is projected with velocity V m/s from a point O at an angle of elevation α.

The object just clears two vertical chimneys of height h metres at horizontal distance of m metres and n metres from O. The acceleration due to gravity is taken as $10\,\text{ms}^{-2}$



i) Show that the expression for the horizontal displacement of the particle after t seconds is $x = Vt\cos\alpha$ and the vertical displacement is $y = Vt\sin\alpha - \frac{1}{2}gt^2$. (2)

ii) Show that : (2)

$$V^2 = \frac{5m^2(1+\tan^2\alpha)}{m\tan\alpha - h}.$$

iii) Show that : (3)

$$\tan\alpha = \frac{h(m+n)}{mn}.$$

End of Paper

a) Prove, using mathematical induction, that $4^n + 14$is divisible by 6 for all $n \geq 1$.
(3)

a) Use the substitution $u = \sin^2 x$ to evaluate $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\sin 2x}{1 + \sin^2 x} dx$, give your answer in simplest form.

Ella was playing fetch with her dog at the park. She threw a tennis ball with a speed of $15ms^{-1}$ and an initial angle of 45° from the horizontal. Assume that $g = 10ms^{-2}$.

(a) Find the velocity vector for the tennis ball.

(b) Find the position vector for the tennis ball.

(c) Determine the maximum height reached by the tennis ball.

(d) How far did the ball land from Ella's position?

Ella was playing fetch with her dog at the park. She threw the tennis ball with a speed of $15m/s$ and initial angle of 45° from the horizontal . Assume that $g = 10ms^{-2}$.

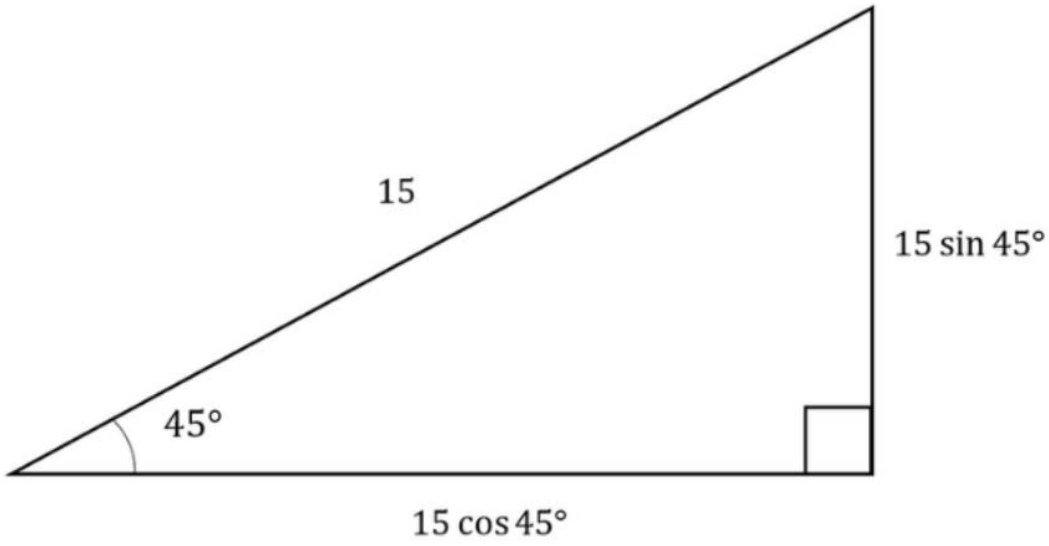
Find the velocity vector for the tennis ball.

Find the position vector for the tennis ball.

Determine the maximum height reached by the tennis ball.

How far did the ball land from Ella's position?

Decomposing initial velocity:



$$
\begin{aligned}
a &= -10j \\
&\text{Integrating a gives} \\
v &= C_1 i + (-10t + C_2)j \\
&\text{Substituting } t = 0 \text{ and } v = 15cos(45^\circ)i + 15sin(45^\circ)j \\
15cos(45^\circ)i + 15sin(45^\circ)j &= C_1 i + (-10 \times 0 + C_2)j \\
C_1 &= 15cos(45^\circ) \\
&= \frac{15}{\sqrt{2}} \\
C_2 &= 15sin(45^\circ) \\
&= \frac{15}{\sqrt{2}} \\
\therefore v &= \frac{15}{\sqrt{2}}i + \left(-10t + \frac{15}{\sqrt{2}}\right)j
\end{aligned}
$$

The maximum height is reached when $\dot{y} = 0$

$$-10t + \frac{15}{\sqrt{2}} = 0$$

$$t = \frac{3}{2\sqrt{2}}$$

$$\text{Substituting } t = \frac{3}{2\sqrt{2}} \text{ into } y = -5t^2 + \frac{15}{\sqrt{2}}t$$

$$y = -5(\frac{3}{2\sqrt{2}})^2 + \frac{15}{\sqrt{2}}(\frac{3}{2\sqrt{2}}) \qquad = \frac{45}{8}$$

$\therefore$ The maximum height reached by the ball is $\frac{45}{8}m$.

The ball lands when $y = 0$

$$-5t^2 + \frac{15}{\sqrt{2}}t = 0$$

$$-5t(t - \frac{3}{\sqrt{2}}) = 0$$

$$t = 0, \frac{3}{\sqrt{2}}$$

$t = 0$ is when the ball is initially in Ella's hands,

so we will use $t = \frac{3}{\sqrt{2}}$

ıbstituting $t = \frac{3}{\sqrt{2}}$ into $x = \frac{15}{\sqrt{2}}t$

$$x = \frac{15}{\sqrt{2}} \times \frac{3}{\sqrt{2}}$$

$$= \frac{45}{2}$$

Find $\frac{d}{dx}(\tan^{-1}x + x)$

Hence evaluate $\int_0^1 \frac{x^2+2}{x^2+1}dx$, leaving your answer in exact form

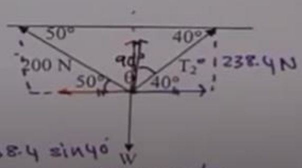
Tension in Strings : Vector Applications

An object is hanging from two ropes. One rope has a tension of 200 N and makes an angle of 50° with the ceiling. The other rope makes an angle of 40° with the ceiling.

a. Calculate the tension in the second rope.

$$\frac{T_2}{\sin 50^\circ} = \frac{200}{\sin 40^\circ}$$

$$T_2 = 200 \cdot \frac{\sin 50^\circ}{\sin 40^\circ} = 238.4\,N$$



b. Calculate weight of the object. $W = 200 \sin 50^\circ + 238.4 \sin 40^\circ$

$= 306.44\,N$ acting downwards.

Sydney Girls High School
Mathematics Faculty

Multiple Choice Answer Sheet
HSC Mathematics Extension 1

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample $2 + 4 = ?$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ~~●~~ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows:

correct

A ~~●~~ B ~~●~~ C ○ D ○

Student Number: 2024 Ext 1 Task 3 Solutions

Completely fill the response oval representing the most correct answer.

1. A ● B ○ C ○ D ○
2. A ○ B ● C ○ D ○
3. A ○ B ○ C ○ D ●
4. A ○ B ○ C ○ D ●
5. A ○ B ○ C ● D ○

2024 Ext 1 Task 3 Solutions

MC

1) $u = x^2 - 2$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$\frac{1}{2}\int \frac{du}{\sqrt{u}}$

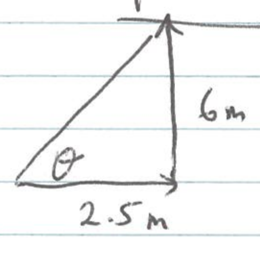
$= \frac{1}{2}\frac{\sqrt{u}}{\frac{1}{2}} + C$

$= \sqrt{x^2-2} + C$ $\boxed{\therefore A}$

2) $f'(x) = \frac{-3}{\sqrt{1-9x^2}} + \cos^{-1}3x - \frac{3x}{\sqrt{1-9x^2}}$

$= \cos^{-1}3x - \frac{3(x+1)}{\sqrt{1-9x^2}}$

$\boxed{\therefore B}$

3) 

$\boxed{\therefore D}$

4)

$$\cos 6x = \cos^2 3x - \sin^2 3x$$

$$= 1 - 2\sin^2 3x$$

$$2\sin^2 3x = 1 - \cos 6x$$

$$\sin^2 3x = \frac{1}{2}(1 - \cos 6x)$$

$$\frac{1}{2}\int 1 - \cos 6x \, dx$$

$$= \frac{1}{2}\left[x - \frac{\sin 6x}{6}\right] + C \qquad \boxed{\therefore \quad D}$$

5)

$$\frac{1}{2}\int \frac{2}{\sqrt{1-(2x)^2}} \, dx$$

$$= \frac{1}{2}\sin^{-1}(2x) + C$$

$$\boxed{\therefore \quad C}$$

Ext 1 Task 3 Q6

6a)

$$\frac{du}{dx} = e^x$$

$$du = e^x\,dx$$

$$\int \frac{dx}{e^x + \frac{9}{e^x}}$$

Many students didn't know how to deal with $9e^{-x}$

$$\int \frac{dx}{\frac{e^{2x}+9}{e^x}}$$

$$\int \frac{e^x\,dx}{e^{2x}+9}$$

$$\int \frac{du}{u^2+9}$$

✓ 1 mark for this step

$$= \frac{1}{3}\tan^{-1}\frac{u}{3} + C$$

$$= \frac{1}{3}\tan^{-1}\frac{e^x}{3} + C$$

✓ 1 mark for correct answer

Some students forgot the fraction at the front.

6b) $A = \int_0^{\frac{\pi}{6}} \cos^2 2x \, dx$

$= \int_0^{\frac{\pi}{6}} \frac{\cos 4x + 1}{2} \, dx$ 1 mark for this expression or equivalent

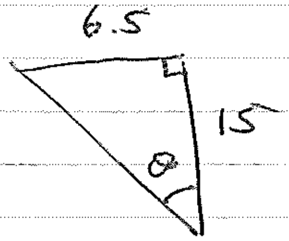
$= \frac{1}{2}\left[\frac{\sin 4x}{4} + x\right]_0^{\frac{\pi}{6}}$

$= \frac{1}{2}\left[\frac{\sqrt{3}}{8} + \frac{\pi}{6}\right]$

$= \frac{\sqrt{3}}{16} + \frac{\pi}{12}$ 1 mark for correct answer

some students did volume.

c)



some students didn't have the correct diagram so they couldn't get the direction right.

$V = \sqrt{15^2 + 6.5^2}$ 1 mark

$= 16.3$ m/s

If only an angle was given 1 mark was deducted.

$\tan\theta = \frac{6.5}{15}$

$\theta = 23^\circ$

direction $N23^\circ W$ 1 mark for correct direction

d) $\frac{du}{dx} = \frac{1}{1+x^2}$

$x=0 \quad u=0$

$x=1 \quad u=\frac{\pi}{4}$

$du = \frac{dx}{1+x^2}$

$\int_0^{\frac{\pi}{4}} u \, du$

1 mark for using the substitution

1 mark for correct integration

1 for correct answer

$= \left[\frac{u^2}{2}\right]_0^{\frac{\pi}{4}}$

$= \frac{\pi^2}{32}$

- some students couldn't deal with the fraction correctly

e) Prove true for $n=2$

LHS $= 2 \times 1$
$= 2$

RHS $= \frac{2(4-1)}{3}$
$= 2$

1 mark was given to these two steps

LHS $=$ RHS $\therefore$ true for $n=2$

Assume true for $n=k$

$2\times1 + 3\times2 + \ldots \; k(k-1) = \frac{k(k^2-1)}{3}$

Prove true for $n = k+1$

$$LHS = \frac{k(k^2-1)}{3} + (k+1)(k+1-1)$$

$$RHS = \frac{(k+1)((k+1)^2-1)}{3}$$

$$LHS = \frac{k(k-1)(k+1) + 3(k+1)(k)}{3}$$

$$= \frac{k(k+1)(k-1+3)}{3}$$

$$= \frac{k(k+1)(k+2)}{3}$$

$$= \frac{(k+1)(k^2+2k)}{3}$$

$$= \frac{(k+1)(k^2+2k+1-1)}{3}$$

$$= \frac{(k+1)((k+1)^2-1)}{3}$$

$$= RHS$$

This section was awarded 2 marks for correct Algebra and steps clearly shown

- many students didn't have brackets in the correct place so they lost a mark

- if had a cubic function and factorised by inspection you lost a mark as this is a show question

- If steps not shown properly a mark was deducted.

Question 7 (a) (i)

Comments
• Some students forgot the negative sign when differentiated. • Many students didn't simplify the denominator $10\sqrt{1-\left(\frac{x-1}{10}\right)^2}$ correctly. 10 can be put under the square root. • $\sqrt{100}\times\sqrt{1-\left(\frac{x-10}{10}\right)^2}=\sqrt{100\times\left(1-\left(\frac{x-10}{10}\right)^2\right)}=\cdots$

Criteria	Marks
• Provides correct solution	2
• Correctly rewrites the derivative in a suitable form and attempts to simplify	1

Sample answer:

$$\frac{d}{dx}\cos^{-1}\left(\frac{x-10}{10}\right)$$

$$=-\frac{\frac{1}{10}}{\sqrt{1-\left(\frac{x-10}{10}\right)^2}} \checkmark$$

$$=-\frac{1}{10\sqrt{\frac{100}{100}-\frac{x^2-20x+100}{100}}}$$

$$=-\frac{1}{10\sqrt{\frac{20x-x^2}{100}}}$$

$$=-\frac{1}{\sqrt{100\times\frac{20x-x^2}{100}}} \checkmark$$

$$=-\frac{1}{\sqrt{20x-x^2}}$$

Question 7 (a) (ii)

Comments
• Some students forgot the negative sign when integrated. • Some students evaluated $\cos^{-1}\left(-\frac{1}{2}\right)$ incorrectly.

Criteria	Marks
• Provides correct solution	2
• Correctly rewrites the integrand in a suitable form and attempts to evaluate	1

Sample answer:

$$\int_5^{10} \frac{1}{\sqrt{20x - x^2}}\,dx$$

$$= -\left[\cos^{-1}\left(\frac{x-10}{10}\right)\right]_5^{10} \checkmark \quad \text{according to part (i)}$$

$$= -\left(\cos^{-1} 0 - \cos^{-1}\left(-\frac{1}{2}\right)\right)$$

$$= -\left(\frac{\pi}{2} - \frac{2\pi}{3}\right) \checkmark$$

$$= \frac{\pi}{6}$$

Note that

$$\cos^{-1}\left(-\frac{1}{2}\right)$$

$$= \pi - \cos^{-1}\left(\frac{1}{2}\right)$$

$$= \pi - \frac{\pi}{3}$$

Question 7 (b)

Comments
• Some students forgot to change the limits

Criteria	Marks
• Provides correct solution	2
• Obtains the correct substitution and limits	1

Sample answer:

$u^2 = 1 + x$

$x = u^2 - 1$

$dx = 2u\, du$

when $x = 1, \qquad u = \sqrt{2}$

when $x = 0, \qquad u = 1$

$$\int_1^{\sqrt{2}} \frac{u^2 - 1}{\sqrt{u^2}}\, 2u\, du$$

$$= 2\int_1^{\sqrt{2}} u^2 - 1\, du \checkmark$$

$$= 2\left[\frac{u^3}{3} - u\right]_1^{\sqrt{2}}$$

$$= 2\left[\left(\frac{2\sqrt{2}}{3} - \sqrt{2}\right) - \left(\frac{1}{3} - 1\right)\right]$$

$$= \frac{-2\sqrt{2} + 4}{3} \checkmark$$

Question 7 (c)

Comments
• Generally done well

Criteria	Marks
• Provides correct solution	3
• Attempts to prove true for $n = k + 1$	2
• Proves true for $n = 1$ and assumes true for $n = k$, showing sufficient working out.	1

Sample answer:

Prove ture for $n = 1$:

$\text{LHS} = 1 \times 1! = 1$

$\text{RHS} = (1 + 1)! - 1 = 2 - 1 = 1 = \text{LHS}$

$\therefore$ True for $n = 1$

✓

Assume true for $n = k$:

$1 \times 1! + 2 \times 2! + 3 \times 3! + \cdots + k \times k! = (k + 1)! - 1$

Prove true for $n = k + 1$

i.e. $\underbrace{1 \times 1! + 2 \times 2! + \cdots + k \times k!}_{(k+1)!-1} + (k + 1) \times (k + 1)! = (k + 2)! - 1$ ✓

$\text{LHS} = (k + 1)! - 1 + (k + 1)(k + 1)!$

$= (k + 1)!\,[1 + (k + 1)] - 1$

$= (k + 1)!\,(k + 2) - 1$ ✓

$= (k + 2)! - 1$

$= \text{RHS} \quad \Longrightarrow$ True for $n = k + 1$

$\therefore$ By mathematical induction, the statement is true for integers $n \geq 1$.

Question 7 (d)

Comments
• Generally done well • Some students didn't answer in vector forms for part (i) (ii) • Some students evaluated answers incorrectly when dealing with surds in part (iv)

Question 7 (d) (i)

Criteria	Marks
• Provides correct solution (vector form)	1

Sample answer:

$$\boldsymbol{v}(\boldsymbol{o}) = \begin{pmatrix} 15\cos 45° \\ 15\sin 45° \end{pmatrix} = \begin{pmatrix} \frac{15\sqrt{2}}{2} \\ \frac{15\sqrt{2}}{2} \end{pmatrix}$$

$\ddot{x} = 0$

$\dot{x} = \int 0\,dt = C_1$

$\dot{x} = 15\cos 45° = \frac{15\sqrt{2}}{2}$

$\ddot{y} = -g = -10$

$\dot{y} = \int -10\ dt$

$\quad = -10t + C_2$

When $t = 0$, $\quad \dot{y}(0) = \frac{15\sqrt{2}}{2}$

$\therefore \dot{y} = -10t + \frac{15\sqrt{2}}{2}$

$$\therefore \boldsymbol{v}(\boldsymbol{t}) = \begin{pmatrix} \frac{15\sqrt{2}}{2} \\ -10t + \frac{15\sqrt{2}}{2} \end{pmatrix} \checkmark$$

Question 7 (d) (ii)

Criteria	Marks
• Provides correct solution (vector form)	1

Sample answer:

$\dot{x} = \dfrac{15\sqrt{2}}{2}$

$x = \displaystyle\int \frac{15\sqrt{2}}{2}\,\mathrm{dt}$

$\quad = \dfrac{15\sqrt{2}}{2}\mathrm{t} + C_3$

When $t = 0, x = 0,$

$\therefore C_3 = 0$

$\therefore x = \dfrac{15\sqrt{2}}{2}\mathrm{t}$

$\dot{y} = -10t + \dfrac{15\sqrt{2}}{2}$

$y = \displaystyle\int -10t + \frac{15\sqrt{2}}{2}\,dt$

$\quad = -5t^2 + \dfrac{15\sqrt{2}}{2}t + C_4$

When $t = 0, y = 0$

$\therefore C_4 = 0$

$\therefore y = -5t^2 + \dfrac{15\sqrt{2}}{2}t$

$$\therefore \boldsymbol{r}(\boldsymbol{t}) = \begin{pmatrix} \dfrac{15\sqrt{2}}{2}t \\ -5t^2 + \dfrac{15\sqrt{2}}{2}t \end{pmatrix} \checkmark$$

Question 7 (d) (iii)

Criteria	Marks
• Provides correct solution	1

Sample answer:

Let $\dot{y} = 0$,

$$-10t + \frac{15\sqrt{2}}{2} = 0$$

$$t = \frac{3\sqrt{2}}{4} \text{ seconds}$$

Then sub t into y:

$$\therefore y_{MAX} = -5\left(\frac{3\sqrt{2}}{4}\right)^2 + \frac{15\sqrt{2}}{2} \times \frac{3\sqrt{2}}{4} = \frac{45}{8} \text{ metres} \quad ✓$$

Question 7 (d) (iv)

Criteria	Marks
• Provides correct solution	1

Sample answer:

Let $y = 0$,

$$-5t^2 + \frac{15\sqrt{2}}{2}\text{t} = 0$$

$$-5t\left(t - \frac{3\sqrt{2}}{2}\right) = 0$$

$$t = 0 \text{ or } t = \frac{3\sqrt{2}}{2}$$

Then sub t into $x = \frac{15\sqrt{2}}{2}t$:

$$\therefore x = \frac{15\sqrt{2}}{2} \times \frac{3\sqrt{2}}{2} = \frac{45}{2} \text{ metres} \quad ✓$$

$\boxed{Q8}$

a) $y = \dfrac{\cos^{-1}x}{x}$

$$y' = \frac{\dfrac{-x}{\sqrt{1-x^2}} - \cos^{-1}x}{x^2}$$

$$y' = \frac{-x - \sqrt{1-x^2}\,\cos^{-1}(x)}{x^2\sqrt{1-x^2}}$$

1 mark for apply or any attempt to apply quotient rule.

2 marks for apply correct quotient rule and provide correct answer.

b) prove by Mathematical induction that

$2^n + (-1)^{n+1}$ is divisible by 3.

Prove it is true for $n = 1$

$2^1 + (-1)^{1+1} = 3$ which is divisible by 3.

Assume it is true for $n = k$

$$2^k + (-1)^{k+1} = 3M \quad (*)$$

Prove it is true for $n = k+1$

$$2^{k+1} + (-1)^{k+2} = 2\left(2^k + (-1)^{k+1}\right) + 3(-1)^k$$

$$= 2(3M) + 3(-1)^k$$

$$= 3\left(2M + (-1)^k\right)$$

This is divisible by 3 which is true for $n = k+1$. Proven b Mathematical induction.

1 mark for prove LHS = RHS when n = 1
2 marks for using the assumption to subs into step prove it is true for n = k +1
3 marks for logic and correct prove.

$\boxed{Q8}$

c) i) $y = x\cos^{-1}x$

$$y' = \cos^{-1}x - \frac{x}{\sqrt{1-x^2}}$$ ✓✓

1 mark for showing some merits in applying product rule.

2 marks for applying the product rule and provide the correct answer.

ii) Note $\int f'(x)\,dx = f(x) + c$

$$\int \cos^{-1}(x)\,dx - \int \frac{x}{\sqrt{1-x^2}}\,dx = x\cos^{-1}(x) + c$$ ✓

$$\int \cos^{-1}x\,dx = \int \frac{x}{\sqrt{1-x^2}}\,dx + x\cos^{-1}x + c$$

$$= -\frac{1}{2}\int \frac{-2x}{\sqrt{1-x^2}}\,dx + x\cos^{-1}x + c$$

$$= -\sqrt{1-x^2} + x\cos^{-1}x + c$$ ✓

1 mark for having $x\cos^{-1}x$

Or use $\int f'(x)\,dx = f(x) + c$

2 marks for correct working and provide correct answer.

d) $$V = V_{cylinder} - \pi\int_0^{\pi/4} \sin^2 2x\,dx$$

$$= \pi(\sqrt{2})^2\frac{\pi}{4} - \frac{\pi}{2}\int_0^{\pi/4}(1-\cos 4x)\,dx$$ ✓

$$= \frac{\pi^2}{2} - \frac{\pi}{2}\left[x - \frac{1}{4}\sin 4x\right]_0^{\pi/4}$$ ✓

$$= \frac{\pi^2}{2} - \frac{\pi}{2}\left[\frac{\pi}{4} - 0\right] = \frac{\pi^2}{2} - \frac{\pi^2}{8} = \frac{3\pi^2}{8} \text{ units}^3$$ ✓

1 mark for finding the volume of the cylinder correctly.

2 marks for recognising Volume is equal to Volume of the cylinder minus the volume of the solid formed when the shaded area rotated about the x-axis and applied double angle formula of sin2x correctly.

3 marks for correct working and answer as $\frac{3\pi^2}{8}$

Q9

a) $-x^2+1=\frac{3x}{2}$

$-2x^2+2=3x$

$2x^2+3x+2=0$

$(x+2)(2x-1)=0$ ✓

$x=-2$ or $\begin{cases} x=\frac{1}{2} \\ y=\frac{3}{4} \end{cases}$

• 1 mark for finding the point of intersection

$$V=\pi\int_0^{3/4}\left(\frac{2y}{3}\right)^2 dy+\pi\int_{3/4}^{1}\left(\sqrt{(1-y)}\right)^2 dy$$

$$=\pi\int_0^{3/4}\frac{4y^2}{9}\,dy+\pi\int_{3/4}^{1}1-y\,dy$$ ✓

1 mark for correct limits with correct functions

$$=\pi\left[\frac{4y^3}{27}\right]_0^{3/4}+\pi\left[y-\frac{y^2}{2}\right]_{3/4}^{1}$$

$$=\pi\left(\frac{4\left(\frac{3}{4}\right)^3}{27}+\left(1-\frac{1}{2}-\left(\frac{3}{4}-\frac{9}{32}\right)\right)\right)$$

$$=\pi\left[\frac{1}{16}+\frac{1}{32}\right]$$ ✓

$$=\frac{3\pi}{32}$$

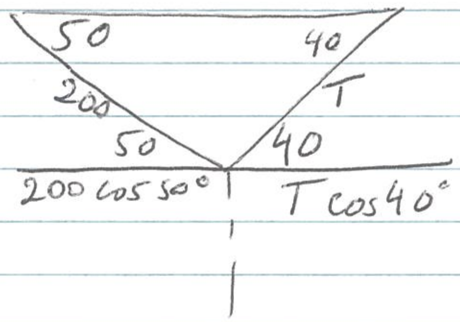
1 mark for the correct answer

• Many students had difficulty with this question

• some rotated around the wrong axis (0 marks)

• some integrated from 0 to 1 so only 1 mark was awarded if they had the integration correctly done.

b)



i) $T\cos 40^\circ = 200\cos 50^\circ$ ✓

$T = \dfrac{200\cos 50^\circ}{\cos 40^\circ}$ ✓

$= 167.8\ N$

2 marks for correct working and answer

ii) $W = 200\sin 50^\circ + T\sin 40^\circ$

$= 261.08$ ✓

1mark for correct answer

- some students using $F = mg$ you don't have to do this in Maths.
- students who used sine rule incorrectly didn't get any marks

c i) $\ddot{x} = 0$

$\dot{x} = \int 0 \, dt$

$= C$

at $t = 0$ $\dot{x} = V\cos\theta$ $\therefore C = V\cos\theta$

$\dot{x} = V\cos\theta$

$x = \int V\cos\theta \, dt$

$= Vt\cos\theta + C$

at $t = 0$ $x = 0$ $\therefore C = 0$

$x = Vt\cos\theta$

1 mark for all the steps shown correctly if any C missing no mark was awarded.

$\ddot{y} = -10$

$\dot{y} = \int -10 \, dt$

$= -10t + C$

at $t = 0$ $\dot{y} = V\sin\theta$ $\therefore C = V\sin\theta$

$\dot{y} = -10t + V\sin\theta$

$y = \int -10t + V\sin\theta \, dt$

$= -5t^2 + Vt\sin\theta + C$

at $t = 0$ $y = 0$ $\therefore C = 0$

$y = -5t^2 + Vt\sin\theta$

Some students only writing the formulas $\therefore$ no mark was awarded. As this was a show question

• 1 mark also for all the steps shown properly

ii) $t = \dfrac{x}{V\cos\alpha}$

$$y = \frac{\cancel{V} x \sin\alpha}{\cancel{V}\cos\alpha} - \frac{5x^2}{V^2\cos^2\alpha}$$

$$= x\tan\alpha - \frac{5x^2\sec^2\alpha}{V^2}$$

at $x = m$ $y = h$ ✓

$$h = m\tan\alpha - \frac{5m^2(1+\tan^2\alpha)}{V^2}$$

$$\frac{5m^2(1+\tan^2\alpha)}{V^2} = m\tan\alpha - h$$

✓

$$V^2 = \frac{5m^2(1+\tan^2\alpha)}{m\tan\alpha - h}$$

* To recieve the 2 marks you had to show all the steps clearly ever changing $\cos^2\alpha$ to $\sec^2\alpha$

iii) $$v^2 = \frac{5m^2(1+\tan^2\alpha)}{m\tan\alpha - h}$$

similarly at $x = n$

$$v^2 = \frac{5n^2(1+\tan^2\alpha)}{n\tan\alpha - h}$$

1 mark to equate these 2

$$\frac{\cancel{5}m^2\cancel{(1+\tan^2\alpha)}}{m\tan\alpha - h} = \frac{\cancel{5}n^2\cancel{(1+\tan^2\alpha)}}{n\tan\alpha - h}$$

$$m^2(n\tan\alpha - h) = n^2(m\tan\alpha - h)$$

$$m^2n\tan\alpha - m^2h = n^2m\tan\alpha - n^2h$$

$$m^2h - n^2h = m^2n\tan\alpha - n^2m\tan\alpha$$

$$h(m^2 - n^2) = mn\tan\alpha(m-n)$$

$$\tan\alpha = \frac{h\cancel{(m-n)}(m+n)}{mn\cancel{(m-n)}}$$

$$= \frac{h(m+n)}{mn}$$

To get full marks all the steps had to be shown properly