TRINITY GRAMMAR SCHOOL
MATHEMATICS DEPARTMENT

YEAR 11 2009 ASSESSMENT TASK 3

MATHEMATICS

(EXTENSION 1)

HALF-YEARLY EXAMINATION

Time allowed- *2 hours plus 5 minutes reading time.*

Date: Tuesday May 5^{th}, 2009

WEIGHTING: 25% towards final result

OUTCOMES referred to: P2, P3, P4, P5, P6, P7, P8

INSTRUCTIONS:

1. Attempt **ALL** questions.
2. There are 6 questions worth 14 marks each (Total marks: 84)
3. Show all necessary working.
4. Begin each question on a **new page.**
5. **Write your name, your teacher's initials and class at the top of each page.**
6. Mark values are shown beside each part.
7. The only calculators permitted are scientific calculators approved by the Board of Studies.
8. Responses must be in black or blue pen but diagrams are to be drawn in pencil.
9. If required, additional writing sheets may be obtained from a teacher upon request.

Name:______________________________

Start each question on a new page.

QUESTION 1 **Marks**

(a) Convert $0.4\dot{7}\dot{2}$ to a fraction in its simplest terms. **2**

(b) Solve $2x^2 = 4x$ **1**

(c) Find $\lim\limits_{x \to 3} \dfrac{6x - 18}{x^2 - 9}$ **2**

(d) Solve $2x^2 - 11x + 7 \geq 2$ **2**

(e) Sketch, showing all essential features:

$$y = |2x + 3|$$ **2**

(f) Simplify

$$\frac{2}{x^2 - x} - \frac{5}{x^2 - 1}$$ **2**

(g) Find the solutions of the following simultaneous equations. **3**

$$x + 2y - z = -5$$

$$2x - 3y + 4z = 28$$

$$4x + 5y - 3z = -10$$

Start each question on a new page.

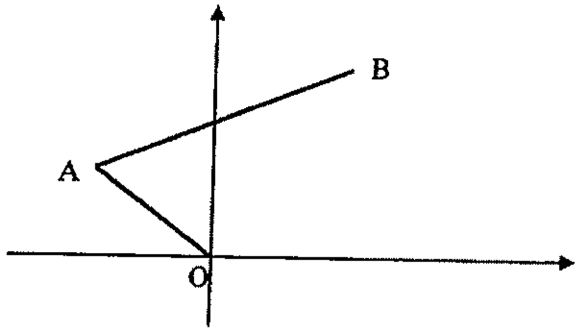
QUESTION 2 **Marks**

(a) Shade the region where $y \le \sqrt{9-x^2}$ **2**

(b) By rationalising the denominators, express $\dfrac{1}{4-\sqrt{3}} + \dfrac{1}{4+\sqrt{3}}$,

in simplest form. **2**

(c)



Not to scale

A is the point (-3,1) and B is the point (2,5). Draw the diagram in your answer booklet.

(i) Show that the equation AB is $4x - 5y + 17 = 0$ **2**

(ii) Show that the length of AB is $\sqrt{41}$ **1**

(iii) Calculate the perpendicular distance from O to AB **2**

(iv) Find the point C(x,y) such that ABCO is a parallelogram. **2**

(v) Find the area of the parallelogram ABCO **1**

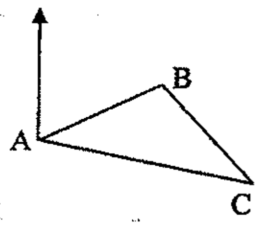
(vi) Find the coordinates of the point of intersection of AC and OB **2**

Start each question on a new page.

QUESTION 3 **Marks**

(a) Solve for x: $\tan x = -1$ for $-180° \le x \le 180°$ **2**

(b)



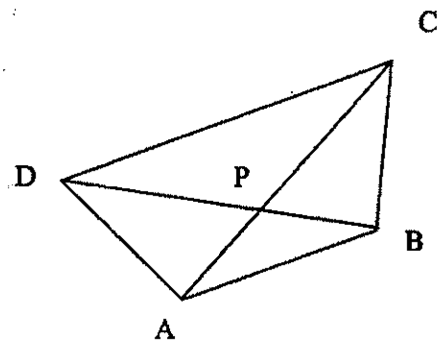
George travels 10 kilometres on a bearing of 40° from A to B. He then travels 15 kilometres on a bearing of 140° from B to C.

Copy the diagram onto your answer sheet.

(i) Show that $\angle ABC = 80°$ **2**

(ii) How far is George from his starting point? (Answer to the nearest metre) **2**

(c)



Not to scale

ABCD is a quadrilateral. The diagonals AC and BD intersect at P.
AD = BC and AC = BD.

Copy the diagram onto your answer sheet.

(i) Show the triangles ABC and ABD are congruent. **2**

(ii) Show that triangle ABP is isosceles. **2**

(iii) Hence show that triangle CDP is isosceles. **2**

(iv) Show AB is parallel to CD **2**

Start each question on a new page.

QUESTION 4 **Marks**

(a) Sketch the following showing all essential features:

(i) $y = 1 + \dfrac{1}{x-2}$ **3**

(ii) $y = (x+2)^2 - 1$ **2**

(b) Given $f(x) = \dfrac{2x^2}{x^2 - 16}$

Is f(x) even, odd or neither? Justify your answer. **2**

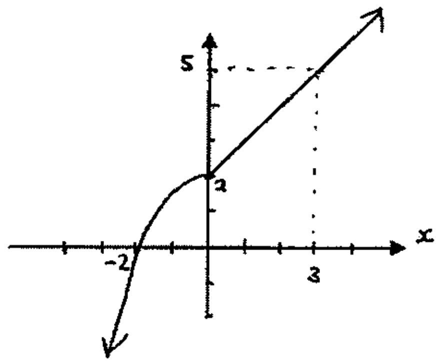
(c)

(i) State the domain and range of $y = \sqrt{2x-7}$ **2**

(ii) If $f(x) = \dfrac{3^x + 3^{-x}}{2}$

Show that $[f(x)]^2 = \frac{1}{2}[f(2x) + 1]$ **3**

(iii)



Find *g(x)* for graph above which is made up of a parabola and a straight line.

$g(x) = \begin{cases} \_\_\_\_\_\_\_\_\_\_, & x < 0 \\ \_\_\_\_\_\_\_\_\_\_, & x \geq 0 \end{cases}$ **2**

Start each question on a new page.

QUESTION 5

Marks

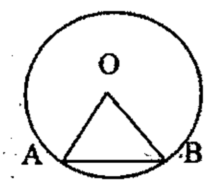
(a) (i) Sketch the graph of $y = \cos x$ for $-360^\circ \leq x \leq 360^\circ$ showing all essential features. **2**

(ii) Hence or otherwise find the values of $\cos x = -\frac{1}{2}$ for $-360^\circ \leq x \leq 360^\circ$ **2**

(iii) Find the value of t if $\tan(2t + 5^\circ) = \cot(3t - 15^\circ)$ **2**

(b) Solve $2\sin^2\theta + \sin\theta - 1 = 0$ for $0^\circ \leq \theta \leq 360^\circ$ **3**

(c)



O is the centre of a circle.

OB =OA= 8cm

$\angle AOB = 102^\circ$

Find the area of triangle OAB to one decimal place. **1**

(d)

(i) Prove that $\dfrac{1 + \cot x}{\operatorname{cosec} x} - \dfrac{1}{\sec x} = \sin x$ **2**

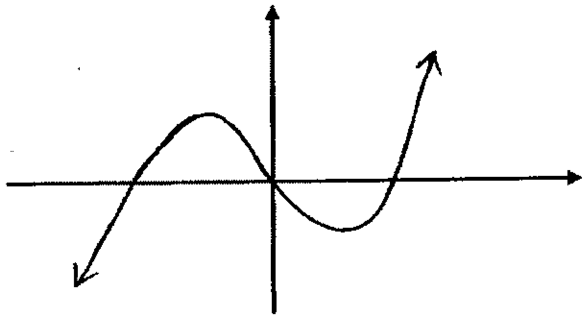
(ii) The angle of depression from the top of an 80m cliff to a boat at the foot of the cliff is $64^\circ\ 19'$. How far out from the cliff is the boat? (answer to the nearest metre) **2**

Start each question on a new page.

QUESTION 6

Marks

(a) The diagram shows the graph of a function. **Copy this graph onto your answer sheet.** On the **same** set of axes, draw a sketch of the derivative f '(x) of the function. 2



(i) Differentiate from first principles to find the gradient of the tangent at any point on the curve $f(x) = x^2 - 4x + 4$ 3

(ii) Hence or otherwise find the gradient of the tangent to the curve

$f(x) = x^2 - 4x + 4$ when $x = 2$ 1

(b) Differentiate with respect to x:

(i) $\dfrac{3}{x^4} + x\sqrt{x}$ 2

(ii) $\dfrac{\sqrt{x+1}}{2x+5}$ 2

(iii) $x^3(2x-1)^2$ 2

(c) Find the equation of the tangent to the curve $y = (2 - 3x)^4$

at the point where x = 1 2

END OF TASK

Solutions YR11 Ext Task 3 2009

QUESTION 1

a) Let $x = 0.47272\ldots$

$$\left.\begin{aligned} 1000x &= 472.7272\ldots \\ 10x &= 4.7272\ldots \end{aligned}\right\} \checkmark$$

$$1000x - 10x = 472.7272\ldots - 4.7272\ldots$$

$$990x = 468$$

$$\therefore x = \frac{468}{990} = \frac{26}{55} \quad \checkmark \text{ (simplified)}$$

(2)

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b)

$$2x^2 - 4x = 0$$

$$2x(x-2) = 0$$

$$\therefore x = 2 \text{ or } 0 \quad \checkmark$$

(both correct)

(1)

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c)

$$\lim_{x \to 3} \frac{6x-18}{x^2-9} = \lim_{x \to 3} \frac{6\cancel{(x-3)}}{\cancel{(x-3)}(x+3)} \quad \checkmark$$

$$= \lim_{x \to 3} \frac{6}{x+3}$$

$$= \frac{6}{6} = 1 \quad \checkmark$$

(2)

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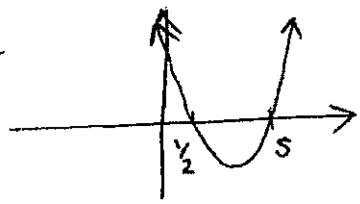
d)

$$2x^2 - 11x + 5 \geq 0$$

$$(2x-1)(x-5) \geq 0 \quad \checkmark$$



$$\left.\begin{aligned} x &\geq 5 \\ x &\leq \tfrac{1}{2} \end{aligned}\right\} \checkmark$$

(2)

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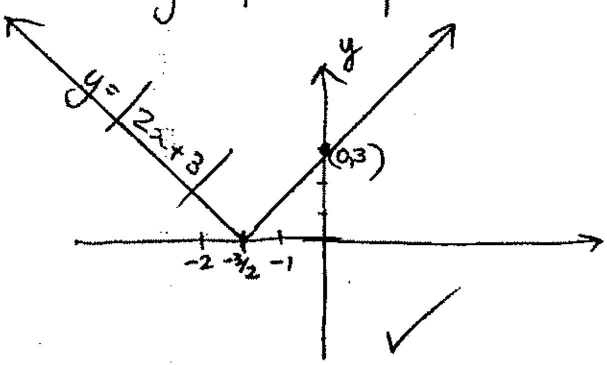
e) $y = |2x+3|$



when $y = 0$

$$2x + 3 = 0$$

$$2x = -3$$

$$x = -3/2$$

$$(-3/2, 0)$$

when $x = 0$,

$$y = |3|$$

$$= 3$$

goes through $(0,3)$

$\checkmark$

$\checkmark$

(2)

Q1 (f) $\dfrac{2}{x^2-x} - \dfrac{5}{x^2-1} = \dfrac{2}{x(x-1)} - \dfrac{5}{(x-1)(x+1)}$

$= \dfrac{2(x+1)}{x(x-1)(x+1)} - \dfrac{5x}{x(x-1)(x+1)}$ ✓

$= \dfrac{2x+2-5x}{x(x-1)(x+1)}$

$= \dfrac{2-3x}{x(x^2-1)}$ ✓ (2)

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g) $x+2y-z=-5$ — (1)
$2x-3y+4z=28$ — (2)
$4x+5y-3z=-10$ — (3)

$4\times$(1) $4x+8y-4z=-20$ — (4) $2\times$(2) $4x-6y+8z=56$ — (5)

(4)−(5) $14y-12z=-76$ — (6)

(5)−(3) $-11y+11z=66$

ie $-y+z=6$ — (7)

(7)$\times 12$ $-12y+12z=72$ — (8)

(6)+(8) $2y=-4$

$\underline{y=-2}$ sub into (6)

$14\times -2 - 12z = -76$
$-28 - 12z = -76$
$-12z=-48$
$\underline{z=4}$

sub $z=4$, $y=-2$ into (1)

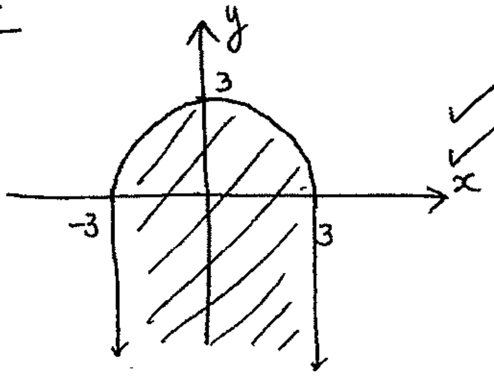
$x-4-4=-5$
$\therefore \underline{x=3}$

check ✓

$\therefore \underline{\underline{x=3}}$ ✓, $\underline{\underline{y=-2}}$ ✓, $\underline{\underline{z=4}}$ ✓ (3 marks)

QUESTION 2

(a)



✓ Boundaries
✓ shading

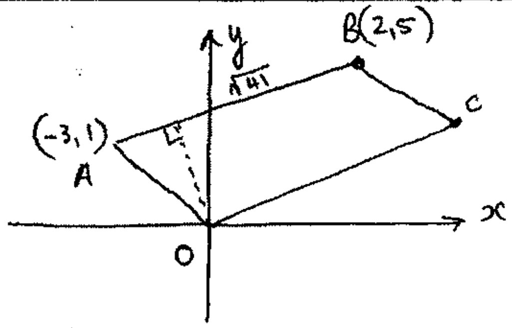
(2)

(b) $\dfrac{1}{4-\sqrt{3}} + \dfrac{1}{4+\sqrt{3}} = \dfrac{4+\sqrt{3}+4-\sqrt{3}}{(4-\sqrt{3})(4+\sqrt{3})}$ ✓

(2)

$= \dfrac{8}{16-3} = \dfrac{8}{13}$ ✓

(c)



(i) $m_{AB} = \dfrac{y_2-y_1}{x_2-x_1}$

$= \dfrac{5-1}{2--3} = \dfrac{4}{5}$ ✓

Eqn AB: $y-y_1 = m(x-x_1)$

$y-1 = \dfrac{4}{5}(x--3)$ ✓

$5y-5 = 4x+12$ (2)

$\therefore\ 4x-5y+17=0$

(ii) $d_{AB} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(2--3)^2+(5-1)^2}$ ✓ (1)

$= \sqrt{25+16}$

$= \sqrt{41}$

(iii) $P.D = \left|\dfrac{Ax_1+By_1+C}{\sqrt{A^2+B^2}}\right|$

$\boxed{(x_1,y_1)=(0,0) \quad A=4, B=-5, C=17}$

$P.D = \left|\dfrac{4\times0+-5\times0+17}{\sqrt{4^2+(-5)^2}}\right|$ ✓

$= \left|\dfrac{17}{\sqrt{41}}\right|$ (2)

$= \dfrac{17}{\sqrt{41}}$

$= \dfrac{17\sqrt{41}}{41}$ ✓

(iv) $C(5,4)$ ✓✓ (2)

(v) $A = bh$ ✓ (1)

$= \sqrt{41}\times\dfrac{17}{\sqrt{41}} = 17$ units

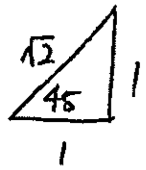
(vi) Point of intersection of diagonals is midpoint AC or OB (diagonals of a llogram bisect)

$M\left(\dfrac{x_2+x_1}{2}, \dfrac{y_2+y_1}{2}\right) = \left(\dfrac{0+2}{2}, \dfrac{0+5}{2}\right)$ (2)

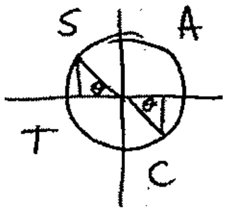
using O(0,0) & B(2,5) $= \left(1, 2\tfrac{1}{2}\right)$ ✓✓

Question 3

a) $\tan x = -1$ for $-180° \le x \le 180°$



Related angle is $45°$



since $\tan x$ is negative, x is in quadrant 2 or 4

$x = -45°,\ 180-45°$

ie $x = -45°,\ 135°$ ✓ ✓

(2)

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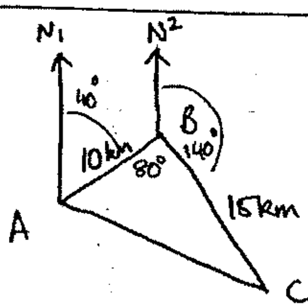
b)



(i) $\angle N_2BA = 140°$ (cointerior $\angle$'s supplementary, $N_1A \parallel N_2B$) ✓

$\therefore \angle ABC = 360 - (140° + 140°)$ (angles at a point add to 360°) ✓

$= 80°$

(2)

(ii) Cosine Rule

$AC^2 = AB^2 + BC^2 - 2 \times AB \times AC \times \cos 80°$ ✓

$= 10^2 + 15^2 - 2 \times 10 \times 15 \times \cos 80°$

$= 272.9055467\ldots$

$\therefore$

$AC = 16.51985\ldots$ km

$AC = 16520$ m to nearest metre. ✓

(2)

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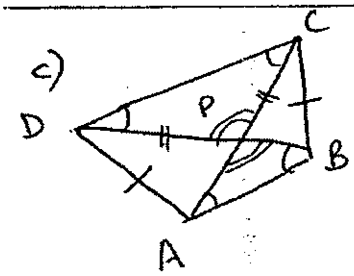
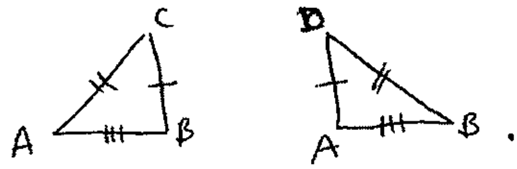
c)



(i)



In Δ's ABC & ΔABD, AB is common

AC = DB (given) ✓

BC = AD (given)

$\therefore \Delta ABC \equiv \Delta ABD$ (SSS) ✓

(2)

(ii) $\angle CAB = \angle DBA$ (corresponding angles in congruent Δ's, $\Delta ABC \equiv \Delta ABD$) ✓

$\therefore \Delta APB$ is isosceles (base angles equal) ✓

(2)

(iii) Since ΔAPB is isosceles, AP = BP ✓

Now AC = BD (given)

$\therefore$ PC = PD ✓

$\therefore \Delta CDP$ is isosceles (2 equal sides)

(2)

Q3 (c)(iv) Show $AB \parallel CD$

Now $\angle APB = \angle DPC$ (Vertically opp. $\angle$'s equal)

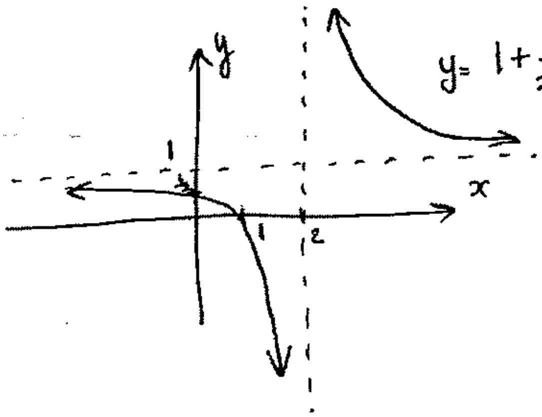
and $\triangle APB$ and $\triangle CDP$ both isosceles, (equal base $\angle$'s)

$\therefore \angle PDC = \angle PCD = \angle PBA = \angle BAP$

Since $\angle PDC = \angle PBA$, $DC \parallel AB$ (alternate $\angle$'s equal on $\parallel$ lines)

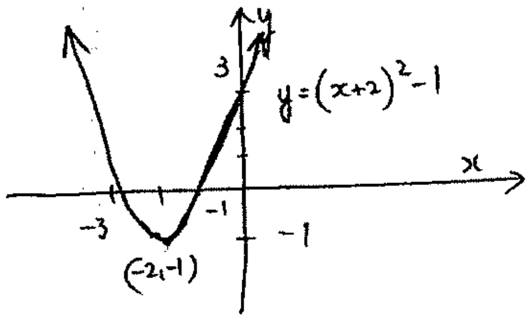
QUESTION 4

a) (i)



asymptotes ✓
shape ✓
x, y axis cut ✓ (3)

(ii)



✓ (vertex)
✓ x/y intercept (2)

b) $f(x) = \dfrac{2x^2}{x^2-16}$ $\quad f(-x) = \dfrac{2(-x)^2}{(-x)^2-16}$

$= \dfrac{2x^2}{2x^2-16} = f(x)$ ✓ (2)

Hence the function is even and is symmetrical about the y-axis. ✓

Q4 c) (i) $2x - 7 \geq 0$

$2x \geq 7$

$x \geq 3\frac{1}{2}$

Domain: $x \geq 3\frac{1}{2}$ ✓

Range: $y \geq 0$ ✓ (2)

(ii)

$$LHS = \left(\frac{3^x + 3^{-x}}{2}\right)^2$$

$$= \frac{(3^x + 3^{-x})(3^x + 3^{-x})}{2 \times 2}$$

$$= \frac{3^{2x} + 3^0 + 3^0 + 3^{-2x}}{4}$$

$$= \frac{3^{2x} + 2 + 3^{-2x}}{4}$$ ✓

$$RHS = \frac{1}{2}\left[f(2x) + 1\right]$$

$$= \frac{1}{2}\left(\frac{3^{2x} + 3^{-2x}}{2} + 1\right)$$

$$= \frac{1}{2}\left(\frac{3^{2x} + 3^{-2x} + 2}{2}\right)$$

$$= \frac{3^{2x} + 3^{-2x} + 2}{4}$$ ✓

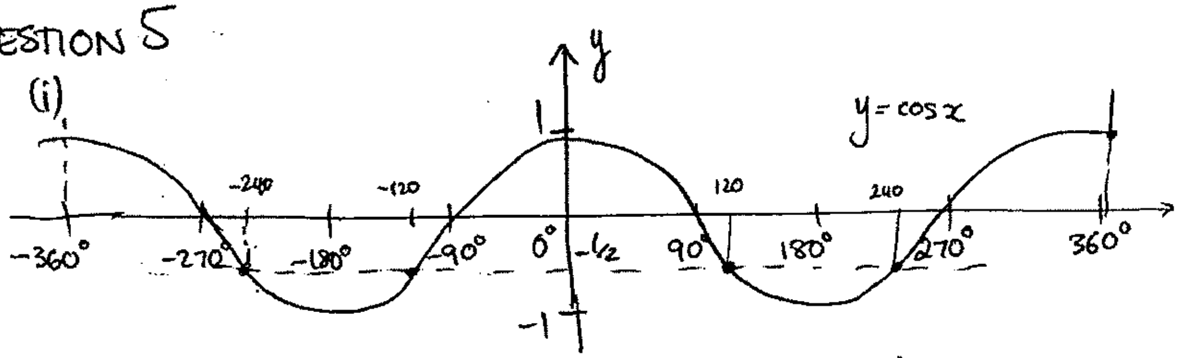
$= LHS$ ✓ (3)

(iii)

$$f(x) = \begin{cases} -\frac{1}{2}x^2 + 2 & \text{for } x \leq 0 \\ x + 2 & \text{for } x > 0 \end{cases}$$ ✓ ✓ (2)

QUESTION 5

(a) (i)



②

(ii) $\cos x = -\frac{1}{2}$ (Related angle $x = 60°$)

$\therefore \quad x = -240°, -120°, 120°, 240°$ ②

(iii) Note:

$$\tan \theta = \cot(90° - \theta)$$

$$\therefore \quad 2t + 5 = 90° - (3t - 15°)$$

$$2t + 5° = 90° - 3t + 15°$$

$$5t = 100°$$

$$\underline{t = 20°}$$ ②

b) $$(2\sin\theta - 1)(\sin\theta + 1) = 0$$

$\sin\theta = \frac{1}{2}$ or $\sin\theta = -1$ ③

(must have all 3)

$\therefore \quad \theta = 30°, 150°$ or $270°$ for $0 \le \theta \le 360°$

c) Area $= \frac{1}{2} bc \sin A$

Area $\triangle AOB = \frac{1}{2} \times 8 \times 8 \times \sin 102°$

$= 31.3 \text{ cm}^2$ (to 1 decimal place) ①

Q5 d) (i) $LHS = \dfrac{1+\cot x}{\operatorname{cosec} x} - \dfrac{1}{\sec x}$

$$= \frac{1+\frac{\cos x}{\sin x}}{\frac{1}{\sin x}} - \frac{1}{\frac{1}{\cos x}}$$

$$= \left(1+\frac{\cos x}{\sin x}\right)\times\frac{\sin x}{1} - \cos x$$

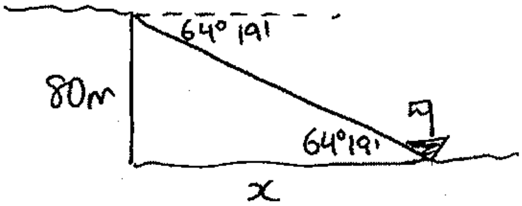
(2)

$$= \sin x + \cos x - \cos x$$

$$= \sin x$$

$$= RHS.$$

(ii)



$$\frac{80}{x} = \tan 64°19'$$

$$x = 80 \div (\tan 64°19')$$

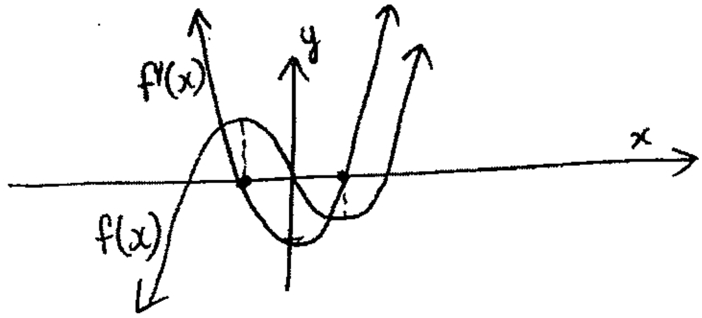
$$= 38.47274288\ldots$$

$\therefore\ x = 38\text{ m}$ (to nearest metre)

(2)

QUESTION 6

a)



✓ shape
✓ x-axis cuts

(2)

b) (i) $f(x) = x^2 - 4x + 4$

$f(x+h) = (x+h)^2 - 4(x+h) + 4$ ✓

$= x^2 + 2xh + h^2 - 4x - 4h + 4$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ ✓

$= \lim_{h \to 0} \frac{(x^2 + 2xh + h^2 - 4x - 4h + 4) - (x^2 - 4x + 4)}{h}$

$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{4x} - 4h + \cancel{4} - \cancel{x^2} + \cancel{4x} - \cancel{4}}{h}$

(3)

$= \lim_{h \to 0} \frac{\cancel{h}(2x + h - 4)}{\cancel{h}}$ ✓

$= 2x - 4$

(ii) when $x = 2$, $f'(2) = 2 \times 2 - 4$ ✓ (1)

$= 0$

c) (i) $\frac{d}{dx}\left(3x^{-4} + x^{3/2}\right) = -12x^{-5} + \frac{3}{2}x^{1/2}$

$= \frac{-12}{x^5} + \frac{3\sqrt{x}}{2}$ ✓✓ (2)

(ii) $\frac{d}{dx}\frac{(x+1)^{1/2}}{2x+5} = \frac{(2x+5)\cdot\frac{1}{2}(x+1)^{-1/2} - (x+1)^{1/2}\cdot 2}{(2x+5)^2}$ ✓

$= \frac{\frac{2x+5}{2\sqrt{x+1}} - 2\sqrt{x+1}}{(2x+5)^2}$ ✓

$= \frac{1}{2\sqrt{x+1}(2x+5)} - \frac{2\sqrt{x+1}}{(2x+5)^2}$ q. (2)

Quotient Rule, $\frac{vu' - uv'}{v^2}$

$u = (x+1)^{1/2}$

$u' = \frac{1}{2}(x+1)^{-1/2}$

$v = 2x+5$

$v' = 2$

Q6 c) (iii) $\frac{d\left(x^3(2x-1)^2\right)}{dx} = (2x-1)^2 . 3x^2 + x^3 2(2x-1)$ ✓

$= x^2(2x-1)\left[3(2x-1)+2x\right]$

$= x^2(2x-1)(8x-3)$ ✓

②

Product Rule

$vu' + uv'$

$u = x^3 \quad v = (2x-1)^2$

$u' = 3x^2 \quad v' = 2(2x-1)$

d) $y = (2-3x)^4$

$\frac{dy}{dx} = 4(2-3x)^3 \times -3$

$= -12(2-3x)^3$ ✓

at $x=1$, $f'(1) = -12(2-3)^3$

$= -12 \times -1$

$= 12$ ✓

②

Equation at $x=1$, $y = (2-3\times1)^4$

$= (-1)^4$

$= 1$

$\therefore$ line passes through $(1,1)$, slope 12

$y-1 = 12(x-1)$

$y-1 = 12x-12$

$y = 12x-11$ ✓