TRINITY GRAMMAR SCHOOL

Mathematics Department

2012

HALF YEARLY EXAMINATION

ASSESSMENT TASK 3

Year 11

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- Show all necessary working in Questions 11 – 14
- Write your Student Name **and** Class Teacher on the writing booklet(s) **or** sheet(s) submitted
- Weighting 30%

Total marks – 70

Section I Pages 1 - 2

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II Pages 3 - 6

60 marks

- Attempt Questions 11 – 14
- Allow about 1 hour 45 minutes for this section

Section I **10 marks**

Shade the correct response on the answer sheet provided
Each question is worth 1 mark

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1. What is the product of $(6\times10^3)(4.6\times10^5)$ *and* (2×10^{-2}) expressed in scientific notation?

A. 55.2×10^6 B. 5.52×10^7 C. 55.2×10^7 D. 5.52×10^{10}

2. The two lines represented by the equations below are graphed on a coordinate plane.

$$x+6y=12$$
$$3(x-2)=-y-4$$

Which statement best describes the two lines?

A. The lines are parallel.
B. The lines are the same.
C. The lines are perpendicular.
D. The lines intersect at an angle other than 90 degrees.

3. The expression $\dfrac{\sin^2\theta+\cos^2\theta}{1-\sin^2\theta}$ is equivalent to

A. $\cos^2\theta$ B. $\sin^2\theta$ C. $\sec^2\theta$ D. $\text{cosec}^2\theta$

4. When $x^{-1}-1$ is divided by $x-1$, the solution is:

A. -1 B. $-\dfrac{1}{x}$ C. $\dfrac{1}{x^2}$ D. $\dfrac{1}{(x-1)^2}$

5. Which ordered pair is in the solution set of the following system of linear equations?

$$y<2x+2$$
$$y\geq -x-1$$

A. $(0,3)$ B. $(2,0)$ C. $(-1,0)$ D. $(-1,-4)$

6. The solution set of the inequality $x^2 - 3x > 10$ is

A. $-2 < x < 5$
B. $0 < x < 3$
C. $x < -5 \text{ or } x > 2$
D. $x < -2 \text{ or } x > 5$

7. If $\frac{ey}{n} + k = t$, rewrite y as the subject of the equation.

A. $y = \frac{tn + k}{e}$ B. $y = \frac{tn - k}{e}$ C. $y = \frac{n(t + k)}{e}$ D. $y = \frac{n(t - k)}{e}$

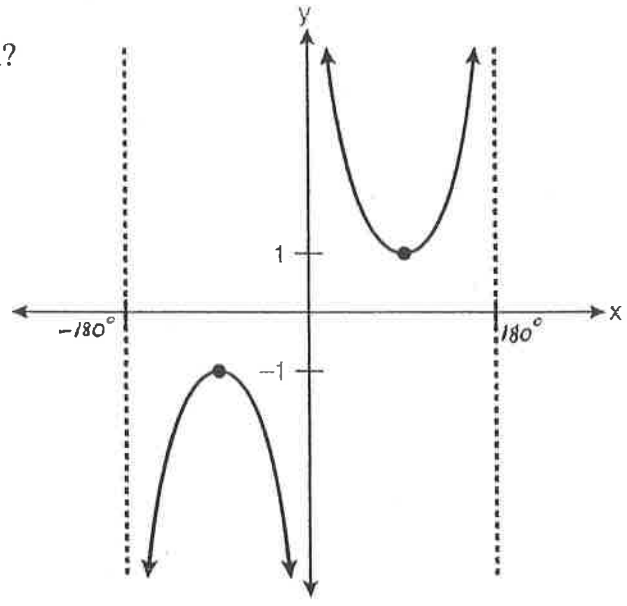
8. The solution set of the equation $\sqrt{x+3} = 3 - x$ is

A. $\{1\}$ B. $\{0\}$ C. $\{1, 6\}$ D. $\{2, 3\}$

9. If $\Delta JKL \equiv \Delta MNO$, which statement is always true?

A. $\angle KLJ = \angle NMO$
B. $\angle KJL = \angle MON$
C. $JL = MO$
D. $JK = ON$

10. Which equation is sketched in the diagram?



A. $y = \sec x$
B. $y = \text{cosec}\, x$
C. $y = \tan x$
D. $y = \cot x$

End of Section I

Section II **60 marks**

Begin each question on a new answer sheet
Show all necessary working
Each question is worth 15 marks

Question 11 **(15 marks)** **Begin a NEW answer booklet or answer sheet**

(a) Simplify $\left(-2\sqrt{3}\right)^3 \times \left(\sqrt{6}\right)$ **1**

(b) Find the exact value of $\cos\left(-210^\circ\right)$. **1**

(c) Find $\lim\limits_{x \to a} \dfrac{x^3 - a^3}{x^2 - a^2}$ **2**

(d) Solve $\dfrac{t+3}{5} - \dfrac{t-4}{3} = 2$ **2**

(e) i) State the domain of the function $y = \sqrt{9 - x^2}$. **1**
ii) State the range of the function $y = 4 - |x|$. **1**

(f) Solve $4^{2x-5} = 8^{3x}$ **2**

(g) Show that $(k-1)x^2 + 2kx + k + 1 = 0$ has real roots for all values of k. **2**

(h) If $x = \sqrt{2} - 1$, simplify $x^2 - \dfrac{1}{x^2}$ giving your answer with a rational denominator. **3**

Question 12 **(15 marks)** **Begin a NEW answer booklet or answer sheet**

(a) Sketch the graph of $y=(x+2)^2-4$ showing the main features including the x and y intercepts. **2**

(b) The points A (1, 2), B (5, 4) and C (-3, 6) are located on a number plane.

(i) What angle does the line AB make with the x - axis? **2**

(ii) Find the equation of the line parallel to AB passing through the point C. **2**

(iii) Find the perpendicular distance of the point C to the line AB. **2**

(iv) Find the area of triangle ABC **2**

(v) Find the circumference of the circle having BC as its diameter, correct to two decimal places. **2**

(c) Find the solutions of the following set of simultaneous equations.

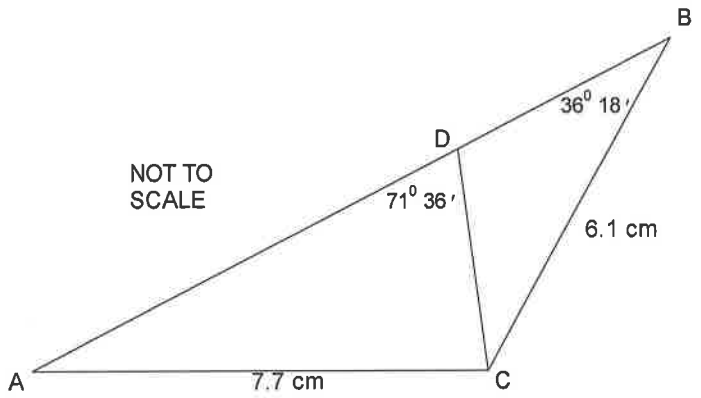
$$5=y-x$$
$$4x^2=4+y-17x$$

3

Question 13 **(15 marks)** **Begin a NEW answer booklet or answer sheet**

(a) Simplify $\dfrac{\operatorname{cosec}\theta}{\cos\theta+\sin\theta\tan\theta}$ **1**

(b) For the diagram below,



(i) Find the length of CD. **2**

(ii) Find the length of AB. **2**

(c) Solve: $\sqrt{3}\tan 2\theta-1=0$ for $0^\circ\le\theta\le 360^\circ$ **3**

(d) If α and β are the roots of $3x^2-2x+5=0$, find the value of $\alpha^2+\beta^2$. **2**

(e) In the quadratic equation $x^2+3x+2p-1=0$, one root is triple another. Find the value of p. **2**

(f) Solve $\left(x+\dfrac{2}{x}\right)^2+4\left(x+\dfrac{2}{x}\right)+3=0$ giving answer in exact form. **3**

Question 14 **(15 marks)** **Begin a NEW answer booklet or answer sheet**

(a) Differentiate $f(x)=3x^2-4x+2$ from first principles. **2**

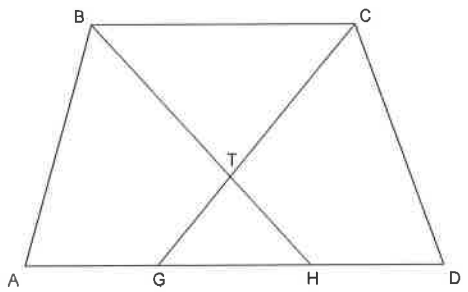
(b) Differentiate $y=\sqrt{8x}-\frac{1}{x}$ with respect to x. **2**

(c) Differentiate $y=3x^2(2x+7)^3$ with respect to x. **2**

(d) Find the equation of the tangent to the curve $y=\frac{x^2}{2x+1}$ at the point where $x=-2$. **3**

(e) Sketch the function $f(x)=\begin{cases}-4, & if\ x<0\\ x^2-4, & if\ 0\le x\le 2\\ x-2, & if\ x>2\end{cases}$ **3**

(f)



In the diagram, $\angle ABH=\angle DCG$, $BT=CT$, $TG=TH$.

Copy the diagram on to your answer sheet.

Prove: $AH=DG$ **3**

End of Section II

End of Examination

TRINITY GRAMMAR SCHOOL
2012, Year 11 Mathematics Extension 1
Half Yearly Examination
SECTION I
ANSWER SHEET

SOLUTIONS

Name: ..... **Class Teacher:** .....

Be sure to write your answers for Section I on this answer sheet. After you have selected an answer, **CIRCLE** the correct answer. To change an answer, erase your previous mark completely, and then record your new answer. Mark only one answer for each question.

Q1.	A	(B)	C	D
Q2.	A	B	C	(D)
Q3.	A	B	(C)	D
Q4.	A	(B)	C	D
Q5.	A	(B)	C	D
Q6.	A	B	C	(D)
Q7.	A	B	C	(D)
Q8.	(A)	B	C	D
Q9.	A	B	(C)	D
Q10.	A	(B)	C	D

TGS Year 11 Half-Yearly Exam April 2012

Q 11. a) $-8 \times 3\sqrt{3} \times \sqrt{6}$

$= -24\sqrt{18}$

$= -72\sqrt{2}$ (1)

b) $\cos(-210^\circ) = \cos 150^\circ$

$= -\frac{\sqrt{3}}{2}$ (1)

c) $\lim_{x \to a} \frac{(x-a)(x^2+ax+a^2)}{(x-a)(x+a)}$ (M1)

$= \frac{3a^2}{2a}$

$= \frac{3a}{2}$ (1)

d) $3t + 9 - 5t + 20 = 30$ (M1)

$-2t = 1$

$t = -\frac{1}{2}$ (1)

e) i) D: $-3 \leq x \leq 3$ (1)

ii) R: $y \leq 4$ (1)

f) $2^{4x-10} = (2^3)^{3x}$

$4x - 10 = 9x$ (M1)

$-10 = 5x$

$x = -2$ (1)

g) $4k^2 - 4(k-1)(k+1) \geq 0$ (M1)

$4k^2 - 4(k^2-1) \geq 0$

$4k^2 - 4k^2 + 4 \geq 0$

$4 \geq 0$ (1)

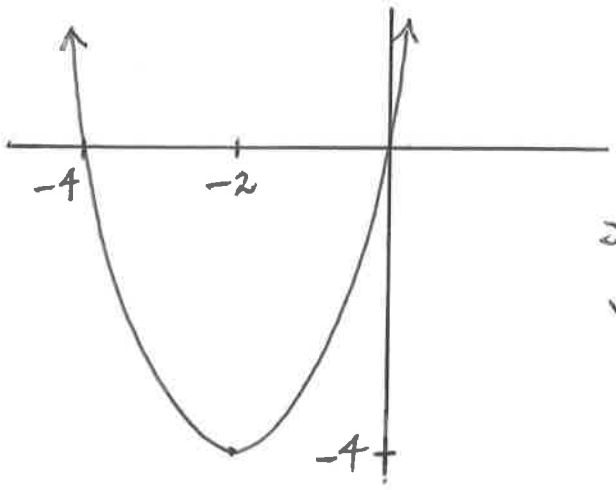
since $\Delta \geq 0$ the roots are real.

h) $3 - 2\sqrt{2} - \frac{1}{(3-2\sqrt{2})} \times \frac{(3+2\sqrt{2})}{(3+2\sqrt{2})}$ (M1)

$= 3 - 2\sqrt{2} - \frac{(3+2\sqrt{2})}{9-8}$ (M1)

$= -4\sqrt{2}$ (1)

Q12. a)



x-intercepts – (1)
Vertex – (1)

b) i) $m_{AB} = \frac{2}{4} = \frac{1}{2}$ (1)

$\theta = \tan^{-1}\left(\frac{1}{2}\right)$, (1)
$= 26° 34'$

ii) $y - 6 = \frac{1}{2}(x+3)$ (M1)
$2y - 12 = x + 3$
$x - 2y + 15 = 0$ (1)

iii) Line AB: $y - 2 = \frac{1}{2}(x-1)$
$2y - 4 = x - 1$ (M1)
$x - 2y + 3 = 0$

$d = \frac{|-3-2(6)+3|}{\sqrt{1+4}} = \frac{12}{\sqrt{5}}$ (1)

iv) $AB = \sqrt{4^2 + 2^2}$
$= \sqrt{20}$ (M1)

$\text{Area} = \frac{1}{2} \times 2\sqrt{5} \times \frac{12}{\sqrt{5}}$
$= 12\ u^2$ (1)

v) $BC = \sqrt{2^2 + (-8)^2}$
$= \sqrt{68}$ (M1)

$C = \pi \times \sqrt{68}$
$= 25.91$ units (1)

c) $y = x + 5$
$4x^2 = 4 + (x+5) - 17x$
$4x^2 + 16x - 9 = 0$
$(2x-1)(2x+9) = 0$ (M1)

$x = \frac{1}{2}$, $y = 5\frac{1}{2}$ (1)

$x = -\frac{9}{2}$, $y = \frac{1}{2}$ (1)

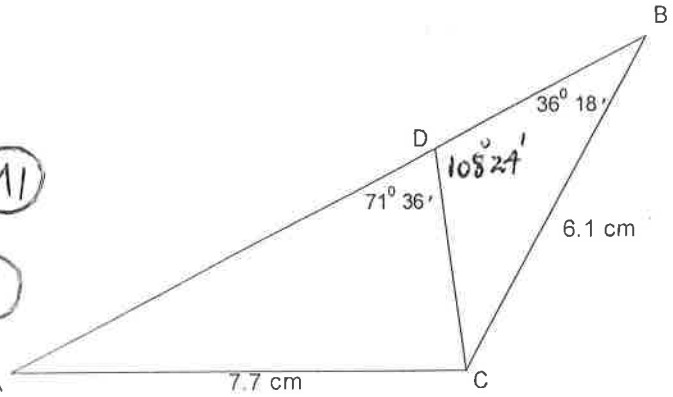
Q13. a) $$\frac{\frac{1}{\sin\theta}}{\cos\theta + \frac{\sin^2\theta}{\cos\theta}} = \frac{\frac{1}{\sin\theta}}{\frac{1}{\cos\theta}}$$

$$= \cot\theta \quad (1)$$

b) i) $$\frac{CD}{\sin 36°18'} = \frac{6.1}{\sin 108°24'}$$

$$CD = \frac{6.1 \times \sin 36°18'}{\sin 108°24'} \quad (M1)$$

$$= 3.805850655 \quad (1)$$



ii) $$\frac{\sin A}{6.1} = \frac{\sin 36°18'}{7.7}$$

$$A = \sin^{-1}\left[\frac{6.1 \times \sin 36°18'}{7.7}\right]$$

$$= 27°58'$$

$$\therefore \angle ACB = 115°44' \quad (M1)$$

$$AB = \sqrt{7.7^2 + 6.1^2 - 2(7.7)(6.1)\cos 115°44'}$$

$$= 11.7 \quad (1)$$

c) $$\tan 2\theta = \frac{1}{\sqrt{3}}$$

$$2\theta = 30°, 210°, 390°, 570° \quad (M1)$$

$$\theta = \underbrace{15°, 105°}_{(1)}, \underbrace{195°, 285°}_{(1)}$$

d) $$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$$

$$= \left(\frac{2}{3}\right)^2 - 2\left(\frac{5}{3}\right) \quad (M1)$$

$$= -\frac{26}{9} \quad (1)$$

e) $$\alpha + 3\alpha = -3$$

$$4\alpha = -3$$

$$\alpha = -\frac{3}{4} \quad (M1)$$

$$3\alpha^2 = 2\beta - 1$$

$$\frac{27}{16} = 2\beta - 1$$

$$\beta = \frac{43}{32} \quad (1)$$

f) $$w = x + \frac{2}{x}$$

$$w^2 + 4w + 3 = 0$$

$$(w+3)(w+1) = 0 \quad (M1)$$

$$x + \frac{2}{x} + 3 = 0 \qquad x + \frac{2}{x} + 1 = 0$$

$$x^2 + 3x + 2 = 0 \qquad x^2 + x + 2 = 0$$

$$(x+2)(x+1) = 0 \qquad \Delta = 1 - 4(1)(2)$$

$$x = -2 \quad x = -1 \quad (1) \qquad = -7$$

(1) No real roots

Q14. a) $f(x+h) = 3(x^2+2xh+h^2) - 4(x+h) + 2$

$= 3x^2 + 6xh + 3h^2 - 4x - 4h + 2$

$f'(x) = \lim_{h\to 0} \dfrac{3x^2+6xh+3h^2-4x-4h+2-3x^2+4x-2}{h}$ (M1)

$= \lim_{h\to 0} \dfrac{h(6x+3h-4)}{h}$

$= 6x - 4$ (1)

b) $y = \sqrt{8}\,x^{1/2} - x^{-1}$

$\dfrac{dy}{dx} = \sqrt{8}\cdot\frac{1}{2}x^{-1/2} + x^{-2}$

$= \sqrt{2}\,x^{-1/2} + \dfrac{1}{x^2}$ (1) (1)

c) $y' = (3x^2)\cdot 3(2x+7)^2(2) + (2x+7)^3(6x)$ (M1)

$= 18x^2(2x+7)^2 + 6x(2x+7)^3$

$= 6x(2x+7)^2(3x+2x+7)$

$= 6x(5x+7)(2x+7)^2$ (1)

d) $y = \dfrac{x^2}{2x+1}$ when $x=-2$ $y = \dfrac{4}{-3}$

$y' = \dfrac{(2x+1)(2x)-(x^2)(2)}{(2x+1)^2}$

$= \dfrac{2x^2+2x}{(2x+1)^2}$

$= \dfrac{2x(x+1)}{(2x+1)^2}$ (M1)

when $x=-2$ $y' = \dfrac{(-4)(-1)}{9}$

$= \dfrac{4}{9}$

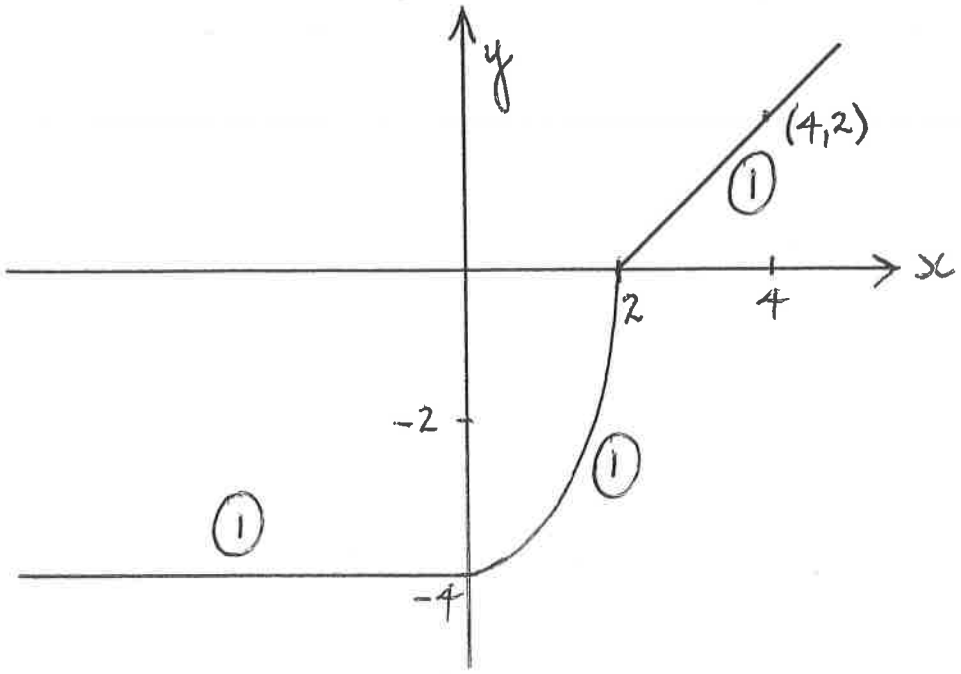
$y + \dfrac{4}{3} = \dfrac{4}{9}(x+2)$ (M1)

$9y + 12 = 4x + 8$

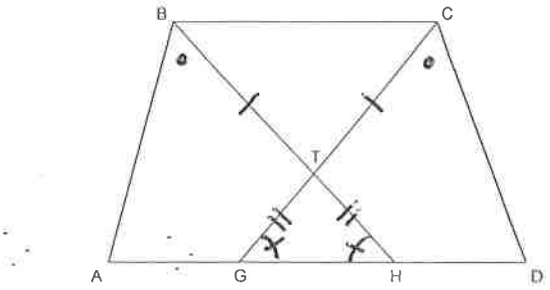
$4x - 9y - 4 = 0$ (1)

Q14

e)



f)



$\angle ABH = \angle DCG$ Given

$BT = CT$ Given

$TG = TH$ Given

$BH = CG$ $BT + TH = CT + TG$

$\angle TGH = \angle THG$ Base $\angle$'s isos. Δ = (1)

$\Delta ABH \equiv \Delta DCG$ A.A.S (1)

$AH = DG$ Matching sides $\equiv \Delta$'s = (1)