TRINITY GRAMMAR SCHOOL
MATHEMATICS DEPARTMENT

YEAR 11 2010 ASSESSMENT TASK 3

MATHEMATICS

(EXTENSION 1)

HALF-YEARLY EXAMINATION

Time allowed – *2 hours plus 5 minutes reading time*

Friday 23rd April, 2010

WEIGHTING: 25% towards final result

OUTCOMES referred to: P2, P3, P4, P5, P6, P7, P8.

INSTRUCTIONS:
1. Attempt **ALL** questions
2. There are 7 questions worth 12 marks each (Total marks: 84)
3. Show all necessary working.
4. Begin each question on a **new page**.
5. **Write your name, teacher's initials and class on the top of each question.**
6. Mark values are shown beside each part.
7. The only calculators permitted are scientific calculators approved by the Board of Studies.
8. Responses must be in blue or black pen but diagrams are to be drawn in pencil.
9. If requested, additional writing sheets may be obtained from a teacher upon request.

Name: ______________________________

Start each question on a new page

QUESTION 1 (12 marks) **Marks**

(a) Evaluate $\dfrac{4.67+12.7^3}{12.56-\sqrt{12}}$ correct to 3 significant figures. **2**

(b) Write $3.2\dot{5}\dot{7}$ as a mixed numeral. (Include all working steps.) **2**

(c) Write $\dfrac{1}{\sqrt[3]{(x-2)^2}}$ in index form with a negative index. **1**

(d) Solve $9^{2x+1}=27$. **2**

(e) Simplify $\dfrac{4}{x-1}-\dfrac{2}{x^2-1}$. **3**

(f) Factorise $\dfrac{4}{x^2}-\dfrac{a^2}{b^2}$. **2**

Start each question on a new page

QUESTION 2 (12 marks) **Marks**

(a) Solve $-4x-9<1-2(x+1)$. **2**

(b)(i) Sketch the graph of $y=|2x-3|$ showing all essential features. **1**

(ii) Hence, or otherwise, solve $|2x-3|\geq 3$. **2**

(c) Solve $x^2+4x-1=0$ by completing the square, giving exact answers in simplest surd form. **2**

(d) Sketch the region $x^2+y^2<4$ and $y\leq -x$. **2**

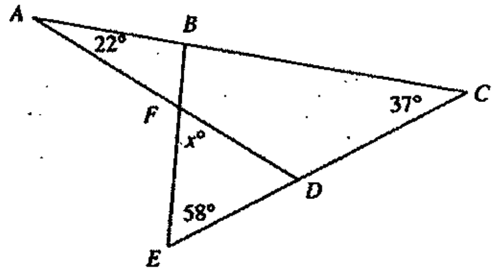
(e) Find the solutions of the following simultaneous equations. **3**

$$\begin{aligned} x+y&=5\\ 2x+y+z&=4\\ x-y-z&=5 \end{aligned}$$

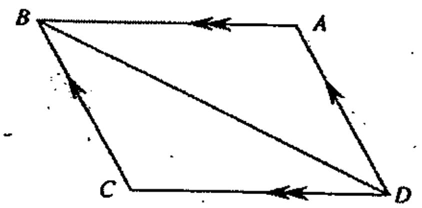
Start each question on a new page

QUESTION 3 (12 marks) **Marks**

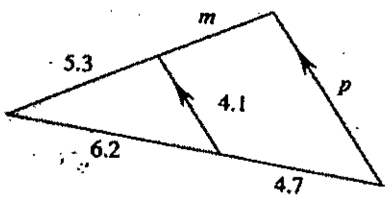
(a) Find the value of x, giving reasons. **2**



(b) Prove that triangles ABD and CDB are congruent. Hence prove that AD=BC. **4**



(c) Find the values of m and p in the diagram below, correct to one decimal place. Give reasons for your answers. **4**



(d) (i) Find the sum of the interior angles of a regular 12-sided polygon. **1**

(ii) How large is each exterior angle of a regular 15-sided polygon? **1**

Start each question on a new page

QUESTION 4 (12 marks) **Marks**

(a) A function is defined by the rule $f(x)=\begin{cases} x+1, & x\geq 1 \\ -1, & -1<x<1 \\ 1-x, & x\leq -2 \end{cases}$.

Find the value of ;

(i) $f(-3)$ **1**

(ii) $f(m+2)$ in terms of m, given that $m+2\leq -2$. **1**

(b) Show that $f(x)=x^3-x$ is an odd function. **1**

(c) Sketch the function $y=\sqrt{25-x^2}$ stating its domain and range. **3**

(d) Sketch the graph of $y=3^x+1$ stating its domain and range. **3**

(e) Find $\lim\limits_{x\to 5}\dfrac{x^3-125}{x-5}$. **3**

Start each question on a new page

QUESTION 5 (12 marks) **Marks**

(a) Find the value of y if $\sec 39^0 = \operatorname{cosec}(90-y)^0$. 1

(b) Find the exact value of ; (i) $\tan 210^0$ 1

(ii) $\cos(-45^0)$ 1

(c) Solve $2\sin 2\theta - 1 = 0$ for $0^0 \le \theta \le 360^0$. 4

(d) Prove that $\dfrac{1-\cos\theta}{\sin^2\theta} = \dfrac{1}{1+\cos\theta}$ 2

(e) A plane flies from Orange for 1800 km on a bearing of 300^0. 3

It then turns and flies 2500 km on a bearing of 205^0.

How far is the plane from Orange, to the nearest km?

Start each question on a new page

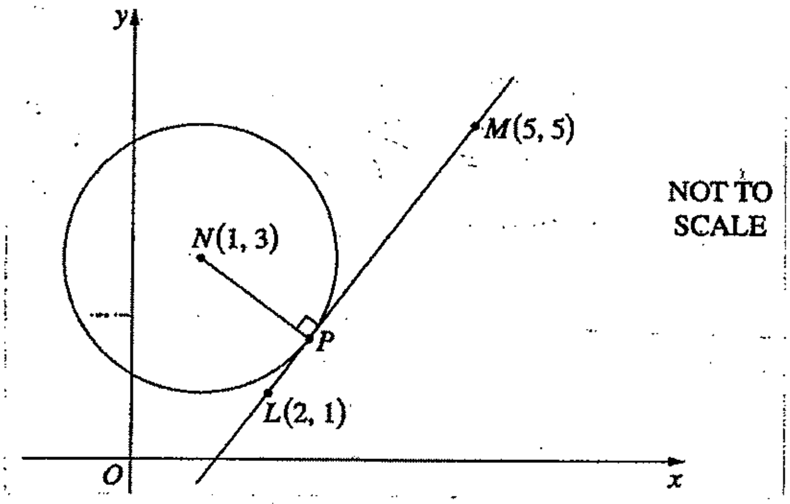
QUESTION 6 (12 marks) **Marks**

(a) If the distance between $(a, 3)$ and $(4, 2)$ is $\sqrt{37}$, find the values of a. **2**

(b) A circle with centre (-2, 5) has one end of a diameter at (4, -3). Find the co-ordinates of the other end of the diameter. **2**

(c) Find the **exact** gradient of the line that makes an angle with the x-axis in the positive direction of 120^0. **2**

(d) Consider the diagram below.



The circle in the diagram has centre N. The line LM is tangent to the circle at P.

(i) Find the gradient of ML. **1**

(ii) Find the equation of LM in the form $ax + by + c = 0$. **2**

(iii) Find the distance NP. **2**

(iv) Find the equation of the circle. **1**

Start each question on a new page

QUESTION 7 (12 marks) **Marks**

(a) Differentiate with respect to x:

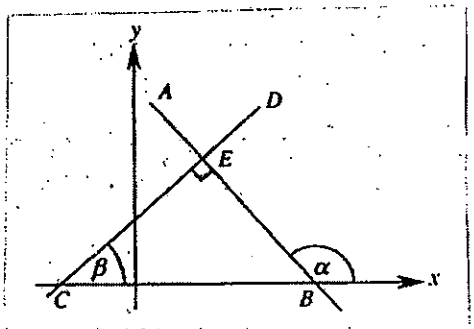
(i) $5x^2+7$ 1

(ii) $6x^3\sqrt{2x-1}$ 2

(iii) $\dfrac{x^2+4}{x^2-5}$ 2

(b) Differentiate from first principles to find the gradient of the tangent at any point on the curve $f(x)=4x^3-11x$. 3

(c) Consider the diagram below.



Let $m_1=\tan\alpha$ and $m_2=\tan\beta$.

(i) Write down an expression for $\tan\beta$ in terms of EB and EC. 1

(ii) Show that $\cot(180^0-\alpha)=\tan\beta$. Hence show that $m_1\times m_2=-1$. 3

End of examination

TRINITY GRAMMAR SCHOOL
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(EXTENSION 1)

HALF-YEARLY EXAMINATION

SOLUTIONS

SOLUTIONS

marked by Kesby

① (a) $225.7119542\ldots$ (calc) ①

$= 226$ (3 s.f.) ①

① (b) Let $x = 3.2575757\ldots$

$10x = 32.575757\ldots$

$1000x = 3257.575757\ldots$ ①

$1000x - 10x = 3225$

$990x = 3225$

$x = \frac{3225}{990}$

$x = 3\frac{17}{66}$ ①

① (c) $\frac{1}{\sqrt[3]{(x-2)^2}} = \frac{1}{((x-2)^2)^{1/3}} = \frac{1}{(x-2)^{2/3}} = (x-2)^{-2/3}$ ①

① (d) $9^{2x+1} = 27$

$(3^2)^{2x+1} = 3^3$ ①

Equate powers, $4x+2 = 3$

$4x = 1$

$x = \frac{1}{4}$ ①

① (e) $\frac{4}{x-1} - \frac{2}{x^2-1} = \frac{4}{x-1} - \frac{2}{(x+1)(x-1)}$ ①

$= \frac{4(x+1) - 2}{(x+1)(x-1)}$ ①

$= \frac{4x+4-2}{(x+1)(x-1)}$

$= \frac{4x+2}{(x+1)(x-1)}$. ①

① (f) $\frac{4}{x^2} - \frac{a^2}{b^2} = \left(\frac{2}{x}\right)^2 - \left(\frac{a}{b}\right)^2$ ①

$= \left(\frac{2}{x} + \frac{a}{b}\right)\left(\frac{2}{x} - \frac{a}{b}\right)$ ①

marked by Stathakis

2 (a) $-4x - 9 < 1 - 2(x+1)$

$-4x - 9 < 1 - 2x - 2$

$-2x - 9 < -1$

$-2x < 8$ 1

$x > \frac{8}{-2}$

$\underline{x > -4}$ 1

2 (b)(i) x-intercept: when $y=0$, $2x - 3 = 0$

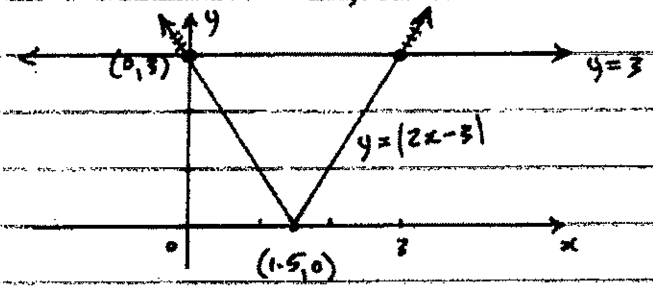
$2x = 3$

$x = 1.5$ $(1.5, 0)$

y-intercept: when $x=0$, $y = |2(0) - 3|$

$y = |-3|$

$y = 3$ $(0, 3)$



1

2(b)(ii) From the graph, $\boxed{x \leq 0 \text{ OR } x \geq 3}$

Alternative method:

Consider $|2x-3| \geq 3$

$2x - 3 \geq 3$ OR $-(2x-3) \geq 3$

$2x \geq 6$ $-2x + 3 \geq 3$

$x \geq 3$ $-2x \geq 0$

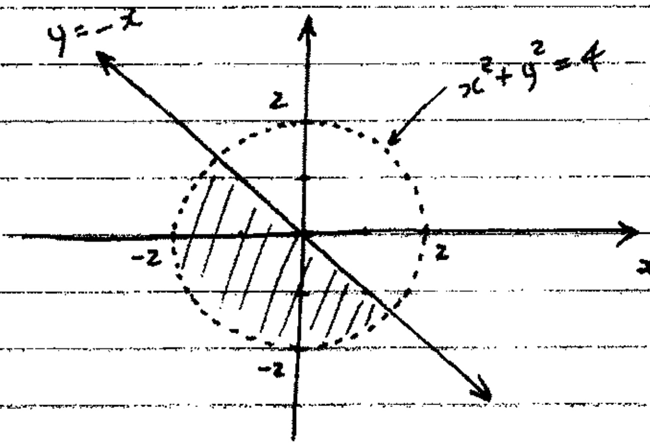
$x \leq 0$

$\therefore$ $x \leq 0$ OR $x \geq 3$

(2) (c)

$$x^2 + 4x - 1 = 0$$
$$x^2 + 4x = 1$$
$\left(\frac{4}{2}\right)^2 = 4$
$$x^2 + 4x + 4 = 1 + 4$$
$$(x+2)^2 = 5 \quad (1)$$
$$x + 2 = \pm\sqrt{5}$$
$$x = \pm\sqrt{5} - 2$$
$$x = \sqrt{5} - 2,\ -\sqrt{5} - 2. \quad (1)$$

(2) (d)



circle (1)

region (1)

$x^2 + y^2 < 4 \Rightarrow$ inside the circle with centre $(0,0)$ & radius 2.

Test $(0,-1)$ in $y \le -x$

$$-1 \le 0$$

True.

(2) (e)

$$x + y = 5 \quad — (1)$$
$$2x + y + z = 4 \quad — (2)$$
$$x - y - z = 5 \quad — (3)$$

(2) + (3): $3x = 9$

$\underline{x = 3}$ (1)

Subst. $x = 3$ into (1): $3 + y = 5$

$\underline{y = 2}$. (1)

Subst. $x = 3$ & $y = 2$ into (2): $2(3) + 2 + z = 4$

$$8 + z = 4$$

$\underline{z = -4}$ (1)

$x = 3$
$y = 2$
$z = -4$

marked by Stathakis

(3) (a) $\angle ADC = 121^\circ$ ($\angle$ sum of ΔACD is 180°) 1 (with reason)

$\angle ADE = 180^\circ - 121^\circ$ (a straight $\angle$ is 180°)

$= 59^\circ$

$x + 58 + 59 = 180$ ($\angle$ sum of ΔDEF is 180°)

$\underline{x = 63}$ 1 (with ALL reasons)

(3) (b) $\angle ABD = \angle CDB$ (alternate $\angle$s, $BA \parallel CD$) (A)

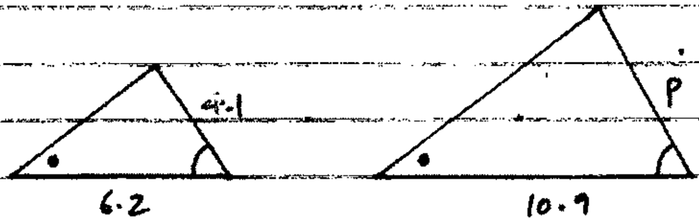
$\angle ADB = \angle CBD$ (alternate $\angle$s, $AD \parallel BC$) (A) 1

BD is common (S) 1

$\therefore \Delta ABD \equiv \Delta CDB$ (AAS) 1

$\therefore AD = BC$ (matching sides of congruent Δs) 1

(3) (c)



$\dfrac{p}{4.1} = \dfrac{10.9}{6.2}$ (matching sides in same ratio in similar Δs) 1

$p = \dfrac{4.1 \times 10.9}{6.2}$

$\underline{p = 7.2 \text{ (1d.p.)}}$ 1

$\dfrac{m}{5.3} = \dfrac{4.7}{6.2}$ (ratio of intercepts theorem) 1

$\underline{m = 4.0 \text{ (1d.p.)}}$ 1

(3) (d) (i) Subst. $n = 12$ in $A = (n-2) \times 180^\circ$

$= (12-2) \times 180^\circ$

$= \underline{1800^\circ}$ 1

(ii) $15 \times E = 360^\circ$

$E = 24^\circ$

$\therefore$ each exterior $\angle$ is $\underline{24^\circ}$ 1 marked by Rogerson

4 (a) (i) $f(-3) = 1-(-3)$

$= 4.$ 1

(ii) $f(x) = 1-x$ when $x \le -2$

$\therefore f(m+2) = 1-(m+2)$ when $m+2 \le -2$

$= 1-m-2$

$= -m-1.$ 1

(b) Show $f(-x) = -f(x)$.

$f(-x) = (-x)^3-(-x)$

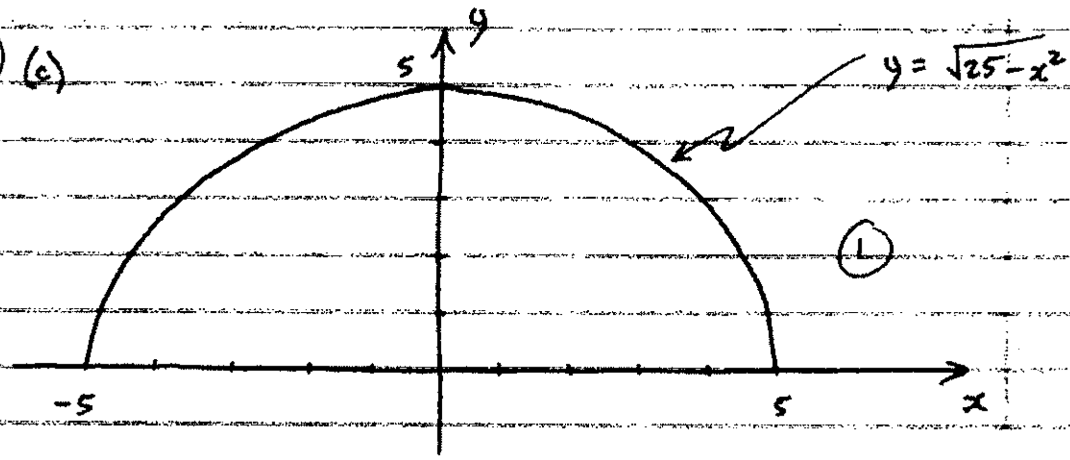
$= -x^3+x$

$= -(x^3-x)$

$= -f(x)$

$\therefore$ $f(x)$ is an odd function. 1

4 (c)

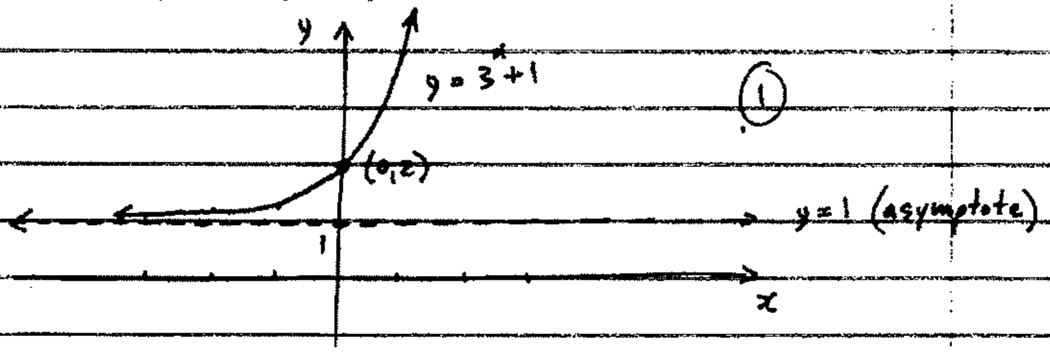


domain: $-5 \le x \le 5$ 1

range: $0 \le y \le 5$ 1

(4) (d) $y = 3^x + 1$

x	-3	-2	-1	0	1	2	3
y	$1\frac{1}{27}$	$1\frac{1}{9}$	$1\frac{1}{3}$	2	4	10	28



1

domain: all real x 1

range: $y > 1$ 1

(4) (e) $$\lim_{x \to 5} \frac{x^3 - 125}{x - 5} = \lim_{x \to 5} \frac{\cancel{(x-5)}(x^2 + 5x + 25)}{\cancel{(x-5)}} \quad 1$$

$$= \lim_{x \to 5} (x^2 + 5x + 25) \quad 1$$

$$= (5)^2 + 5(5) + 25$$

$$= \underline{75} \quad 1$$

marked by Rogerson

5 (a) $y = 39$ (1)

(b) (i) $\tan 210^\circ = \tan(180^\circ + 30^\circ)$

$= \tan 30^\circ$

$= \frac{1}{\sqrt{3}}$. (1)

(ii) $\cos(-45^\circ) = \cos 45^\circ$

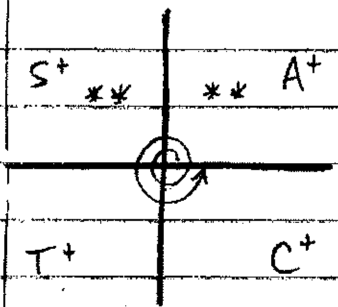
$= \frac{1}{\sqrt{2}}$ (1)

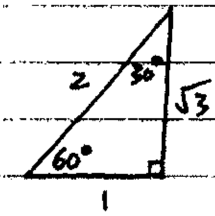
(c) $2\sin 2\theta - 1 = 0$ $\qquad 0^\circ \le \theta \le 360^\circ$

$2\sin 2\theta = 1$ $\qquad 0^\circ \le 2\theta \le 720^\circ$ (1)

$\sin 2\theta = \frac{1}{2}$

$\sin 30^\circ = \frac{1}{2}$





(1)

$2\theta = 30^\circ,\ 180^\circ - 30^\circ,\ 360^\circ + 30^\circ,\ 360^\circ + 150^\circ$

$2\theta = 30^\circ,\ 150^\circ,\ 390^\circ,\ 510^\circ$ (1)

$\underline{\theta = 15^\circ,\ 75^\circ,\ 195^\circ,\ 255^\circ}$. (1)

5 (d) $LHS = \frac{1 - \cos\theta}{\sin^2\theta}$ $\qquad \sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

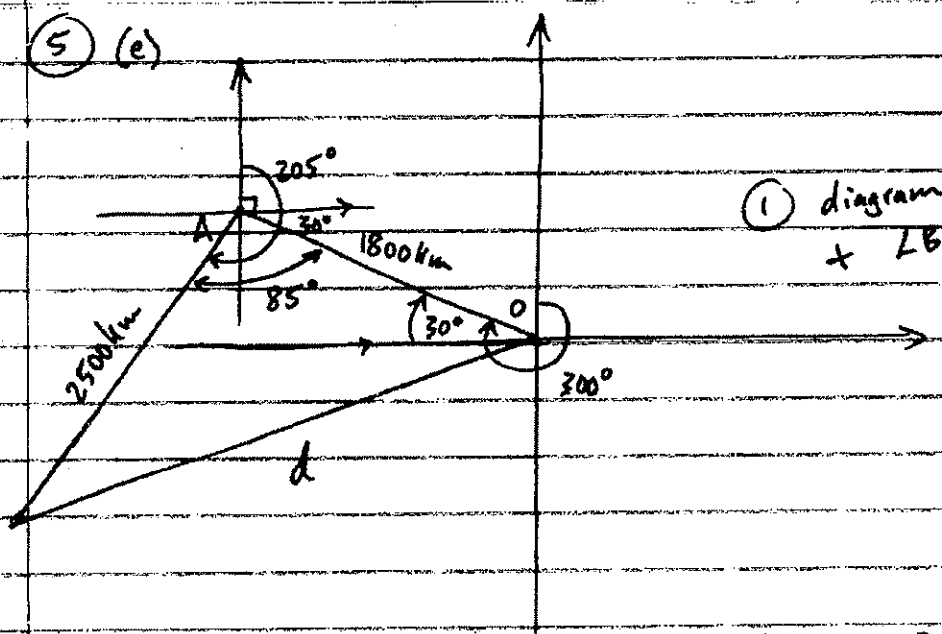
$= \frac{1 - \cos\theta}{1 - \cos^2\theta}$ (1)

$= \frac{\cancel{1 - \cos\theta}}{\cancel{(1 - \cos\theta)}(1 + \cos\theta)}$

$= \frac{1}{1 + \cos\theta}$ (1)

$= RHS$.

5 (e)



1 diagram + $\angle BAO = 85°$

Using the Cosine Rule, $d^2 = ((1800)^2 + (2500)^2 - 2(1800)(2500)\cos 85°$ 1

$d^2 = 8705598.315$

$d = 2950.525091...$

$d \doteqdot 2951$ km 1 (nearest km)

marked by Geddes

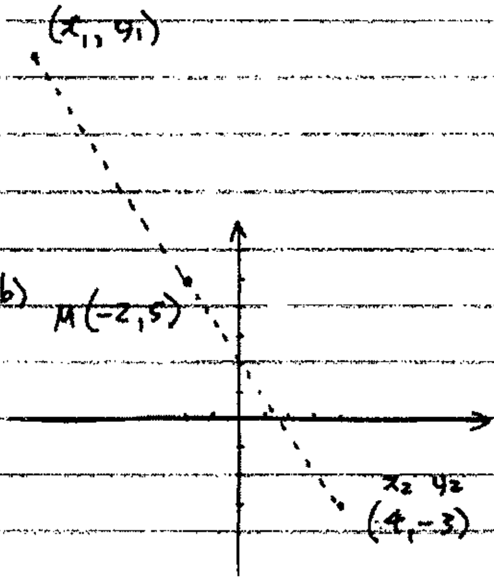
6 (a) $\overset{x_1\ y_1}{(a, 3)}$ $\overset{x_2\ y_2}{(4, 2)}$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(4 - a)^2 + (2 - 3)^2} = \sqrt{37} \quad (1)$$

$$(4 - a)^2 + (-1)^2 = 37$$
$$(4 - a)^2 + 1 = 37$$
$$(4 - a)^2 = 36$$
$$4 - a = \pm\sqrt{36}$$
$$4 - a = \pm 6$$
$$4 = \pm 6 + a$$
$$4 \pm 6 = a$$
$$\underline{a = 10, -2} \quad (1)$$

6 (b)



$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = M(-2, 5)$$

$$\therefore M\left(\frac{x_1 + 4}{2}, \frac{y_1 + (-3)}{2}\right) = M(-2, 5)$$

$$M\left(\frac{x_1 + 4}{2}, \frac{y_1 - 3}{2}\right) = M(-2, 5) \quad (1)$$

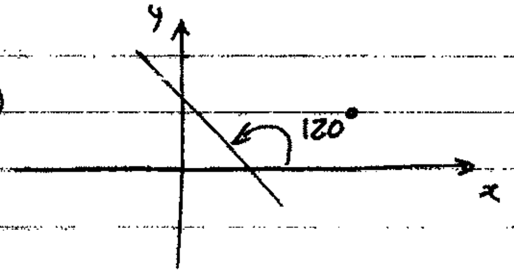
$$\therefore \frac{x_1 + 4}{2} = -2 \qquad \frac{y_1 - 3}{2} = 5$$

$$x_1 + 4 = -4 \qquad y_1 - 3 = 10$$

$$x_1 = -8 \qquad y_1 = 13$$

$\therefore$ other end of diameter has co-ordinates $(-8, 13)$. (1)

6 (c)



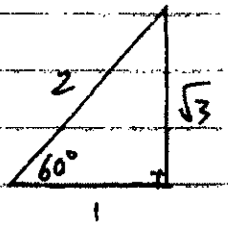
$m = \tan\theta$

$m = \tan 120°$ 1

$m = \tan(180° - 60°)$

$m = -\tan 60°$

$\underline{m = -\sqrt{3}}$ 1



$\tan 60° = \sqrt{3}$.

6 (d) (i) $\overset{x_1\ y_1}{M(5,5)}$ $\overset{x_2\ y_2}{L(2,1)}$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{1-5}{2-5}$$

$$m = \frac{-4}{-3}$$

$$m = \frac{4}{3} \quad 1$$

(ii) $\overset{x_1\ y_1}{L(2,1)}$ $\overset{x_2\ y_2}{M(5,5)}$

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{y-1}{x-2} = \frac{5-1}{5-2}$$

$$\frac{y-1}{x-2} = \frac{4}{3} \quad 1$$

$$3(y-1) = 4(x-2)$$

$$3y - 3 = 4x - 8$$

$$0 = 4x - 3y - 5$$

ie. $\boxed{4x - 3y - 5 = 0}$ 1

6 (d) (iii)

$$d_{NP} = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$4x - 3y - 5 = 0$

$a = 4$
$b = -3$
$c = -5$

$N(\overset{x_1}{1}, \overset{y_1}{3})$

$$d_{NP} = \frac{|4(1) + (-3)(3) + (-5)|}{\sqrt{(4)^2 + (-3)^2}} \quad (1)$$

$$d_{NP} = \frac{|4 - 9 - 5|}{\sqrt{16 + 9}}$$

$$d_{NP} = \frac{|-10|}{\sqrt{25}}$$

$$d_{NP} = \frac{10}{5}$$

$d_{NP} = 2$ units. (1)

6 (d) (iv) Centre: $N(\overset{x_1}{1}, \overset{y_1}{3})$ radius = NP = 2 units.

$$(x - a)^2 + (y - b)^2 = r^2 \quad \left\} \begin{array}{l} \text{Centre } (a, b) \\ \text{radius} = r. \end{array} \right.$$

$$\therefore \boxed{(x - 1)^2 + (y - 3)^2 = 4} \quad (1)$$

marked by Geddes

7 (a) (i) $\frac{d}{dx}\left(5x^2+7\right) = 10x.$ 1

(ii) $\frac{d}{dx}\left(6x^3\sqrt{2x-1}\right) = \frac{d}{dx}\left(6x^3(2x-1)^{\frac{1}{2}}\right)$

$u = 6x^3 \quad v = (2x-1)^{\frac{1}{2}}$

$u' = 18x^2$ 1 $\quad v' = \frac{1}{2}(2x-1)^{-\frac{1}{2}} \times 2$

$= (2x-1)^{-\frac{1}{2}}$

$= u'v + uv'$

$= 18x^2(2x-1)^{\frac{1}{2}} + 6x^3(2x-1)^{-\frac{1}{2}}$

$= 18x^2.\sqrt{2x-1} + \frac{6x^3}{\sqrt{2x-1}}$ 1

(iii) $\frac{d}{dx}\left(\frac{x^2+4}{x^2-5}\right) = \frac{u'v - uv'}{v^2}.$

$u = x^2+4 \quad v = x^2-5$

$u' = 2x \quad v' = 2x$

$= \frac{2x(x^2-5) - (x^2+4).2x}{(x^2-5)^2}$ 1

$= \frac{\cancel{2x^3} - 10x - \cancel{2x^3} - 8x}{(x^2-5)^2}$

$= \frac{-18x}{(x^2-5)^2}$ 1

7 (b) $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\left(4(x+h)^3 - 11(x+h)\right) - \left(4x^3 - 11x\right)}{h} \quad 1$$

$$= \lim_{h \to 0} \frac{\left(4(x^3 + 3x^2h + 3xh^2 + h^3\right) - 11x - 11h - 4x^3 + 11x}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{4x^3} + 12x^2h + 12xh^2 + 4h^3 \cancel{- 11x} - 11h \cancel{- 4x^3} \cancel{+ 11x}}{h} \quad 1$$

$$= \lim_{h \to 0} \frac{12x^2h + 12xh^2 + 4h^3 - 11h}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}\left(12x^2 + 12xh + 4h^2 - 11\right)}{\cancel{h}}$$

$$= \lim_{h \to 0} 12x^2 + 12xh + 4h^2 - 11$$

$$= 12x^2 - 11. \quad 1$$

7 (c) (i) $\tan\beta = \dfrac{EB}{EC}$ (1)

(ii) $\angle EBC = 180^\circ - \alpha$

Now, $\tan(180^\circ - \alpha) = \dfrac{EC}{EB}$

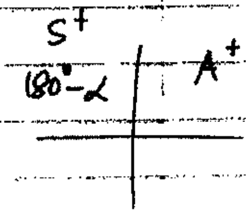
$\therefore \cot(180^\circ - \alpha) = \dfrac{1}{\tan(180^\circ - \alpha)}$

$= \dfrac{EB}{EC}$ (1)

from (i).

$= \tan\beta$

Now, $\tan\beta = \cot(180^\circ - \alpha)$



$= \dfrac{1}{\tan(180^\circ - \alpha)}$

$= \dfrac{1}{-\tan\alpha}$ (1) (tan is negative in 2nd quadrant)

$\therefore \tan\beta \times -\tan\alpha = 1$

$\tan\beta \times \tan\alpha = -1$

$\tan\alpha \times \tan\beta = -1$

$\therefore m_1 \times m_2 = -1$ (1)