Trinity Grammar School

Mathematics Department

2016

COMMON

HSC ASSESSMENT TASK 3

Year 12

Mathematics Extension 1

General Instructions

- Readin time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Only approved calculators for this course are allowed in this task
- A reference sheet for this course is supplied.
- Show all necessary working
- Write your Board of Studies Student Number (Year 12 HSC) or Name (Year 11) and your Class teacher on the question paper and on any answer sheets or writing booklets used to write your responses to the questions submitted
- If you do not attempt a question you must submit an answer sheet or writing booklet for that question clearly indicating **N/A** and your Student Number or Name.
- **HSC Assessment Weighting:** 30%

Board of Studies Student Number
(Year 12 only)

Class Teacher:

.....

Name:
(Year 11 only)

.....

Do NOT write solutions on this question paper. Any working on this question paper will NOT be marked.

Date of examination:
Wednesday, 4 May 2016

Total marks – 70

Section I

10 marks

- Attempt Questions 1 – 10
- Allow about **10** minutes for this section

Section II

60 marks

- Attempt Questions 11 – 14
- Allow about 110 minutes for this section

For examinations and in-class Assessment tasks

The School defines malpractice, or cheating, as "dishonest behaviour by a student that gives them an unfair advantage over others". I certify that my attempt at this Assessment task does not involve any malpractice or cheating.

Student signature _____ Date _____

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Section I **10 marks**

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section
- Shade the correct response on the multiple-choice answer sheet for Questions 1 – **10**
- Each question is worth 1 mark

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1 A curve has parametric equations $x = t + 1$ and $y = 2t^2$.
What is Cartesian equation of this curve?

(A) $y = 2\sqrt{x-1}$

(B) $y = 2\sqrt{x+1}$

(C) $y = 2(x-1)^2$

(D) $y = 2(x+1)^2$

2 What is the exact value of $\tan 75°$?

(A) $2-\sqrt{3}$

(B) $4-\sqrt{3}$

(C) $2+\sqrt{3}$

(D) $4+\sqrt{3}$

3 If $t = \tan\dfrac{\theta}{2}$ which of the following expressions is equivalent to $\sec\theta$?

(A) $\dfrac{1+t^2}{1-t^2}$

(B) $\dfrac{2t}{1+t^2}$

(C) $\dfrac{1-t^2}{1+t^2}$

(D) $\dfrac{2t}{1-t^2}$

4 The curves $y = x^2$ and $y = (x-2)^2$ meet at the point $(1,1)$. What is the acute angle between the tangents to curves at this point, to the nearest degree?

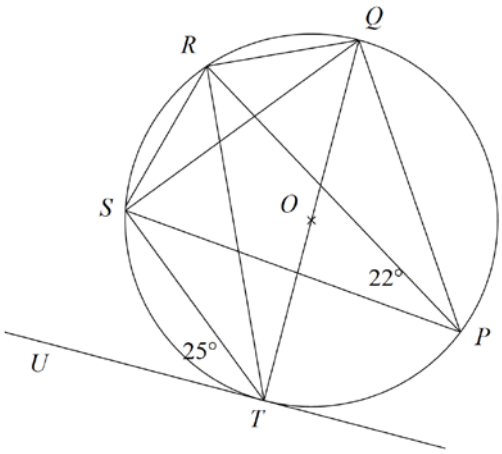
(A) 0°

(B) 37°

(C) 53°

(D) 65°

5 A circle with centre *O* has a tangent *TU*, diameter *QT*, $\angle \text{STU} = 25^\circ$ and $\angle \text{RPS} = 22^\circ$.



What is the size of $\angle RTQ$?

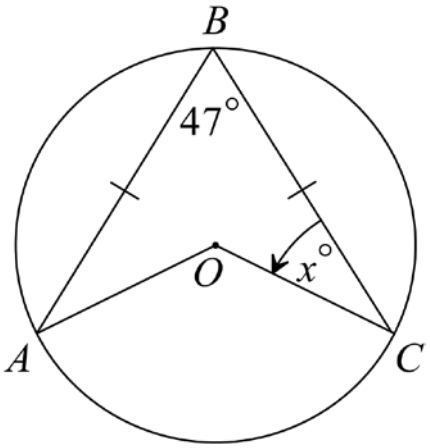
(A) 22°

(B) 25°

(C) 43°

(D) 47°

6 What is the value of *x*?



NOT TO SCALE

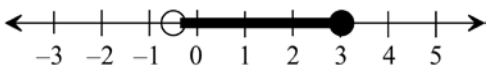
(A) $x = 21 \cdot 5$

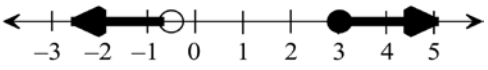
(B) $x = 23 \cdot 5$

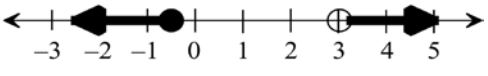
(C) $x = 43$

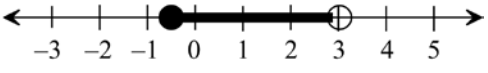
(D) $x = 66 \cdot 5$

7 Which diagram displays the solution to $\frac{7}{3-x} \geq 2$?

(A) 

(B) 

(C) 

(D) 

8 Use one application of Newton's method to find an approximate root of the equation $2x^3 - x - 2 = 0$, correct to 2 decimal places, using $x = 1.2$ as the first approximation.

(A) $x = 1{\cdot}16$

(B) $x = 1{\cdot}34$

(C) $x = 1{\cdot}14$

(D) $x = 1{\cdot}17$

9 A polynomial equation has $f(-1) < 0$ and $f(-2) > 0$. By halving the interval, it is found that $f(-1\frac{1}{2}) < 0$. A root of the equation lies between:

(A) $x = -2,$ and $x = -1\frac{1}{2}$

(B) $x = -1\frac{1}{2},$ and $x = -1$

(C) $x = -1\frac{1}{2},$ and $x = -1\frac{1}{4}$

(D) $x = -1\frac{3}{4},$ and $x = -1\frac{1}{2}$

10 Which of the following expressions is $\int \cos^2 3x \, dx$?

(A) $2\cos 3x + C$

(B) $\cos^3 3x \sin^2 3x + C$

(C) $\frac{1}{2}(x + \frac{1}{3}\sin 3x) + C$

(D) $\frac{1}{2}(x + \frac{1}{6}\sin 6x) + C$

End of Section I

Section II commences on the next page

Section II **60 marks**

- Attempt Questions 11 – 14
- Allow about 1 hour and 45 minutes for this section
- Answer each question in a SEPARATE writing booklet or answer sheet. Extra writing booklets are available.
- In Questions 11 – 14, your responses should include all relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet

(a) Calculate the size of the acute angle between the lines $y = 2x + 5$ and $y = 4 - 3x$. **2**

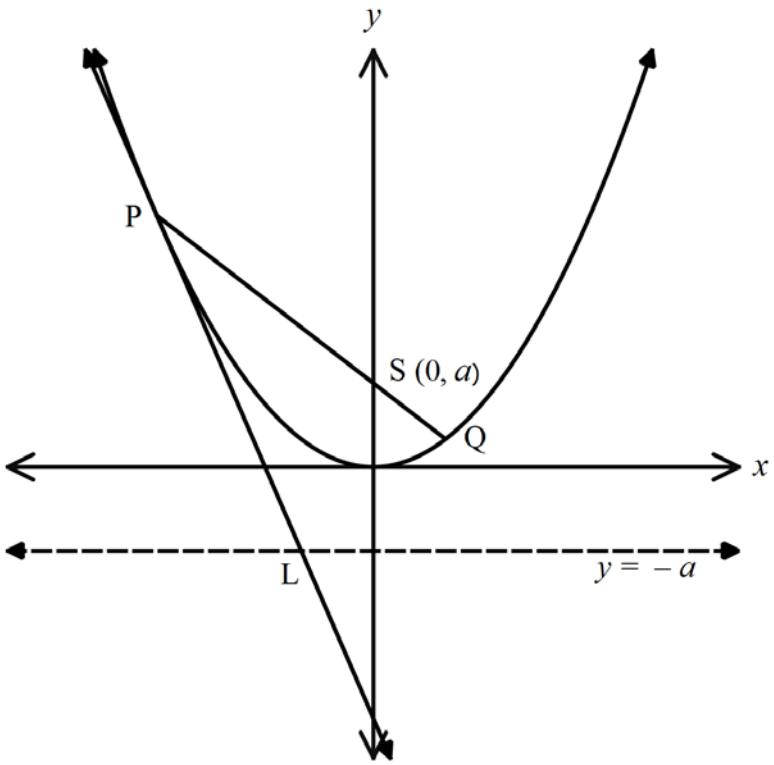
(b) Solve the inequality $\frac{x}{x-5} \geq 2$. **3**

(c) Find $\int \sin^2 x \, dx$. **2**

(d) Write $x^2 - 7x + 4$ in the form $A(x) \cdot Q(x) + R(x)$, given that $A(x) = x - 1$. **2**

Question 11 continues on page 7

(e) $x^2 = 4ay$ is a parabola with focus $S(0, a)$.



NOT TO SCALE

$P(2ap, ap^2)$ and $Q(2aq, aq^2)$ are the endpoints of a focal chord to the parabola.
L is the point where the tangent at P meets the directrix of the parabola.
You may assume without proof that the tangent at P has equation $y = px - ap^2$.

(i) Show that $SP = a(p^2 + 1)$. **3**

(ii) Show that L has coordinates $\left(ap - \frac{a}{p}, -a\right)$. **2**

(iii) Find the equation of the locus of L. **1**

End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet

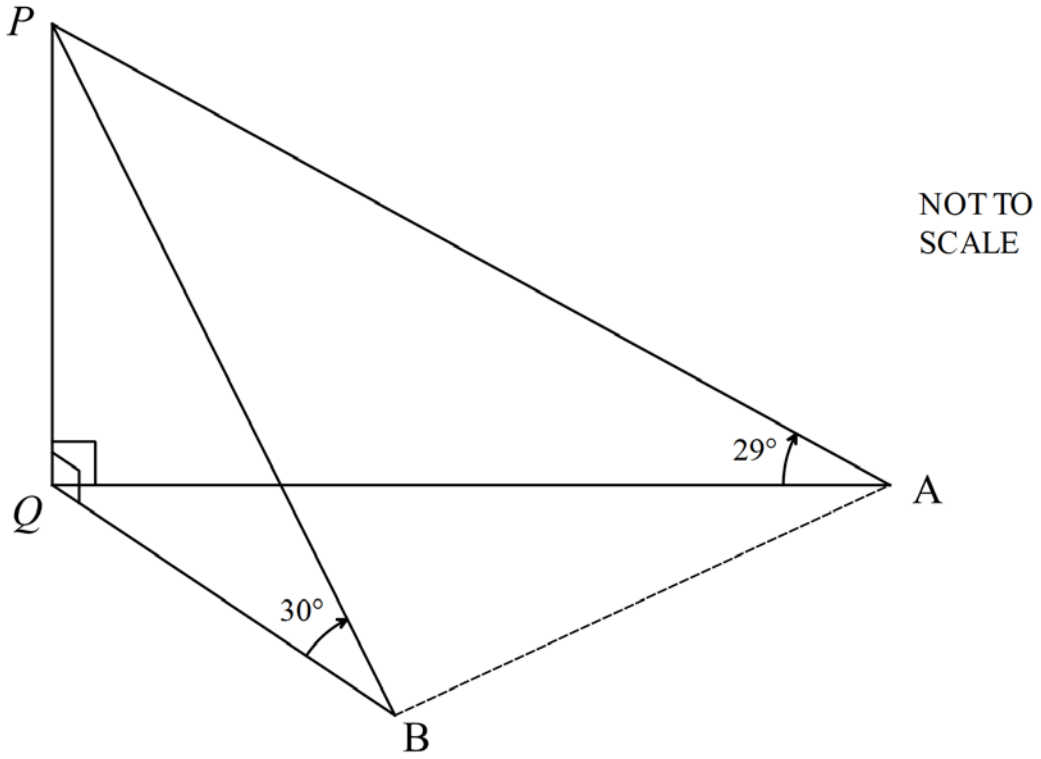
(a) Using the substitution $u = 1 + 2x$, find $\int \frac{6\,dx}{\sqrt{(1+2x)^3}}$. **3**

(b) Solve the equation $\cos 2x + \sin 2x + 1 = 0$ for $0 \le x \le 360°$. **4**

(c) If α, β and γ are roots of the equation $x^3 + 2x^2 + 3x + 5 = 0$, find the value of $\frac{1}{\alpha\beta} + \frac{1}{\alpha\gamma} + \frac{1}{\beta\gamma}$. **2**

(d) The polynomial $P(x) = 2x^3 + 6x^2 - 8$ has three roots: $1, -2$ and α. Determine the value of α, and explain the geometrical significance of this value. **2**

(e) In the diagram below, *P* is 60 metres above the horizontal plane. *PQ* is vertical. The elevation of *P* from *A* is 29° and the elevation of *P* from *B* is 30°. *A* is due East of *Q* and *B* is on a bearing of 137° from *Q*.



(i) Show that $QA = \frac{60}{\tan 29°}$ and $QB = \frac{60}{\tan 30°}$. **1**

(ii) Show that $\angle AQB = 47°$. **1**

(iii) Calculate the distance from *A* to *B*, to the nearest metre. **2**

Question 13 (15 marks) Use a SEPARATE writing booklet

(a) Evaluate $\int_0^1 x^3\sqrt{x^4+1}\,dx$ using the substitution $u = x^4 + 1$. **3**

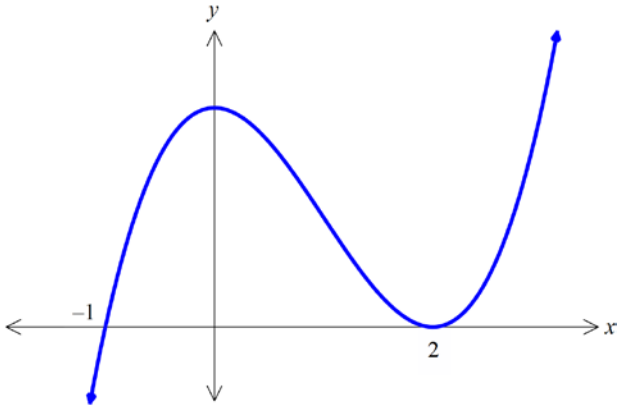
(b) Prove that $\dfrac{\sin 4\theta}{\cos^2\theta - \sin^2\theta} = 4\sin\theta\cos\theta.$ **3**

(c) Use Newton's Method once to solve $x^2 - \sqrt{x} - 2 = 0$, using a first approximation of $x = 2$. Answer to two decimal places. **3**

(d) Find $\int \sin x \cos x\,dx.$ **2**

(e) Write the general solution to $2\sin\theta - 1 = 0$. **2**

(f) The diagram below shows the graph of the monic cubic polynomial $P(x)$.
Write an equation that could represent this graph. **2**

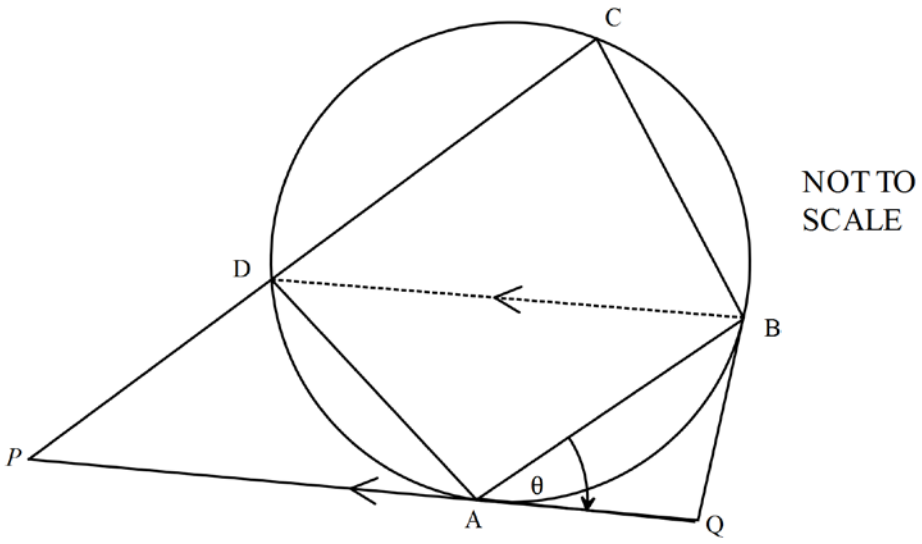


NOT TO SCALE

Question 14 (15 marks) Use a SEPARATE writing booklet

(a) Solve $3\cos\theta - \sqrt{3}\sin\theta = 0$ for $0 \le \theta \le 360°$. **4**

(b) In the diagram below, *ABCD* is a cyclic quadrilateral. The tangents from *Q* touch the circle at *A* and *B*. The diagonal *DB* is parallel to the tangent *AQ*. *QA* produced intersects with *CD* produced at *P*. Let $\angle QAB = \theta$.



(i) Given that *PD* = 5 cm and *DC* is 7 cm in length, calculate the exact length of *AP*. **1**

(ii) Show that $\angle BCD = 2\theta$. **3**

(iii) Show that *PQBC* is a cyclic quadrilateral. **2**

(c) *A*, *B* have coordinates $(-1, 7)$ and $(5, -2)$ respectively. *P* divides the interval *AB* in the ratio $k:1$.

(i) Write down the coordinates of *P* in terms of *k*. **2**

(ii) Hence find $AP:PB$ when *P* lies on the line $5x - 4y - 1 = 0$. **3**

End of Section II
End of Examination

Extension 1 Task 3 Solutions 2016

1. $t = x-1$

$y = 2(x-1)^2$ C

2. $\tan(30+45)$

$$\frac{\tan 30 + \tan 45}{1 - \tan 30 \tan 45}$$

$$\frac{\frac{1}{\sqrt{3}} + 1 \times \sqrt{3}}{1 - \frac{1}{\sqrt{3}} \times 1 \times \sqrt{3}}$$

$$\frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$\frac{3+2\sqrt{3}+1}{2}$$

$$\frac{4+2\sqrt{3}}{2}$$

$2+\sqrt{3}$ C

3. $\sec\theta = \frac{1}{\cos\theta}$

$\cos\theta = \frac{1-t^2}{1+t^2}$

$\sec\theta = \frac{1+t^2}{1-t^2}$ A

4. $y = x^2$ $\quad y = (x-2)^2$

$y' = 2x$ $\quad y' = 2(x-2)$

At $(1,1)$

$m_1 = 2$ $\quad m_2 = -2$

$\tan\theta = \left| \frac{2+2}{1-4} \right|$ C

$\tan\theta = \frac{4}{3}$ $\quad \theta = 53°$

5. $\angle RPS = \angle STR = 22$ ($\angle$ in same segment)

$\angle UTS = 90$ (tangent is perpendicular to radius through the point of contact)

$\therefore \angle RTQ = 90 - 25 - 22$

$= 43$ C

6. $x = 23.5$ B

7. Test D

8. $f'(x) = -6x^2 - 1$

$$x = 1.2 - \frac{2(1.3)^3 - 1.2 - 2}{6(1.2)^2 - 1}$$ D

$x \approx 1.1665$

$x \approx 1.17$

9. $x = -2$ and $x = -1\frac{1}{2}$ A

10. $\int \frac{1}{2}(1+\cos 6x)\,dx$

$= \frac{1}{2}\left(x + \frac{1}{6}\sin 6x\right) + C$ D

Question 11

(a) $m_1 = 2 \quad m_2 = -3$

$\tan\theta = \left|\frac{2+3}{1-6}\right|$ ✓

$= 1$

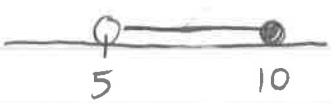
$\therefore \theta = 45°$ ✓

(b) $\frac{x}{x-5} \geq 2 \quad \boxed{x \neq 5}$ ✓

$x = 2(x-5)$

$x = 2x - 10$

$x = 10$ ✓



$5 < x \leq 10$ ✓

(c) $\int \frac{1}{2}(1-\cos 2x)\,dx$ ✓

$= \frac{1}{2}(x - \frac{1}{2}\sin 2x) + C$ ✓

(d) $x-1 \overline{)\,x^2 - 7x + 4}$ ✓

$x-6$

$x^2 - x$

$-6x + 4$

$-6x + 6$

-2

$\therefore (x-1)(x-6) - 2$ ✓

(e) (i) $SP = \sqrt{(2ap)^2 + (ap^2 - a)^2}$ ✓

$= \sqrt{4a^2p^2 + a^2p^4 - 2a^2p^2 + a^2}$

$= \sqrt{2a^2p^2 + a^2p^4 + a^2}$ ✓

$= \sqrt{a^2(2p^2 + p^4 + 1)}$

$= \sqrt{a^2(p^2+1)^2}$ ✓

$= a(p^2+1)$

(ii) $y = px - ap^2 \quad \ldots ①$

$y = -a \quad \ldots ②$

$-a = px - ap^2$ ✓

$px = ap^2 - a$

$x = ap - \frac{a}{p}$ ✓

$\therefore L\left(ap - \frac{a}{p}, -a\right)$

(iii) $y = -a$ ✓

Question 12

(a) $u = 1 + 2x$

$du = 2\,dx$

$3\int \frac{du}{u^{3/2}}$ ✓

$3\int u^{-3/2}\,du$

$= 3x - 2u^{-1/2} + C$ ✓

$= -6u^{-1/2} + C$

$\therefore = \frac{-6}{\sqrt{1+2x}} + C$ ✓

(b) $1 - 2\sin^2 x + 2\sin x\cos x + 1 = 0$ ✓

$2 - 2\sin^2 x + 2\sin x\cos x = 0$

$2(1 - \sin^2 x + \sin x\cos x) = 0$

$2(\cos^2 x + \sin x\cos x) = 0$

$2\cos x(\cos x + \sin x) = 0$ ✓

$\cos x = 0 \qquad \tan x = -1$

$x = 90, 270 \qquad x = 135, 315$

$\therefore x = 90, 135, 270, 315$ ✓ ✓

(c) $\alpha+\beta+\gamma=-2$

$\alpha\beta+\alpha\gamma+\beta\gamma=3$

$\alpha\beta\gamma=-5$

$\frac{\gamma+\beta+\alpha}{\alpha\beta\gamma}$ ✓

$=\frac{-2}{-5}$

$=\frac{2}{5}$ ✓

(d) $\alpha+\beta+\gamma=\frac{-6}{2}$

$1+(-2)+\alpha=-3$

$-1+\alpha=-3$

$\alpha=-2$ ✓

-2 is a double root

(e)

(i) $\tan 29=\frac{60}{QA}$

$\therefore QA=\frac{60}{\tan 29^\circ}$

$\therefore QB=\frac{60}{\tan 30^\circ}$ ✓

(ii) $\angle AQB=137-90$

$=47^\circ$ ✓

(iii)

$AB^2=QA^2+QB^2-2\times QA\times QB\times\cos 47$ ✓

$AB=84.69366\ldots$

$=85$ metres ✓

Question 13

(a)

$u=x^4+1$

$du=4x^3\,dx$

When $x=1$ $u=2$

When $x=0$ $u=1$

$\therefore \frac{1}{4}\int_1^2 \sqrt{u}\,du$ ✓

$=\frac{1}{4}\left[\frac{2u^{\frac{3}{2}}}{3}\right]_1^2$ ✓

$=\frac{1}{4}\left[\left(\frac{2\times 2^{\frac{3}{2}}}{3}\right)-\left(\frac{2\times 1}{3}\right)\right]$

$=0.305$ ✓

(b) LHS $=\frac{\sin 4\theta}{\cos^2\theta-\sin^2\theta}$

$=\frac{\sin 2(2\theta)}{\cos 2\theta}$ ✓

$=\frac{2\sin 2\theta\cos 2\theta}{\cos 2\theta}$

$=2\sin 2\theta$ ✓

$=2\times(2\sin\theta\cos\theta)$ ✓

$=4\sin\theta\cos\theta$

$=$ RHS

(c) $f(x)=x^2-x^{\frac{1}{2}}-2$

$f'(x)=2x-\frac{1}{2}x^{-\frac{1}{2}}$ ✓

$x=2-\frac{2^2-\sqrt{2}-2}{2\times 2-\frac{1}{2}\times 2^{-\frac{1}{2}}}$ ✓

$=1.84$ ✓ (2 d.p)

(d) $\frac{1}{2}\int 2\sin x\cos x\,dx$ ✓

$\frac{1}{2}\int\sin 2x\,dx=-\frac{1}{4}\cos 2x+C$ ✓

OR $-\frac{\cos^2 x}{2}+C$ OR $\frac{\sin 2x}{2}+C$

(e) $2\sin\theta - 1 = 0$

$2\sin\theta = 1$

$\sin\theta = \frac{1}{2}$

$\theta = 30^\circ$ ✓

$\therefore \theta = 180n + (-1)^n \times 30$ ✓

(f) $P(x) = (x+1)(x-2)^2$ ✓ ✓

Question 14

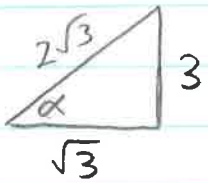
a) $3\cos\theta - \sqrt{3}\sin\theta = 0$ (cos, sin)

$R\sin(\theta - \alpha) = 0$

$R = \sqrt{3^2 + (\sqrt{3})^2}$

$= \sqrt{12}$

$= 2\sqrt{3}$ ✓



$\tan\alpha = \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$\tan\alpha = \sqrt{3}$

$\alpha = 60$ ✓

$\therefore 2\sqrt{3}\sin(\theta - 60) = 0$

$\sin(\theta - 60) = 0$

$\theta - 60 = 0, 180, 360$

$\theta = 60, 240$ ✓ ✓

(b) (i) $AP^2 = 5 \times 12$

$AP = \sqrt{60} = 2\sqrt{15}$ cm ✓

(ii)

$\angle ADB = \theta$ ($\angle$ b/w the tangent and the chord is equal to the $\angle$ in alt segm) ✓

$\angle ABD = \theta$ (alternate $\angle$s in $BD \parallel QP$) ✓

$\therefore \angle BAD = 180 - 2\theta$ ($\angle$ sum of Δ) ✓

$\therefore \angle BCD = 180 - (180 - 2\theta)$

$= 2\theta$ ✓

(opp $\angle$s in cyclic quad are supp)

(iii)

$AQ = BQ$ (tangent from an external point are equal)

$\therefore \angle QAB = \angle QBA = \theta$ (isosceles Δ) ✓

$\therefore \angle AQB = 180 - 2\theta$ ($\angle$ sum of Δ)

$\angle PCB + \angle PQB = 2\theta + (180 - 2\theta)$

$= 180$

$\therefore$ PQBC is a cyclic quad (opposite angles are supplementary) ✓

(c) $(-1, 7)$ $(5, -2)$

$k : 1$

(i) $P\left(\frac{5k-1}{k+1}, \frac{-2k+7}{k+1}\right)$ ✓ ✓

(ii) $5\left(\frac{5k-1}{k+1}\right) - 4\left(\frac{7-2k}{k+1}\right) - 1 = 0$ ✓

$25k - 5 - 28 + 8k - k - 1 = 0$

$33k - 34 = 0$ ✓

$32k = 34$

$k = \frac{34}{32} = \frac{17}{16}$

$k : 1$

$\therefore \frac{17}{16} : 1$

$\therefore 17 : 16$ ✓