



Trinity Grammar School

Mathematics Department

2015

HALF-YEARLY EXAMINATION

PRELIMINARY ASSESSMENT TASK 3

Year 11

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using **black** or **blue** pen
- Only approved calculators for this course are allowed in this task.
- Show all necessary working
- Write your **Name** and your **Class teacher** on the question paper and on any answer sheets or writing booklets used to write your responses to the questions submitted.
- If you do not attempt a question you must submit an answer sheet or writing booklet for that question clearly indicating **N/A** and your Name.
- **Preliminary Assessment Weighting:**
30%

Class Teacher:

.....

Name:

.....

Do NOT write solutions on this question paper. Any working on this question paper will NOT be marked.

Date of examination:

Wednesday, 1 April 2015

Total marks – 70

Section I

10 marks

- Attempt all Questions
- Allow about **20** minutes for this section

Section II

60 marks

- Attempt all Questions
- Allow about **1** hours **40** minutes for this section

The School defines malpractice, or cheating, as "dishonest behaviour by a student that gives them an unfair advantage over others". I certify that my attempt at this Assessment task does not involve any malpractice or cheating.

Student signature

Date

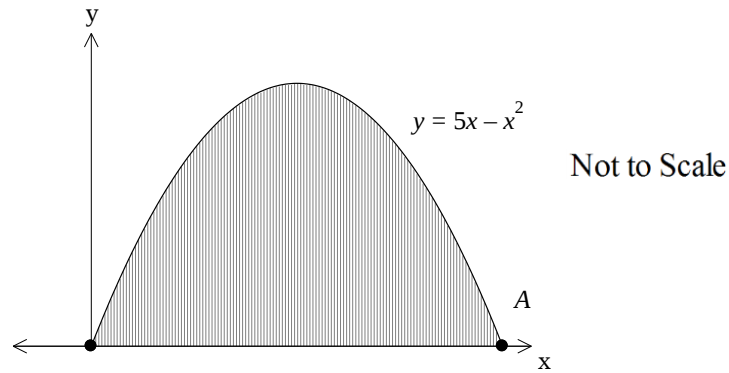
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Section I 10 marks

- Attempt Questions 1 – 10.
 - Allow about 20 minutes for this section.
 - Shade the correct response on the multiple-choice answer sheet for Questions 1 – 10.
 - Each question is worth 1 mark.
-

- 1 What is the value of $f(2a+1)$ if $f(x) = 3x - 2$?
- (A) $6a - 2$
(B) $6a - 1$
(C) $6a + 1$
(D) $6a + 2$
- 2 Which of the following is true for the function $f(x) = 3x^2 - x$?
- (A) Even function
(B) Odd function
(C) Neither odd or even
(D) Zero function
- 3 The formula $H = 5m(Y - X)$ is used to calculate the heat (H) required to raise the temperature of a steel rod, of mass m , from a temperature of X to a temperature of Y . Rearrange the formula to make X the subject.
- (A) $X = \frac{5m - H}{Y}$
(B) $X = \frac{H - 5m}{Y}$
(C) $X = \frac{H - 5mY}{5m}$
(D) $X = \frac{5mY - H}{5m}$
- 4 What is the centre and radius of the circle with the equation $x^2 + y^2 + 6x - 8y - 11 = 0$?
- (A) Centre $(-3, -4)$ and radius 36
(B) Centre $(-3, 4)$ and radius 36
(C) Centre $(-3, -4)$ and radius 6
(D) Centre $(-3, 4)$ and radius 6

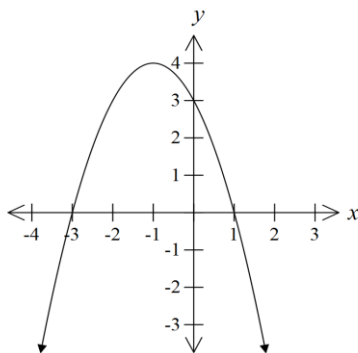
- 5 The diagram shows the graph of the function $y = 5x - x^2$.



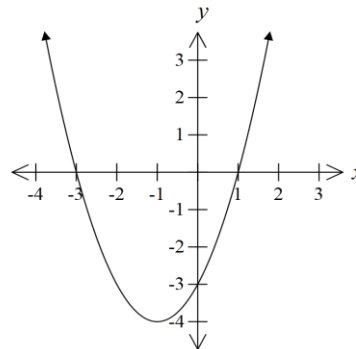
What pair of inequalities specify the shaded region?

- (A) $y \leq 5x - x^2$ and $y \geq 0$.
 (B) $y \leq 5x - x^2$ and $y \geq 0$.
 (C) $y \geq 5x - x^2$ and $y \leq 0$.
 (D) $y \geq 5x - x^2$ and $y \geq 0$.
- 6 Which graph best represents $y = x^2 + 2x - 3$?

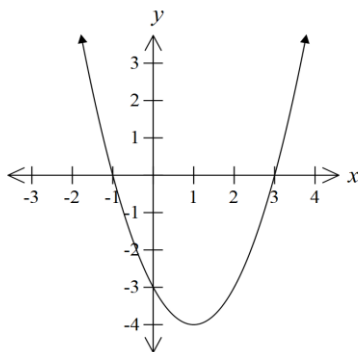
(A)



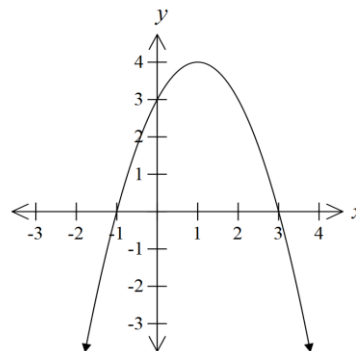
(B)



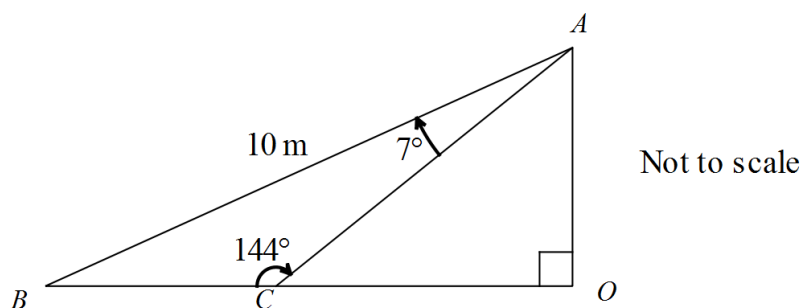
(C)



(D)



Not to Scale



Which trigonometric formula would be most useful in calculating the length of side BC ?

(A) $c^2 = a^2 + b^2 - 2ab \cos C$

(B) $A = \frac{1}{2} ab \sin C$

(C) $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

(D) $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

8 What is solution to the equation $\cos\left(\frac{\theta}{2} + 20^\circ\right) = \sin \theta$ for $0^\circ \leq \theta \leq 90^\circ$?

(A) $\theta = 20^\circ$

(B) $\theta = 40^\circ$

(C) $\theta = \frac{160^\circ}{3}$

(D) $\theta = \frac{140^\circ}{3}$

9 What is the solution to the inequality $\frac{x^2 - 4}{2x} > 0$?

(A) $-2 < x < 0$ or $x > 2$

(B) $-2 < x < 0$ or $x > 4$

(C) $-4 < x < 0$ or $x > 2$

(D) $-4 < x < 0$ or $x > 4$

10 What is the value of $\lim_{x \rightarrow c} \frac{x^3 - c^3}{x - c}$?

(A) Undefined

(B) c^2

(C) $3c^2$

(D) c^3

End of Section I

Section II commences on the next page

Section II 60 marks

- Attempt **Questions 11 – 14**
 - Allow about 1 hour and 40 minutes for this section
 - Answer each question in a SEPARATE writing booklet or answer sheet. Extra writing booklets are available.
 - In Questions 11 – 14, your responses should include all relevant mathematical reasoning and/or calculations.
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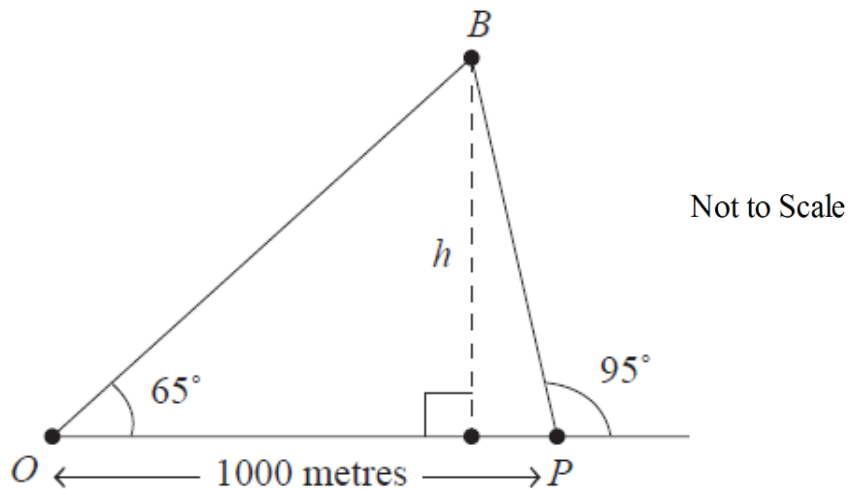
Question 11 (15 marks)	Use a SEPARATE writing booklet	Marks
(a) Evaluate $\log_{10} 70 - \log_{10} 10$, correct to 3 significant figures.		2
(b) Solve: $ 2n+1 = -n+1$		2
(c) Solve: $(x+1)(x-1) = 15$		2
(d) Find the exact value of $\sec 135^\circ$.		1
(e) Simplify $\log_{10} a + \log_{10} b$.		1
(f) Prove: $\frac{1}{\sin \theta + 1} - \frac{1}{\sin \theta - 1} \equiv 2 \sec^2 \theta$		3
(g) Solve: $\frac{2}{x-5} \geq 3$		3
(h) Solve: $\log_2 16 = x^2$		1

Question 12 (15 marks)

Use a SEPARATE writing booklet

Marks

- (a) If A balloon B is in the same vertical plane as two observers O and P . O and P are 1000 metres apart. OB makes an angle of 65° to the horizontal and PB an angle of 95° to the horizontal (see diagram).



- (i) Using the Sine Rule, show that $OB = 2000 \sin 85^\circ$. 2
- (ii) Find the height of the balloon (h) to the nearest metre. 2
- (b) Solve: $2 \log_3(x\sqrt{3}) = \log_3(6 - 7x)$ 3
- (c) If $\sin \alpha = -\frac{3}{5}$ and $\cos \alpha < 0$, what is the exact value of $\tan \alpha$? 2
- (d) Solve: $2^{x^2-4x} - \frac{1}{8} = 0$ 2
- (e) Graph show the region where the following inequalities hold simultaneously: 4
- $$y+1 \leq 0; \quad x+y-1 < 0; \quad x \geq 2$$

Question 13 (15 marks)

Use a SEPARATE writing booklet

Marks

(a) (i) Show: $\frac{\cos^2 \theta}{1 - \sin \theta} - \frac{\cos^2 \theta}{1 + \sin \theta} = 2 \sin \theta$ 2

(ii) Hence, solve $\frac{\cos^2 \theta}{1 - \sin \theta} - \frac{\cos^2 \theta}{1 + \sin \theta} = 1$, for $0 \leq \theta \leq \frac{\pi}{2}$. 1

(b) Solve for x : $\left(x + \frac{1}{x}\right)^2 - 7\left(x + \frac{1}{x}\right) + 6 = 0$ 4

(c) Solve: $(x + y) + \sqrt{3x - y} = 3 + \sqrt{5}$ 2

(d) Write the exact value of $\cos 45^\circ - \sin 120^\circ$ with a rational denominator. 2

(e) If $f(x) = \begin{cases} 1 - x^2 & \text{if } x > 1 \\ x^2 - 1 & \text{if } x \leq 1 \end{cases}$

(i) Find the value of $f(2) + f(-1) - f(0)$. 2

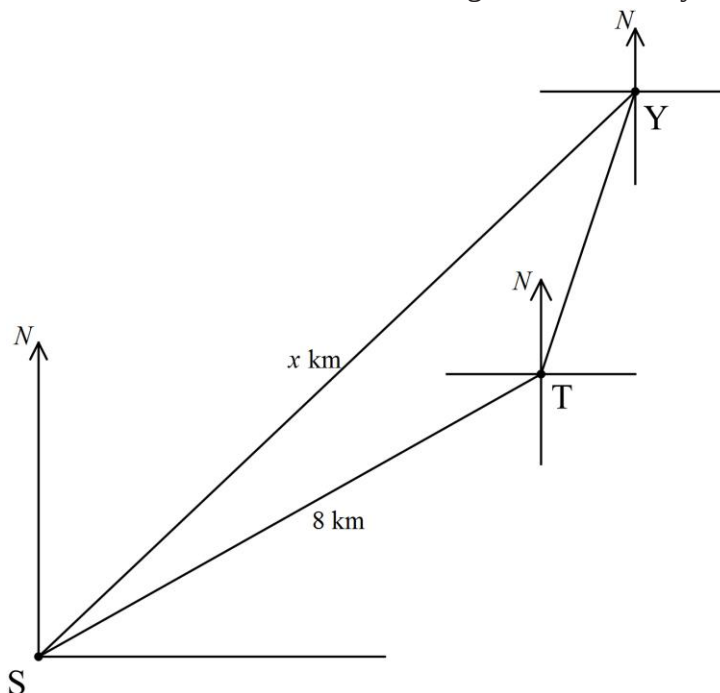
(ii) Sketch the curve. 2

Question 14 (15 marks)

Use a SEPARATE writing booklet

Marks

- (a) A yacht sails from Sydney for 8 km on a course of $N64^\circ E$ and then changes course to $N15^\circ E$ and travels in this direction until it is on a bearing $S35^\circ W$ from Sydney.



- (i) Find $\angle STY$. 1
- (ii) Find $\angle TYS$. 1
- (iii) How far is the yacht from Sydney? (Calculate the value of x) 2
- (b) Consider the function $f(x) = \frac{x}{4-x^2}$.
- (i) Find the domain of the function. 1
- (ii) Show that the function is an odd function. 2
- (iii) Sketch the graph of the function showing clearly the coordinates of any points of intersection with the x -axis or the y -axis and the equations of any asymptotes. 4
- (c) Solve: $\sin \theta = \frac{-1}{\sqrt{2}}$ for $-360^\circ \leq \theta \leq 360^\circ$ 2
- (d) State the domain and range of the function $f(x) = \frac{3+2^x}{4}$. 2

End of Section II
End of Examination

2015 IIMAX Task3 Half-Yearly

Section I

$$1. f(2a+1) = 3(2a+1) - 2 \\ = 6a + 3 - 2 \\ = 6a + 1$$

[C]

$$2. f(-x) = 3(-x)^2 - (-x) \\ = 3x^2 + x$$

Neither odd nor even

[C]

3.

$$H = 5m(Y - X)$$

$$\frac{H}{5m} = (Y - X)$$

$$X = Y - \frac{H}{5m}$$

[D]

$$X = \frac{5mY - H}{5m}$$

$$4. \quad \begin{matrix} (+9) & (+16) & (+9) & (+16) \\ x^2 + 6x + y^2 - 8y = 11 \end{matrix}$$

$$(x+3)^2 + (y-4)^2 = 36$$

$$(-3, 4) \quad r=6$$

[D]

5. Test (2,1)

Must be true

$$1 = 5 \times 1 - 1^2$$

$$1 \geq 0$$

[B]

$$1 = 4$$

$$1 \leq 4$$

$$y \leq 5x - x^2 \quad y \geq 0$$

$$6. \quad y = (x+3)(x-1)$$

$$x = -3 \quad x = 1$$

concave up

[B]

7. Sine Rule

[D]

8.

$$\cos\left(\frac{\theta}{2} + 20\right) = \sin(90 - \theta)$$

$$\frac{\theta}{2} + 20 = 90 - \theta$$

$$\frac{\theta}{2} + \theta = 70$$

$$3\theta = 140$$

$$\theta = \frac{140}{3}$$

[D]

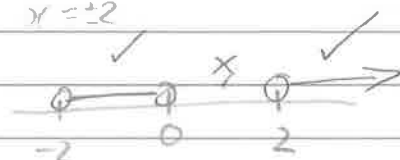
9.

$$x \neq 0$$

$$x^2 - 4 = 0$$

$$(x-2)(x+2) = 0$$

$$x = \pm 2$$



[A]

10.

$$\lim_{x \rightarrow c} \frac{(x-c)(x^2 + xc + c^2)}{(x-c)}$$

$$c^2 + c^2 + c^2 = 3c^2$$

[C]

Section II

Question 11

(a) $0.84509804 \checkmark$
 $0.845 \checkmark$

(b) $|2n+1| = -n+1$
 $2n+1 = -n+1$ or $-2n-1 = -n+1$
 $3n = 0$ $-n = 2$
 $n = 0 \checkmark$ or $n = -2 \checkmark$

(c) $x^2 - 1 = 15$
 $x^2 = 16$
 $x = \pm 4 \checkmark \checkmark$

(d) $\frac{1}{\cos 135} = \frac{1}{-\frac{1}{\sqrt{2}}} = -\sqrt{2} \checkmark$
 $\cos 135 = \cos(180-45)$

(e) $\log_{10} ab \checkmark$

(f)

$$\begin{aligned} \text{LHS} &= \frac{1(\sin \theta - 1) - (\sin \theta + 1)}{\sin^2 \theta - 1} \\ &= \frac{-2}{-\cos^2 \theta} \\ &= \frac{2}{\cos^2 \theta} \\ &= 2 \sec^2 \theta \\ &= \text{RHS} \end{aligned}$$

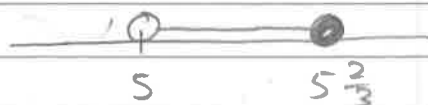
(g) $\frac{2}{x-5} \geq 3$ $x \neq 5$

$$\frac{2}{x-5} = 3$$

$$2 = 3x - 15 \checkmark$$

$$17 = 3x$$

$$x = \frac{17}{3} = 5\frac{2}{3}$$



After testing

$$5 < x \leq 5\frac{2}{3} \checkmark \checkmark$$

(h) $\log_2 16 = x^2$

$$2^{x^2} = 16$$

$$2^{x^2} = 2^4$$

$$x^2 = 4$$

$$x = \pm 2 \checkmark$$

Question 12

(a) $\angle OBP = 95 - 65$

(i) $= 30^\circ$ ✓

$\angle OBP = 85^\circ$ ✓

$$\frac{\sin 85}{OB} = \frac{\sin 30}{1000} \quad \checkmark$$

$$OB = \frac{1000 \sin 85}{\sin 30} \quad \sin 30 = \frac{1}{2}$$

$$OB = 2000 \sin 85$$

(ii) $\sin 65 = \frac{h}{OB}$

$$\begin{aligned} h &= OB \sin 65 \quad \checkmark \\ &= 2000 \sin 85 \times \sin 65 \\ &= 1805.718025 \\ &= 1806 \quad \checkmark \end{aligned}$$

(b) $\log_3 (x\sqrt{3})^2 = \log_3 (6-7x)$

$$3x^2 = 6 - 7x$$

$$3x^2 + 7x - 6 = 0 \quad \checkmark$$

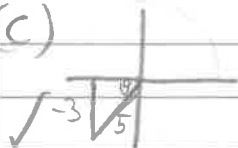
$$(3x-2)(x+3) = 0$$

$$x = \frac{2}{3} \quad x = -3 \quad \checkmark$$

but $x > 0$

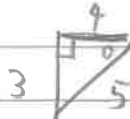
$$\therefore x = \frac{2}{3} \quad \checkmark$$

(c)



$$x = \sqrt{5^2 - 3^2} \quad \checkmark$$

$$x = 4$$



$$\therefore \tan \alpha = \frac{3}{4} \quad \checkmark$$

(d)

$$2^{x^2-4x} = 2^{-3} \quad \checkmark$$

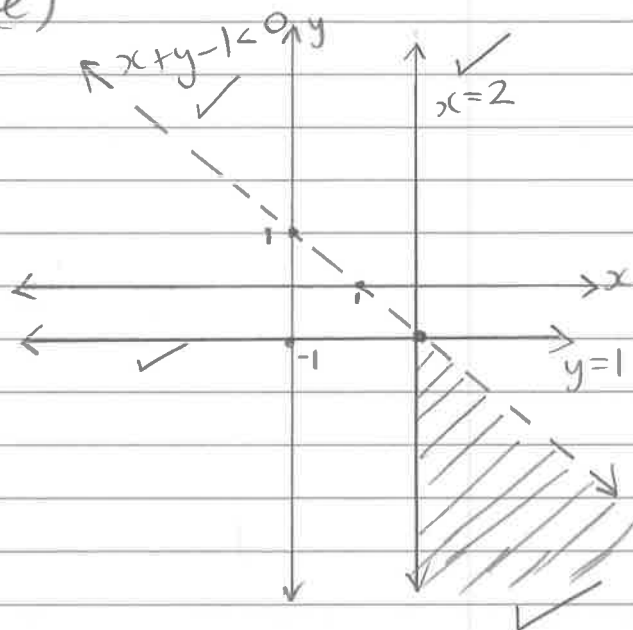
$$x^2 - 4x = -3$$

$$x^2 - 4x + 3 = 0$$

$$(x-3)(x-1) = 0$$

$$x = 3 \quad x = 1 \quad \checkmark$$

(e)



(d) See below

Question 13

(a) (i)

$$\text{LHS} = \frac{\cos^2 \theta (1 + \sin \theta) - \cos^2 \theta (1 - \sin \theta)}{1 - \sin^2 \theta}$$

$$= \frac{\cos^2 \theta [1 + \sin \theta - 1 + \sin \theta]}{\cos^2 \theta}$$

$$= 2 \sin \theta$$

(ii) $2 \sin \theta = 1$

$$\sin \theta = \frac{1}{2}$$

$$\theta = 30^\circ$$

(b) $u^2 - 7u + 6 = 0$

$$(u - 6)(u - 1) = 0$$

$$u = 6 \quad u = 1$$

$$x + \frac{1}{x} = 6$$

$$x^2 + 1 = 6x$$

$$x^2 - 6x + 1 = 0$$

using formula

$$x = 3 \pm 2\sqrt{2}$$

$$x + \frac{1}{x} = 1$$

$$x^2 + 1 = x$$

$$x^2 - x + 1 = 0$$

no solution

(c) $x + y = 3 \dots (1)$

$$\sqrt{3}x - y = \sqrt{5} \dots (2)$$

$$3x - y = 5 \dots (3)$$

$(1) + (2)$

$$4x = 8$$

$$\text{sub } x = 2 \rightarrow (1)$$

$$2 + y = 3$$

$$y = 1$$

$$\therefore x = 2 \quad y = 1$$

(e)

(i)

$$f(2) = 1 - 2^2 = -3$$

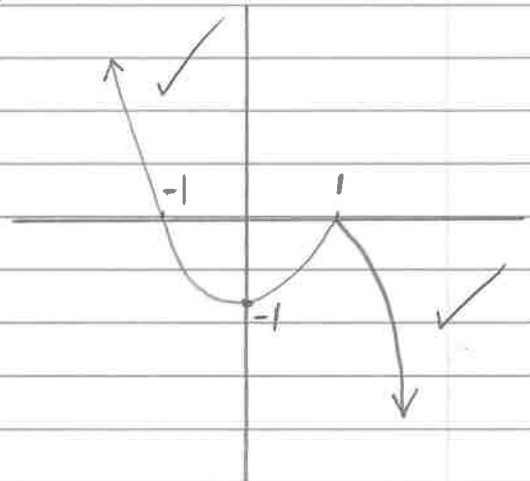
$$f(-1) = (-1)^2 - 1 = 0$$

$$f(0) = 0^2 - 1 = -1$$

$$f(2) + f(-1) - f(0)$$

$$-3 + 0 - 1 = -4$$

(ii)



(d) $\frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2}$

$$\frac{(2 - \sqrt{6}) \times \sqrt{2}}{2\sqrt{2} \sqrt{2}}$$

$$\frac{2\sqrt{2} - \sqrt{12}}{4}$$

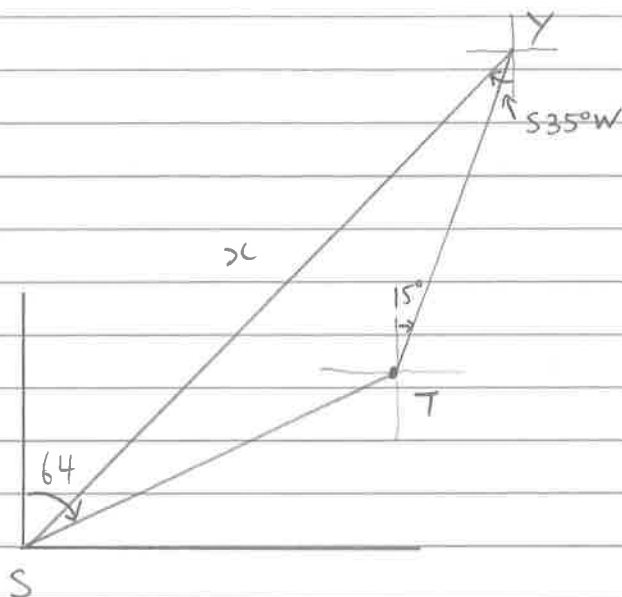
$$\frac{2\sqrt{2} - 2\sqrt{3}}{4}$$

$$\frac{2(\sqrt{2} - \sqrt{3})}{4}$$

$$= \frac{\sqrt{2} - \sqrt{3}}{2}$$

Question 14

(a)



$$(i) \angle STY = 15^\circ + 90^\circ + (90 - 64) = 131^\circ \checkmark$$

$$(ii) \angle TYS = 35^\circ - 15^\circ = 20^\circ \checkmark$$

$$(iii) \frac{8}{\sin 20} = \frac{x}{\sin 131} \checkmark$$

$$x = \frac{8 \sin 131}{\sin 20} = 17.65 \checkmark$$

$$(b) (i) D: \text{All real } x; x \neq \pm 2 \checkmark$$

$$(ii) f(-x) = \frac{-x}{4 - (-x)^2} \checkmark$$

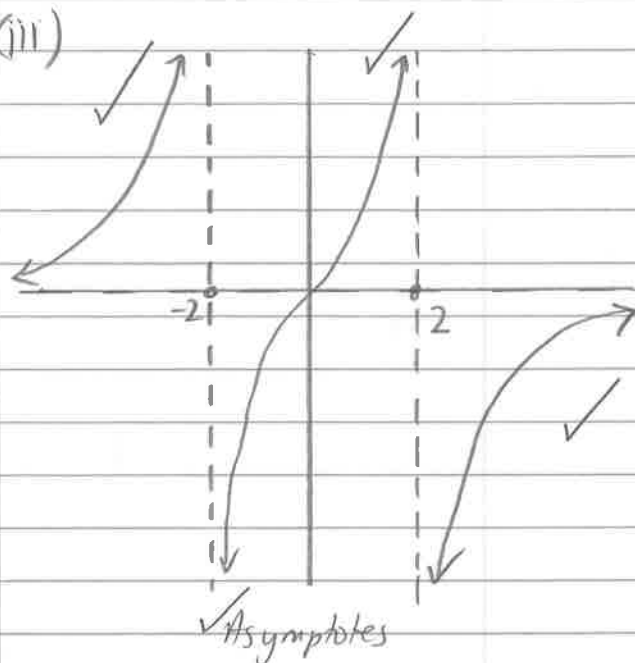
$$= \frac{-x}{4 + x^2}$$

$$= \frac{x}{-(4 - x^2)} \checkmark$$

$$f(-x) = -f(x)$$

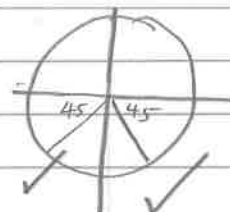
odd

(iii)



$$(c) \sin \theta = -\frac{1}{\sqrt{2}}$$

$$\theta = 45^\circ \checkmark$$



$$\theta = -45^\circ, -135^\circ, 225^\circ, 315^\circ \checkmark$$

$$(d) D: \text{All real } x \checkmark$$

$$R: y > \frac{3}{4} \checkmark$$