DETUR GLORIA SOLI DEO

Trinity Grammar School

Mathematics Department

2015

HALF-YEARLY EXAMINATION

HSC ASSESSMENT TASK 3

Year 12

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Only approved calculators for this course are allowed in this task
- A table of Standard Integrals (if required) in this course is supplied
- Show all necessary working
- Write your Board of Studies Student Number (Year 12 HSC) or Name (Year 11) and your Class teacher on the question paper and on any answer sheets or writing booklets used to write your responses to the questions submitted
- If you do not attempt a question you must submit an answer sheet or writing booklet for that question clearly indicating **N/A** and your Student Number or Name.

- **HSC Assessment Weighting:** 30%

Board of Studies Student Number
(Year 12 only)

Class Teacher:

....

Name:
(Year 11 only)

....

Do NOT write solutions on this question paper. Any working on this question paper will NOT be marked.

Date of examination:

Tuesday, 28 April 2015

Total marks – 70

Section I

10 marks

- Attempt all Questions
- Allow about **15** minutes for this section

Section II

60 marks

- Attempt all Questions
- Allow about **1** hours **45** minutes for this section

The School defines <u>malpractice, or cheating, as</u> "dishonest behaviour by a student that gives them an unfair advantage over others". I certify that my attempt at this Assessment task does not involve any malpractice or cheating.

Student signature   Date

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Section I 10 marks

- Attempt **Questions 1 – 10**
- Allow about 15 minutes for this section
- Shade the correct response on the multiple-choice answer sheet for **Questions 1 – 10**
- Each question is worth **1 mark**

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1 The polynomial $P(x) = x^3 - 4x^2 - 6x + k$ has a factor $x - 2$.

What is the value of k?

(A) 2

(B) 12

(C) 20

(D) 36

2 Which integral is obtained when the substitution $u = 1 + 2x$ is applied to $\int x\sqrt{1+2x}\,dx$?

(A) $\frac{1}{4}\int (u-1)\sqrt{u}\,du$

(B) $\frac{1}{2}\int (u-1)\sqrt{u}\,du$

(C) $\int (u-1)\sqrt{u}\,du$

(D) $2\int (u-1)\sqrt{u}\,du$

3 The angle θ satisfies $\sin\theta = \frac{5}{13}$ and $\frac{\pi}{2} < \theta < \pi$.

What is the value of $\sin 2\theta$?

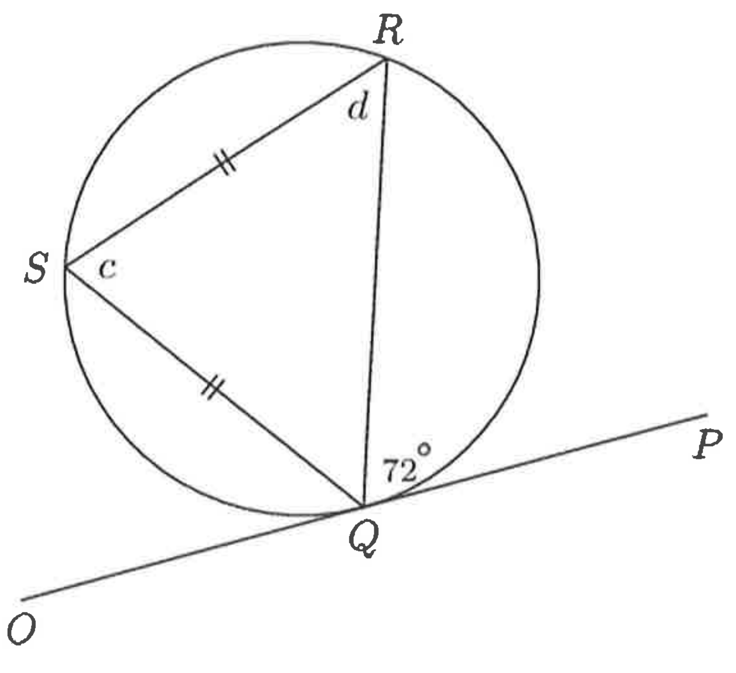
(A) $\frac{10}{13}$

(B) $-\frac{10}{13}$

(C) $\frac{120}{169}$

(D) $-\frac{120}{169}$

4 In the diagram below, Q is the point of contact of the tangent OP. Also, chords QS and SR are equal in length and $\angle PQR = 72^0$.



The values of c and d **respectively** are:

(A) 36^0 *and* 72^0

(B) 36^0 *and* 54^0

(C) 72^0 *and* 36^0

(D) 72^0 *and* 54^0

5 The zeros of the polynomial function $P(x) = x^3 - 5x^2 + 6x$ are …

(A) 3, 2, -3

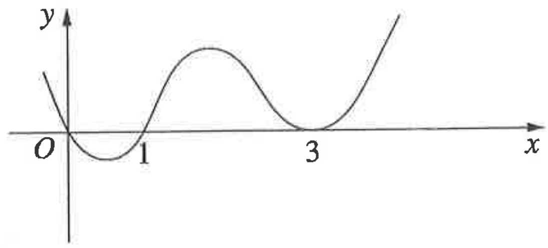
(B) 3, 2

(C) 0, -3, -2

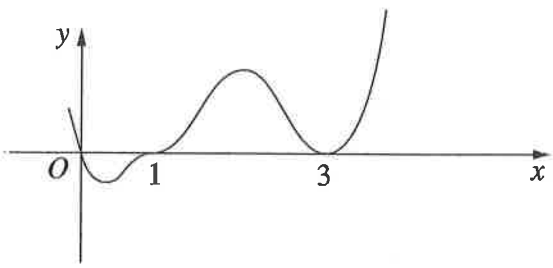
(D) 0, 3, 2

6 Which diagram best represents the graph $y = x\,(1 - x)^3\,(3 - x)^2$?

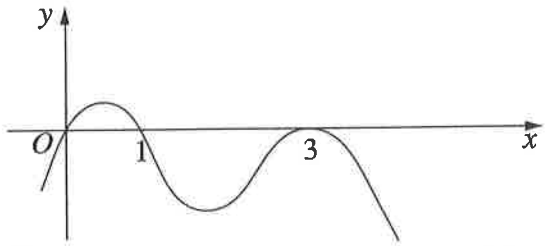
(A)



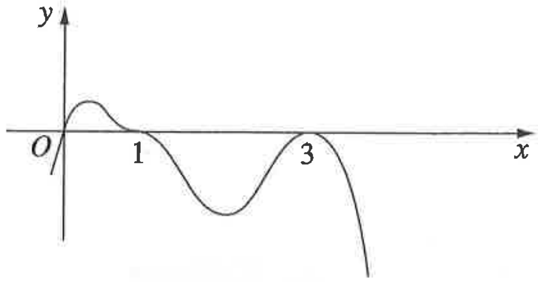
(C)



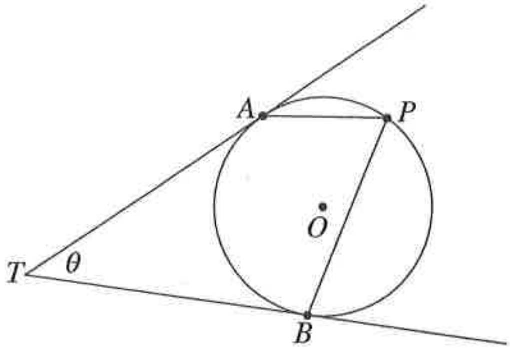
(B)



(D)



7 The points A, B and P lie on a circle centred at O. The tangents to the circle at A and B meet at the point T, and $\angle ATB = \theta$.



What is $\angle APB$ in terms of θ?

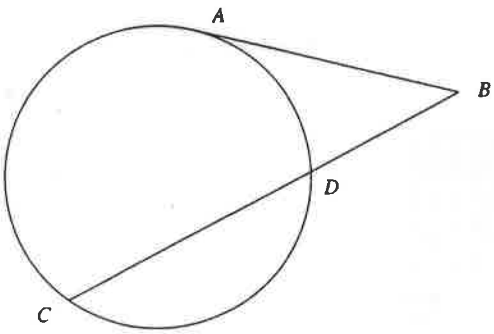
(A) $\dfrac{\theta}{2}$

(B) $90° - \dfrac{\theta}{2}$

(C) θ

(D) $180° - \theta$

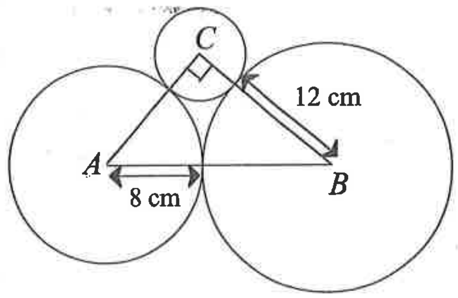
8 In the diagram below, AB is a tangent to circle ACD, and $AB = 7.2$ cm, $BC = 9.3$ cm.



The length of chord CD, correct to one decimal place is …

(A) 5.5 cm
(B) 5.6 cm
(C) 3.7 cm
(D) 3.8 cm

9 Three circles with centres A, B and C touch externally in pairs with $\angle BCA = 90°$, as shown in the diagram. The circles with centres A and B have radii 8cm and 12cm respectively.



The length of the radius of the circle with centre C is ...

(A) 2 cm
(B) 4 cm
(C) 6 cm
(D) 8 cm

10 The complete solution to the inequality $\dfrac{2}{x} \geq x-1$ is

(A) $x \leq 0 \textit{ and } 1 < x \leq 2$

(B) $x \leq -1 \textit{ and } 1 < x < 2$

(C) $x \leq -1 \textit{ and } 0 < x \leq 2$

(D) $x < -1 \textit{ and } 0 < x \leq 2$

End of Section I
Section II commences on the next page

Section II 60 marks

- Attempt **Questions 11-14**
- Allow about 1 hour and 45 minutes for this section
- Answer each question in a SEPARATE writing booklet or answer sheet. Extra writing booklets are available.
- In **Questions 11 – 14**, your responses should include all relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet **Marks**

(a) Solve $\dfrac{2x}{x+3} \geq 1$ **3**

(b) Use the table of standard integrals to find $\int \sec 3x \tan 3x \, dx$. **1**

(c) Evaluate $\displaystyle\lim_{x \to 0} \frac{\sin 5x}{2x}$ **1**

(d) Find the acute angle between the lines $y = 6x - 1$ and $y = \dfrac{5}{7}x + 2$. **2**

(e) Let α, β and γ be the roots of the polynomial equation $x^3 - 3x^2 - 2x + 1 = 0$.

Find; (i) $\alpha\beta + \beta\gamma + \alpha\gamma$ **1**

(ii) $\alpha\beta\gamma$ **1**

(iii) $\dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma}$ **1**

(f) Use the substitution $u = x^2 - 1$ to evaluate $\displaystyle\int_0^1 3x(x^2 - 1)^5 dx$. **3**

(g) *A(-5,6)* and *B(1,3)* are two points. Find the coordinates of the point P which divides the interval externally in the ratio 5:2. **2**

Question 12 **(15 marks)** Use a SEPARATE writing booklet **Marks**

(a) Evaluate: $\sum_{n=0}^{3}\left(\frac{1}{2}\right)^{n} n^{2}$ **1**

(b) The function $g(x) = 20\ln x - x^{2}$ has a zero near $x = 5$. **3**

Use Newton's method with one application to find a better approximation to the zero. Express your answer correct to 4 significant figures.

(c) For what values of x will $1 - \tan^{2} x + \tan^{4} x - \tan^{6} x + \ldots$ have a limiting sum for $0 \le x \le 2\pi$? **4**

(d) Find

(i) $\int \frac{\cos x}{2 + \sin x}\, dx$ **1**

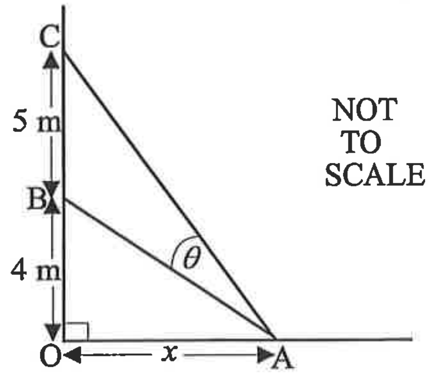
(ii) $\int_{0}^{1} (e^{2x} - e^{-x})^{2}\, dx$ **3**

(e) (i) Prove that $\frac{1}{1+\cos x} = \frac{1}{2}\sec^{2}\frac{x}{2}$ **1**

(ii) Hence evaluate $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{dx}{1+\cos x}$ **2**

Question 13 **(15 marks)** Use a SEPARATE writing booklet **Marks**

(a)



A building is being constructed from pre-cast concrete slabs. A vertical wall panel is being supported by two metal struts which are anchored at the variable point A on the horizontal floor. The distance OB is 4 metres, the distance BC is 5 metres, the distance AO is x metres and the angle BAC is θ radians.

(i) Use the difference formula for tan to show that **2**

$$\tan\theta = \frac{5x}{x^2+36}$$

(ii) Find how far along the horizontal floor the struts should be anchored so that $\tan\theta$ (and hence θ) is a maximum. **3**
Justify your answer.

(b) (i) Express $\sqrt{2}\cos 2x - \sqrt{6}\sin 2x$ in the form $R\cos(2x+\alpha)$ where α is in radians. **2**

(ii) Hence find the solutions, in exact form, to the equation **3**
$\sqrt{2}\cos 2x - \sqrt{6}\sin 2x = \sqrt{2}$ for $0 \le x \le 2\pi$.

(c) Consider the function $f(x) = e^x - x$.

(i) Show that the curve $y = f(x)$ is concave up for all values of x. **1**

(ii) Find the coordinates and nature of the stationary point on the curve $y = f(x)$. **2**

(iii) Hence show that $e^x \ge x+1$ for all values of x. **2**

Question 14 **(15 marks)** Use a SEPARATE writing booklet **Marks**

(a) Use 't' results to prove $\dfrac{1+\sin x-\cos x}{1+\sin x+\cos x}=\tan\dfrac{x}{2}$. **2**

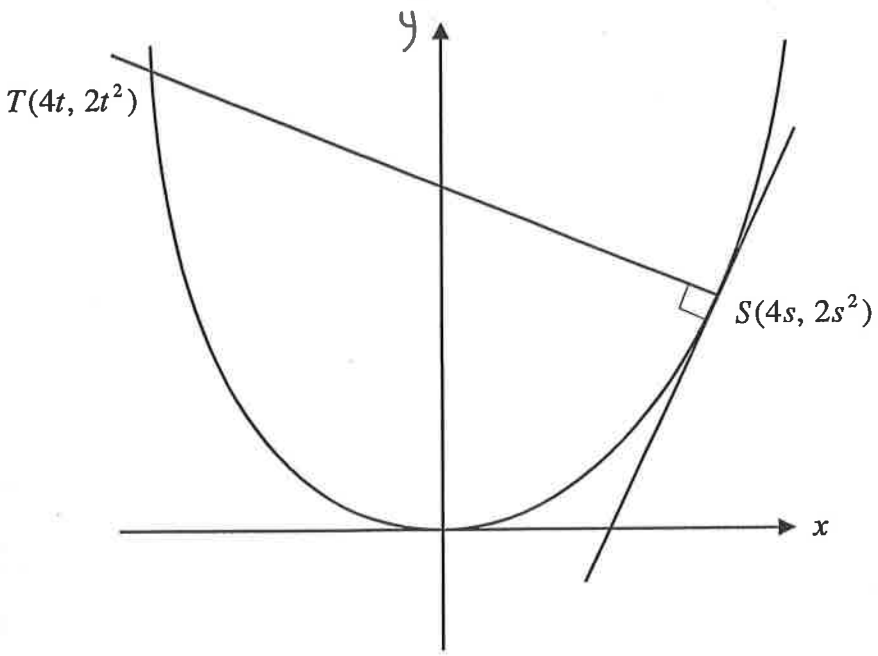
(b) For the polynomial $P(x)=x^3-kx^2-x+2$

(i) Find the value of k if $x-1$ is a factor of $P(x)$. **1**

(ii) Hence factorise $P(x)$ completely. **2**

(c) The roots of the equation $x^3-6x^2+3x+m=0$ are consecutive terms in an arithmetic sequence. Find m. **3**

(d)



The point $S(4s,2s^2)$, $s>0$, lies on the parabola as shown.
The normal at S intersects the parabola at the point $T(4t,2t^2)$, $t<0$.

(i) Find the Cartesian equation of the parabola. **1**

(ii) Show that the chord ST passes through the point $(0,4+2s^2)$. **3**

(iii) Show that $t=-(s+\dfrac{2}{s})$. **2**

(iv) Explain geometrically why $s\neq 0$. **1**

End of Section II
End of Examination

Trinity Grammar School
Mathematics Department
2015
Year 12 Mathematics Extension 1
HALF-YEARLY EXAMINATION

HSC Assessment Task 3
Section I
Multiple Choice Answer Sheet

NSW Board of Studies Student Number
(Year 12 only)

Name: ____
(Year 11 only)

Class Teacher: ______________________________________

- Write your answers for **Section I** on this answer sheet.
- ***Shade completely only ONE answer for each question.***
- To change an answer, erase your previous mark completely and then record your new answer.

1	(A)	(B)	(C)	(D)
2	(A)	(B)	(C)	(D)
3	(A)	(B)	(C)	(D)
4	(A)	(B)	(C)	(D)
5	(A)	(B)	(C)	(D)
6	(A)	(B)	(C)	(D)
7	(A)	(B)	(C)	(D)
8	(A)	(B)	(C)	(D)
9	(A)	(B)	(C)	(D)
10	(A)	(B)	(C)	(D)

A table of Standard Integrals, if required, is printed on the reverse side of this answer sheet

Standard Integrals

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; x \neq 0, \text{if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x \ , \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

Note $\ln x = \log_e x, \quad x > 0$

Year 12 Mathematics Extension 1 - Half Yearly examination solutions (2015)

Section I - solutions

M. Choice

① (C)

$$P(2) = (2)^3 - 4(2)^2 - 6(2) + k = 0$$

$$-20 + k = 0$$

$$k = 20.$$

② (A)

$u = 1 + 2x$

$\frac{du}{dx} = 2$

$du = 2.\,dx$

$\frac{1}{2}\,du = dx$

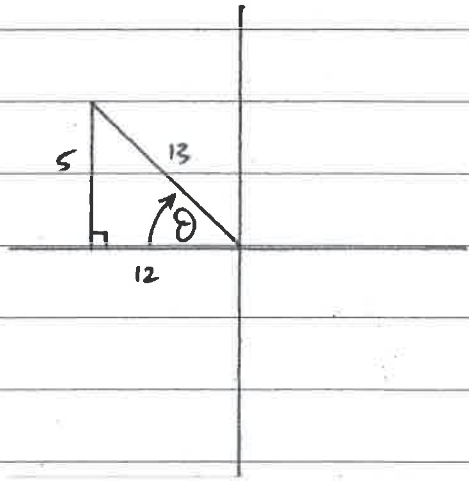
$2x = u - 1$

$x = \frac{1}{2}(u-1)$

$$\int x\sqrt{1+2x}\,.dx = \int \tfrac{1}{2}(u-1).\sqrt{u}\,.\tfrac{1}{2}du$$

$$= \frac{1}{4}\int (u-1).\sqrt{u}\,du.$$

③ (D)



$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$= \frac{2}{1} \times \frac{5}{13} \times \frac{-12}{13}$$

$$= \frac{-120}{169}.$$

④ (D)

Do NOT write anything, or make any marks below this line or outside borders

(5) (D)

$P(x) = x^3 - 5x^2 + 6x$

$P(x) = x(x^2 - 5x + 6)$

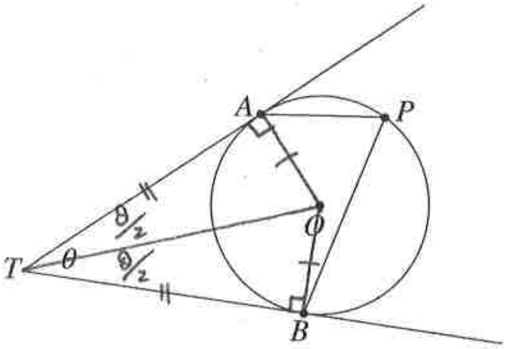
$P(x) = x(x-2)(x-3)$

$\therefore$ when $P(x) = 0$, $x(x-2)(x-3) = 0$

(6) (D)

$x = 0, 2, 3.$

(7) (B)



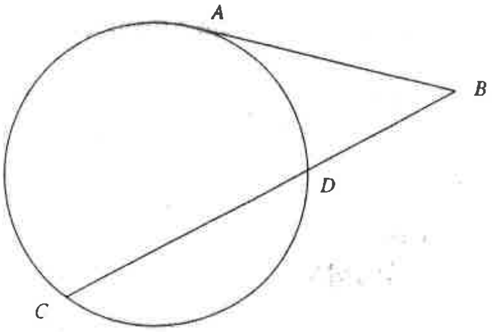
$\angle AOB = 2 \times (90° - \frac{\theta}{2})$

$= 180° - \theta.$

$\therefore \angle APB = \frac{1}{2} \times (180° - \theta)$

$= 90° - \frac{\theta}{2}.$

(8) (C)



$AB^2 = BC \times BD$

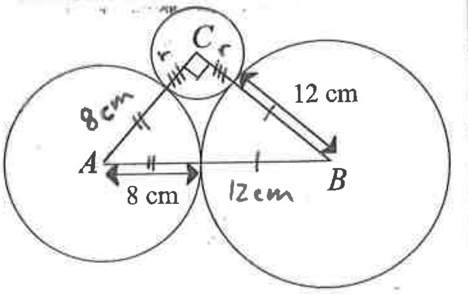
$(7.2)^2 = 9.3 \times BD$

$BD = 5.574193548...$

$CD = 9.3 - 5.57419...$

$\underline{CD = 3.7 \text{ cm}}$ (1d.p.)

(9) (B)



$(r+8)^2 + (r+12)^2 = 20^2$

$r^2 + 16r + 64 + r^2 + 24r + 144 = 400$

$2r^2 + 40r + 208 = 400$

$2r^2 + 40r - 192 = 0$

$r^2 + 20r - 96 = 0$

$(r-4)(r+24) = 0$

(10) (C)

$r = 4, -24$

$\therefore r = 4 \quad (r > 0).$

Do NOT write anything, or make any marks below this line or outside borders

Section II – solutions

11 (a) $(x+3)^{\cancel{2}} \times \dfrac{2x}{\cancel{(x+3)}} \geqslant 1 \times (x+3)^2 \qquad (x \neq -3)$

$$2x(x+3) \geqslant (x+3)^2 \qquad 1$$

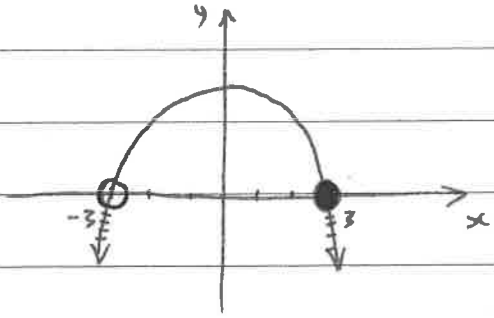
$$0 \geqslant (x+3)^2 - 2x(x+3)$$

$$(x+3)^2 - 2x(x+3) \leqslant 0$$

$$(x+3)\big((x+3) - 2x\big) \leqslant 0$$

$$(x+3)(3-x) \leqslant 0 \qquad 1$$

Consider $y = (x+3)(3-x)$ $\qquad \therefore \boxed{x < -3 \quad \text{or} \quad x \geqslant 3}$ 1



11 (b) $\displaystyle\int \sec 3x . \tan 3x \, dx = \frac{1}{3}\sec 3x + C$ 1

(integral sheet)

11 (c) $\displaystyle\lim_{x\to 0} \frac{\sin 5x}{2x} = \lim_{x \to 0} \frac{\sin 5x}{5x} \times \frac{5}{2}$

$$= \lim_{\theta \to 0} \frac{\sin\theta}{\theta} \times \frac{5}{2}$$

(where $\theta = 5x$)

$$= 1 \times \frac{5}{2}$$

$$= \frac{5}{2}. \qquad 1$$

Do NOT write anything, or make any marks below this line or outside borders

11 (d) Let α be the required angle.

We have $\tan\alpha = \dfrac{m_1 - m_2}{1 + m_1 m_2}$

where $m_1 = 6$ and $m_2 = \dfrac{5}{7}$.

So $$\tan\alpha = \frac{6 - \frac{5}{7}}{1 + 6 \times \frac{5}{7}}$$ **(1 mark)**

$$= 5\frac{2}{7} \div \left(1 + \frac{30}{7}\right)$$

$$= \frac{37}{7} \times \frac{7}{37}$$

$$\tan\alpha = 1$$

So $\alpha = 45^\circ$ **(1 mark)**

11 (e)

(i) $\alpha\beta + \beta\gamma + \alpha\gamma = -2.$ 1

(ii) $\alpha\beta\gamma = -1.$ 1

(iii) $\dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma} = \dfrac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma}$

$= \dfrac{-2}{-1}$

$= 2.$ 1

11 (f)

$$\int_0^1 3x(x^2-1)^5\,dx = \int_{-1}^{0} \frac{3}{2}\frac{du}{dx}u^5\,dx$$

Let $u = x^2 - 1$, so, $\dfrac{du}{dx} = 2x$

Also, since $x = 1$, $u = 0$ and since $x = 0, u = -1$

(1mark) for function **(1 mark)** for terminals (limits)

$$= \frac{3}{2}\int_{-1}^{0} u^5\,du$$

$$= \frac{3}{2}\left[\frac{u^6}{6}\right]_{-1}^{0}$$

$$= -\frac{1}{4}$$ **(1 mark)**

$x_1 \; y_1 \qquad x_2 \; y_2 \qquad m : n$

11 (g) $A(-5,6) \qquad B(1,3) \qquad +5 : -2$

$$P\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

$$= P\left(\frac{5(1) + (-2)(-5)}{5+(-2)}, \frac{5(3) + (-2)(6)}{5+(-2)}\right)$$

$$= P\left(\frac{5+10}{3}, \frac{15-12}{3}\right)$$

1 1

$$= P(5, 1).$$

Do NOT write anything, or make any marks below this line or outside borders

12 (a) $\sum_{n=0}^{3} \left(\frac{1}{2}\right)^n . n^2 = \left(\frac{1}{2}\right)^0 . 0^2 + \left(\frac{1}{2}\right)^1 . 1^2 + \left(\frac{1}{2}\right)^2 . 2^2 + \left(\frac{1}{2}\right)^3 . 3^2$

$= 0 + \frac{1}{2} + \frac{1}{4} \times 4 + \frac{1}{8} \times 9$

$= 0 + \frac{1}{2} + 1 + \frac{9}{8}$

$= 2\frac{5}{8} \left(= \frac{21}{8} = 2.625 \right)$ 1

(b) Now, $g(x) = 20\log_e x - x^2$

So, $g'(x) = 20 \times \frac{1}{x} - 2x$

$= \frac{20}{x} - 2x$ **(1 mark)**

Put $x_1 = 5$

So, $g(x_1) = 20\log_e 5 - 25$

$g'(x_1) = 4 - 10 = -6$

$x_2 = x_1 - \frac{g(x_1)}{g'(x_1)}$

$= 5 - \frac{20\log_e 5 - 25}{-6}$

$= 6.198126$........... **(1 mark)**

So a better approximation to the zero would be 6.198 **(1 mark)**
(correct to 4 significant figures).

(c) $1 - \tan^2 x + \tan^4 x - \ldots$

A limiting sum exists if $|r| < 1$

ie. $-1 < -\tan^2 x < 1$ 1

$1 > \tan^2 x > -1$

$-1 < \tan^2 x < 1$

But $\tan^2 x$ is always positive

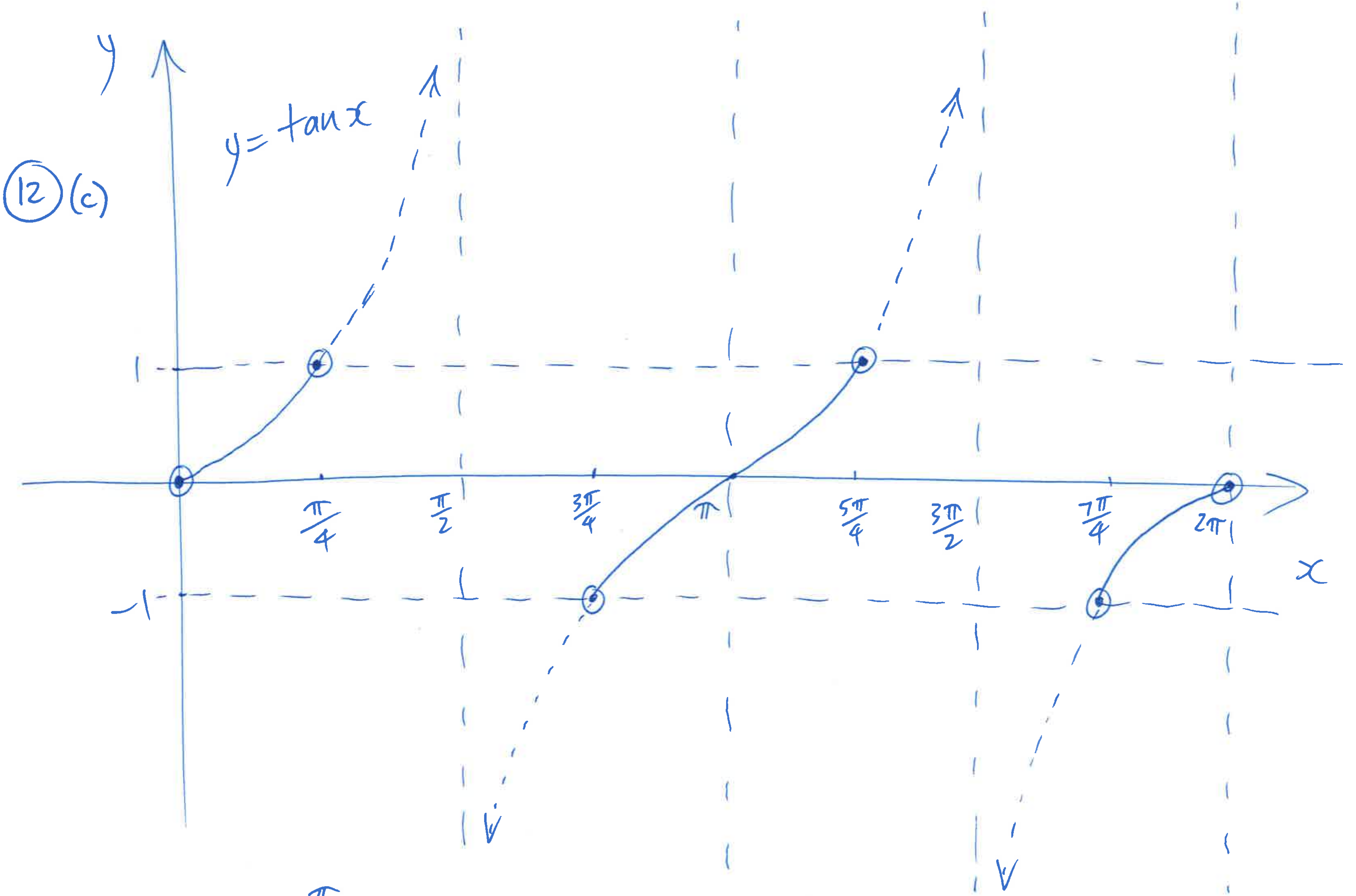
So solve $\tan^2 x < 1$ 1

ie. $-1 < \tan x < 1$

$0 < x < \frac{\pi}{4}$ or $\frac{3\pi}{4} < x < \frac{5\pi}{4}$ or $\frac{7\pi}{4} < x < 2\pi$. 2

Do NOT write anything, or make any marks below this line or outside borders

12 (c)



$0 < x < \frac{\pi}{4}$ OR $\frac{3\pi}{4} < x < \frac{5\pi}{4}$ OR $\frac{7\pi}{4} < x < 2\pi$

(12) (d) (i) Method 1

$$\int \frac{\cos x}{2+\sin x}\,dx = \log_e(2+\sin x)+c$$

Since $\dfrac{d}{dx}(2+\sin x)=\cos x$

Method 2

$$\int \frac{\cos x}{2+\sin x}\,dx = \int \frac{du}{dx}\cdot u^{-1}\,dx \qquad \text{where } u = 2+\sin x$$

$$= \int u^{-1}\,du \qquad \frac{du}{dx}=\cos x$$

$$= \log_e u + c$$

$$= \log_e(2+\sin x)+c$$

(1 mark)

(ii)

$$\int_0^1 \left(e^{2x}-e^{-x}\right)^2 dx = \int_0^1 \left(e^{2x}-e^{-x}\right)\left(e^{2x}-e^{-x}\right)dx$$

$$= \int_0^1 \left(e^{4x}-e^{x}-e^{x}+e^{-2x}\right)dx$$

$$= \int_0^1 \left(e^{4x}-2e^{x}+e^{-2x}\right)dx \qquad \textbf{(1 mark)}$$

$$= \left[\frac{e^{4x}}{4}-2e^{x}-\frac{e^{-2x}}{2}\right]_0^1 \qquad \textbf{(1 mark)}$$

$$= \left\{\left(\frac{e^4}{4}-2e^1-\frac{e^{-2}}{2}\right)-\left(\frac{e^\circ}{4}-2e^\circ-\frac{e^\circ}{2}\right)\right\}$$

$$= \frac{e^4}{4}-2e-\frac{e^{-2}}{2}-\frac{1}{4}+2+\frac{1}{2}$$

$$= \frac{e^4}{4}-2e-\frac{1}{2e^2}+\frac{9}{4} \qquad \textbf{(1 mark)}$$

(12) (e) (i)

$$\cos 2x = 2\cos^2 x - 1$$

$$1+\cos 2x = 2\cos^2 x$$

$$\therefore\ 1+\cos x = 2\cos^2\left(\frac{x}{2}\right)$$

$$\therefore\ \frac{1}{1+\cos x} = \frac{1}{2\cos^2\left(\frac{x}{2}\right)} = \frac{1}{2}\sec^2\left(\frac{x}{2}\right).$$

1

(e)(ii)

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{dx}{1+\cos x} = \frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sec^2\frac{x}{2}\,.\,dx$$

$$= \left[\tan\frac{x}{2}\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}} \quad 1$$

$$= \tan\frac{\pi}{4}-\tan\frac{\pi}{6}$$

$$= 1-\frac{1}{\sqrt{3}}. \quad 1$$

$$= \frac{\sqrt{3}-1}{\sqrt{3}}.$$

$$= \frac{3-\sqrt{3}}{3}.$$

Do NOT write anything, or make any marks below this line or outside borders

13 (a) (i) Let $\angle BAO = \alpha$

Let $\angle CAO = \delta$

$$\text{So, } \theta = \delta - \alpha$$

$$\text{So, } \tan\theta = \tan(\delta - \alpha)$$

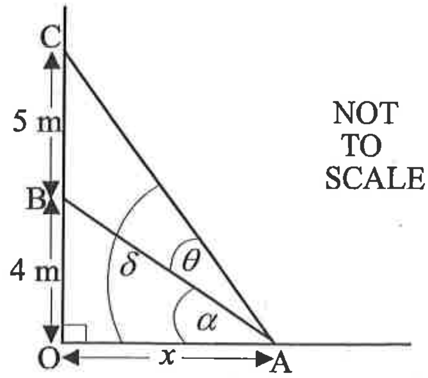
$$= \frac{\tan\delta - \tan\alpha}{1 + \tan\delta\tan\alpha} \quad \textbf{(1 mark)}$$

$$= \frac{\frac{9}{x} - \frac{4}{x}}{1 + \frac{9}{x} \times \frac{4}{x}}$$

$$= \frac{5}{x} \div \left(1 + \frac{36}{x^2}\right)$$

$$= \frac{5}{x} \div \left(\frac{x^2 + 36}{x^2}\right)$$

$$= \frac{5}{x} \times \frac{x^2}{x^2 + 36}$$



So, $\tan\theta = \dfrac{5x}{x^2 + 36}$ as required. **(1 mark)**

(ii) Now $\tan\theta = \dfrac{5x}{x^2 + 36}$

$$\frac{d(\tan\theta)}{dx} = \frac{(x^2 + 36)(5) - 2x \times 5x}{(x^2 + 36)^2}$$

$$= \frac{5x^2 + 180 - 10x^2}{(x^2 + 36)^2}$$

$$= \frac{180 - 5x^2}{(x^2 + 36)^2}$$

(1 mark)

$$\text{Let } \frac{d(\tan\theta)}{dx} = 0$$

$$\text{So, } \frac{180 - 5x^2}{(x^2 + 36)^2} = 0$$

$$\text{Now, } (x^2 + 36)^2 \neq 0$$

$$\text{So, } 180 - 5x^2 = 0$$

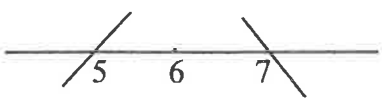
$$5(36 - x^2) = 0$$

$$x = \pm 6$$

Reject $x = -6$ since the struts can only support from one side of the wall in this problem. So $x = 6$ **(1 mark)**

Check for min/max.

When $x = 5$, $\dfrac{d(\tan\theta)}{dx} = 0.147$



When $x = 7$, $\dfrac{d(\tan\theta)}{dx} = -0.0899$

So we have a max at $x = 6$. So the anchor point A, should be located 6m from the base of the vertical wall so that θ is a maximum. **(1 mark)**

13 (b) (i) $\sqrt{2}\cos 2x - \sqrt{6}\sin 2x \equiv R\cos(2x+\alpha)$

Now, $R = \sqrt{a^2+b^2}$

$= \sqrt{2+6}$

$= \sqrt{8}$

$= 2\sqrt{2}$ 1

Also, $\tan\alpha = \dfrac{\sqrt{6}}{\sqrt{2}}$

$\tan\alpha = \sqrt{3}$

So, $\alpha = \dfrac{\pi}{3}$ $\quad$ α is a 1st quadrant angle

So $\sqrt{2}\cos 2x - \sqrt{6}\sin 2x \equiv 2\sqrt{2}\cos\left(2x+\dfrac{\pi}{3}\right)$ 1

(ii) Now, $\sqrt{2}\cos 2x - \sqrt{6}\sin 2x = \sqrt{2}$ $\quad 0 \le x \le 2\pi$

becomes $2\sqrt{2}\cos\left(2x+\dfrac{\pi}{3}\right) = \sqrt{2}$ from part (i)

So, $\cos\left(2x+\dfrac{\pi}{3}\right) = \dfrac{\sqrt{2}}{2\sqrt{2}}$

$\cos\left(2x+\dfrac{\pi}{3}\right) = \dfrac{1}{2}$ 1

Now, $0 \le x \le 2\pi$, so $\dfrac{\pi}{3} \le \left(2x+\dfrac{\pi}{3}\right) \le \dfrac{13\pi}{3}$

So,

$2x+\dfrac{\pi}{3} = \dfrac{\pi}{3}, \dfrac{5\pi}{3}, \dfrac{7\pi}{3}, \dfrac{11\pi}{3}, \dfrac{13\pi}{3}$ 1 progress

$2x = 0, \dfrac{4\pi}{3}, \dfrac{6\pi}{3}, \dfrac{10\pi}{3}, \dfrac{12\pi}{3}$

$x = 0, \dfrac{2\pi}{3}, \pi, \dfrac{5\pi}{3}, 2\pi$ 1 all solutions

☐ Tick this box if you have continued this answer in another booklet.

13 (c) (i) $f(x) = e^{x} - x$

$f'(x) = e^{x} - 1$

$f''(x) = e^{x} > 0$ for all values of x 1

$\therefore$ $f(x)$ is concave up for all values of x.

(c) (ii) Stat. pts occur when $f'(x) = 0$

ie. $e^{x} - 1 = 0$

$e^{x} = 1$

ie. $x = 0$

Now, $f(0) = e^{0} - 0 = 1$

$\therefore$ stat. pt at $(0, 1)$. 1

When $x=0$, $f''(0) = e^{0} = 1 > 0$ $\therefore$ min t.p. at $(0,1)$. 1

(c) (iii) $f(0) = e^{0} - 0 = 1 - 0 = 1$. 1

and since $f(x) = e^{x} - x$ is concave up for all x

then $e^{x} - x \geqslant 1$

ie. $e^{x} \geqslant x + 1$ for all values of x. 1

Do NOT write anything, or make any marks below this line or outside borders

(14) (a)

$$LHS = \frac{1+\sin x - \cos x}{1+\sin x + \cos x}$$

$$= \frac{1 + \frac{2t}{1+t^2} - \frac{1-t^2}{1+t^2}}{1 + \frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2}}$$

$$= \frac{1+t^2 + 2t - 1 + t^2}{1+t^2 + 2t + 1 - t^2} \quad (1)$$

$$= \frac{2t^2 + 2t}{2 + 2t}$$

$$= \frac{2t(t+1)}{2(1+t)}$$

$$= t \quad (1)$$

$$= \tan\frac{x}{2}$$

$$= RHS$$

---

(14) (b)(i) If $(x-1)$ is a factor then $P(1) = 0$

ie. $0 = 1 - k - 1 + 2$

$\underline{k = 2}$ (1)

(b)(ii)

$$P(x) = x^3 - 2x^2 - x + 2$$

$$= x^2(x-2) - (x-2) \quad (1)$$

$$= (x-2)(x^2-1)$$

$$= (x-2)(x-1)(x+1). \quad (1)$$

Alternative method:

Long division was accepted.

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(14) (c) $x^3 - 6x^2 + 3x + m = 0$

(1) Let the roots be $\alpha - d, \alpha, \alpha + d$

$\therefore \; 3\alpha = 6$

$\alpha = 2$ (1)

Subst. $\alpha = 2$ in eqn for x:

$8 - 24 + 6 + m = 0$

$m = 10.$ (1)

14 (b) (i) The point $S(4s, 2s^2)$ lies on the parabola.

So, $x = 4s$ and $y = 2s^2$. So, $s = \frac{x}{4}$ and $y = \frac{2x^2}{16}$

$y = \frac{x^2}{8}$ is the Cartesian equation of the parabola. **(1 mark)**

(ii) Find the equation of the chord ST.

The gradient of the tangent at any point on the parabola is given by $\frac{dy}{dx} = \frac{2x}{8}$

$$= \frac{x}{4}$$

At S, $\frac{dy}{dx} = \frac{4s}{4}$

$$= s$$

The gradient of the normal at S is $-\frac{1}{s}$.

So the gradient of ST is $-\frac{1}{s}$. **(1 mark)**

The equation of ST is $y - 2s^2 = -\frac{1}{s}(x - 4s)$

(other equations accepted)

$$y = -\frac{x}{s} + 4 + 2s^2$$ **(1 mark)**

When $x = 0$, $y = 4 + 2s^2$

So the chord passes through the point $(0, 4 + 2s^2)$ as required. **(1 mark)**

(iii) The gradient of ST is $-\frac{1}{s}$. So, $\frac{2s^2 - 2t^2}{4s - 4t} = -\frac{1}{s}$ **(1 mark)**

$$\frac{(s-t)(s+t)}{2(s-t)} = -\frac{1}{s}$$

$$\frac{s+t}{2} = -\frac{1}{s} \quad \text{since } s \neq t$$

$$s(s+t) = -2$$

$$t = -(s + \frac{2}{s})$$ as required **(1 mark)**

(iv) If $s = 0$, then S is the point $(0,0)$. The normal to S would be the y-axis. The point T would not exist since it would have to lie on the y-axis and could not therefore lie on the parabola.

(1 mark)

(Many answers accepted)

Do NOT write anything, or make any marks below this line or outside borders