Trinity Grammar School

Mathematics Department

2016

HALF-YEARLY EXAMINATION

PRELIMINARY ASSESSMENT TASK 3

Year 11

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- BOSTES approved calculators for this course may be used
- A Reference Sheet for this course is supplied
- In Questions 11-14 , show all relevant mathematical reasoning and/or calculations
- Write your BOSTES Student Number (Year 12 HSC) or Name (Year 11) and your Class teacher on the question paper and on any answer sheets or writing booklets used to write your responses to the questions submitted
- If you do not attempt a question you must submit an answer sheet or writing booklet for that question clearly indicating **N/A** and your Student Number or Name.

- **Preliminary Assessment Weighting:** 35%

BOSTES Student Number
(Year 12 only)

Class Teacher:

……

Name:
(Year 10 or 11 only)

……

Do NOT write solutions on this question paper. Any working on this question paper will NOT be marked.

Date of examination:

Tuesday, 3 May 2016

Total marks – 70

Section I

10 marks

- Attempt all Questions
- Allow about **15** minutes for this section

Section II

60 marks

- Attempt all Questions
- Allow about **1** hours **45** minutes for this section

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Section I 10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section
- Shade the correct response on the multiple-choice answer sheet for Questions 1 – 10
- Each question is worth 1 mark

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1 The formula for the volume of a cone with radius r and perpendicular height h is given by $V = \frac{1}{3}\pi r^2 h$. Which of the following correctly expresses r as the subject of this formula?

(A) $r = \sqrt{\frac{3V}{\pi h}}$

(B) $r = \frac{\sqrt{3V}}{\pi h}$

(C) $r = \sqrt{\frac{Vh}{3\pi}}$

(D) $r = \sqrt{\frac{\pi h}{3V}}$

2 The solutions to $3x^2 - 4x - 4 = 0$ are

(A) $-\frac{2}{3},\ 2$

(B) $-\frac{2}{3},\ -2$

(C) $-\frac{3}{2},\ 2$

(D) $-\frac{3}{2},\ -2$

3 What is the value of $\lim_{x \to 3} \frac{x-3}{x^2-x-6}$?

(A) $-\frac{1}{5}$

(B) 0

(C) $\frac{1}{5}$

(D) 3

4 For the function $f(x) = \frac{2}{3x-1}$ in the domain $-5 \leq x \leq 5$, state the range and the equation of the vertical asymptote.

(A) $y \leq -\frac{1}{8}$ and $y \geq \frac{1}{7}$, $x = -\frac{1}{3}$

(B) $y \leq -\frac{1}{8}$ and $y \geq \frac{1}{7}$, $x = \frac{1}{3}$

(C) $-\frac{1}{8} \leq y \leq \frac{1}{7}$, $x = -\frac{1}{3}$

(D) $-\frac{1}{8} \leq y \leq \frac{1}{7}$, $x = \frac{1}{3}$

5 The solution to the equation $3^{2x-1} = 81$ is:

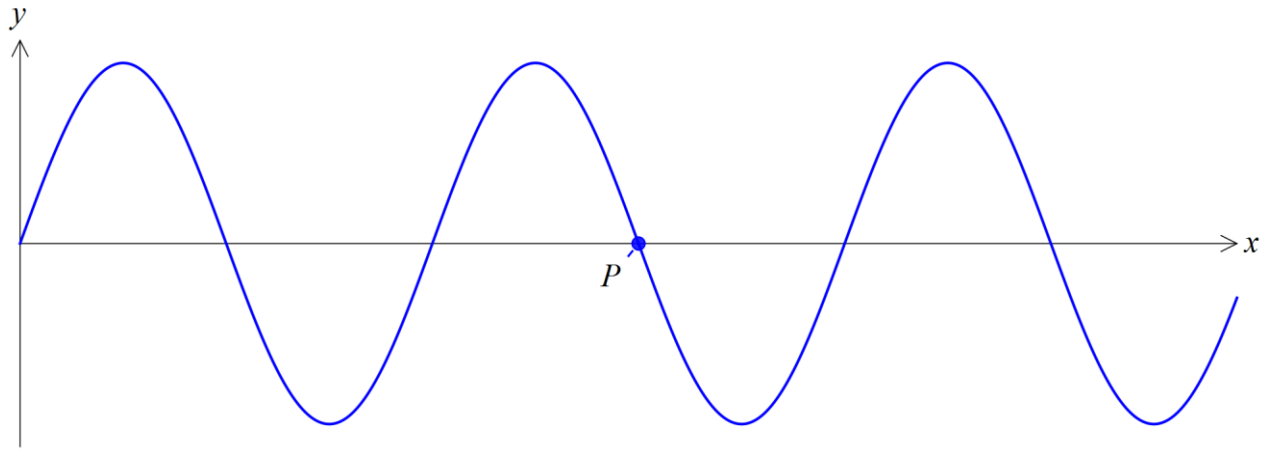
(A) $x = \frac{3}{2}$

(B) $x = 2$

(C) $x = \frac{5}{2}$

(D) $x = 6$

6 The graph of the function $y = \sin x$ is shown below. What are the coordinates of the point P?



(A) (180°, 0)

(B) (360°, 0)

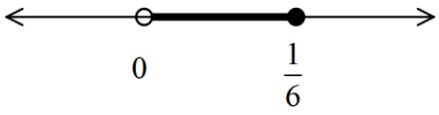
(C) (540°, 0)

(D) (720°, 0)

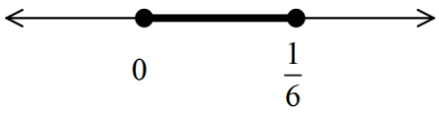
7 The solution to $\frac{1}{x} \geq 6$, when graphed on a number line, is:

(A)

(B)

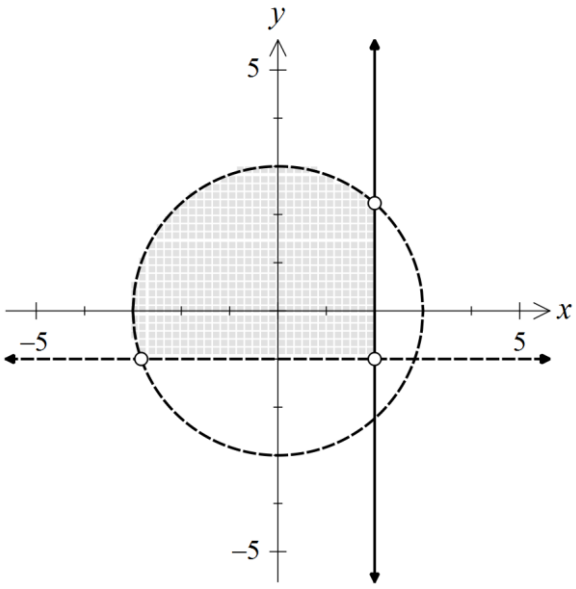


(C)



(D)

8 The inequalities that best describes the region below are:



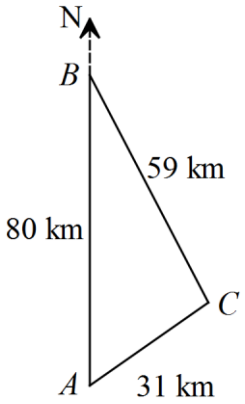
(A) $x^2 + y^2 > 3\ , y > -1\ ,\ x < 2$

(B) $x^2 + y^2 < 3, y > -1, x \leq 2$

(C) $x^2 + y^2 < 3\ ,\ y > -1\ , x < 2$

(D) $x^2 + y^2 > 3\ , y < -1, x \geq 2$

9 Town *B* is 80 km due north of Town *A* and 59 km from Town *C*. Town *A* is 31 km from Town *C*.



What is the bearing of Town *C* from Town *B*?

(A) $019°T$

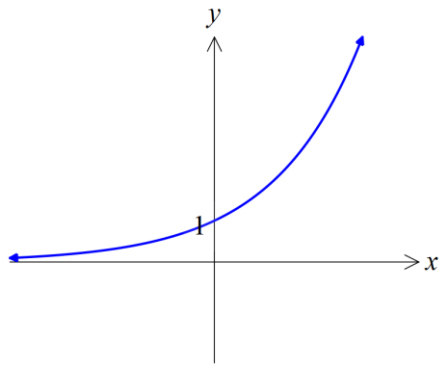
(B) $122°T$

(C) $161°T$

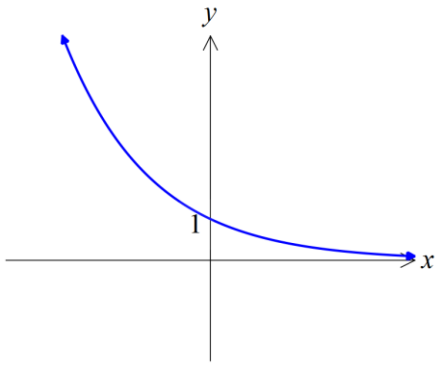
(D) $341°T$

10 Which graph best represents the equation $y = -4^{-x}$?

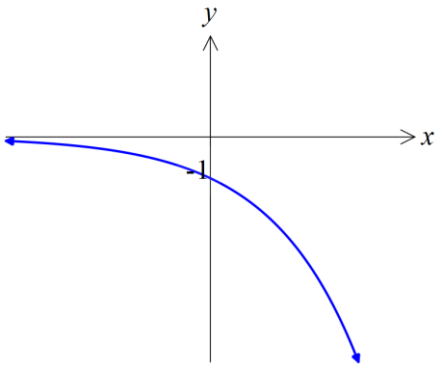
(A)



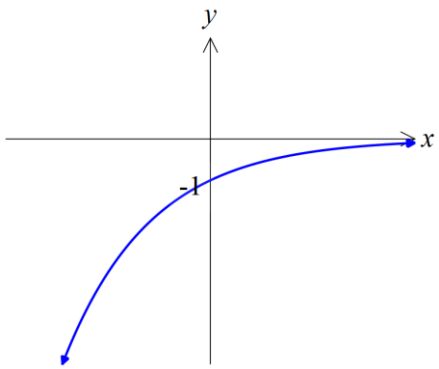
(B)



(C)



(D)



End of Section I

Section II commences on the next page

Section II 60 marks

- Attempt Questions 11-14
- Allow about 1 hours and 45 minutes for this section
- Answer each question in a SEPARATE writing booklet or answer sheet. Extra writing booklets are available.
- In Questions 11 – 14, your responses should include all relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet

(a) Find u in $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$ if $f = 2.34$ and $v = 7.61$. Give your answer correct to 3 significant figures. **1**

(b) Simplify $\frac{2^{4n} \times 3^{2n}}{8^n \times 6^n} + 3^n$ **3**

(c) Solve the following system of equations simultaneously. **4**

$3u + v - 4w = -4$
$u - 2v + 7w = -7$
$4u + 3v - w = 9$

(d) Simplify, writing your answer with a rational denominator: **3**

$$\frac{5}{6-\sqrt{3}}-\frac{3}{5+\sqrt{3}}$$

(e) A lighthouse is situated at the top of a cliff, and a boat is 150 m from the base of the cliff. A person on the boat measures the angles of elevation to the bottom and top of the lighthouse to be 30° and 40° respectively.

(i) Draw a diagram to represent this information. **1**

(ii) Find the height of the lighthouse, correct to the nearest metre. **3**

End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet

(a) Simplify $\dfrac{8x^2+14x+3}{8x^2-10x+3} \times \dfrac{12x^2-6x}{4x^2+5x+1} \div \dfrac{18x^2-6x}{4x^2+x-3}$ **4**

(b) (i) Solve $|2x| = |x+3|$. **2**

(ii) On the same set of axes, sketch the graphs of $y = |2x|$ and $y = |x+3|$, including any points of intersection and intercepts. **2**

(iii) On your diagram, shade the region where $y \leq |2x|$ and $y \geq |x+3|$ hold simultaneously. **1**

(c) Prove $\dfrac{\sin^3 x + \cos^3 x}{\sin x + \cos x} \equiv 1 - \sin x \cos x$ **3**

(d) Solve $x - \dfrac{1}{x} \leq 0$ and graph the solution on a number line. **3**

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet

(a) If $\cos\theta = -\frac{3}{4}$ and $\sin\theta > 0$, find the exact value of $\cot\theta$. **2**

(b) Solve $\sqrt{3}\sec 2\theta = 2$ for the domain $0° \leq \theta \leq 360°$. **3**

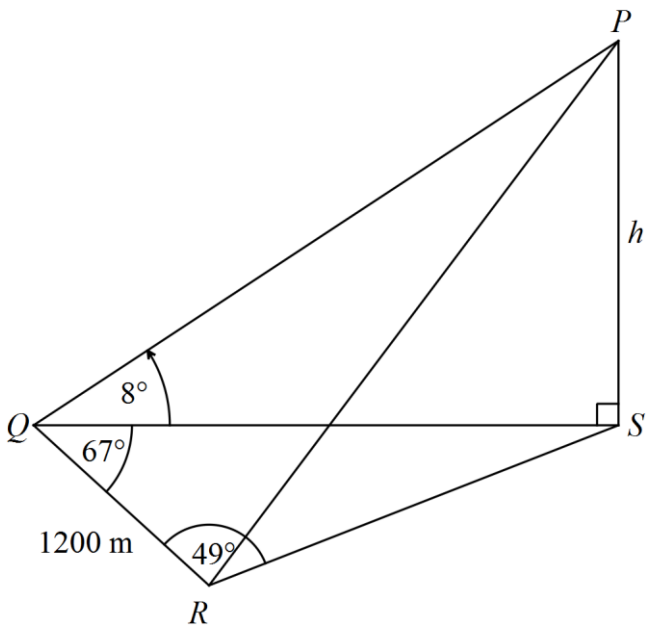
(c) Given $f(x) = \begin{cases} x^2 & x < -2 \\ 3x & -2 \leq x \leq 2 \\ \frac{1}{x} & x > 2 \end{cases}$

(i) Evaluate $f(-1) + f\left(2\frac{1}{2}\right)$. **2**

(ii) State the range. **1**

(iii) Sketch the function. **2**

(d) In the diagram below Q, R, and S, are on level ground and SP is a tower, i.e $\angle PSQ = \angle PSR = 90°$. From Q, the elevation of the top of the tower, P, is 8°. Also, the distance between Q and R is 1200 m, $\angle SQR = 67°$ and $\angle SRQ = 49°$.



(i) Show that $QS = \frac{1200\sin 49°}{\sin 64°}$ **2**

(ii) Find the height of the tower, h, to the nearest metre. **1**

(iii) Find the angle of elevation of P from R, to the nearest degree. **2**

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet

(a) (i) Factorise $3x^3 + 3x^2 - x - 1$. **1**

(ii) Hence, or otherwise, solve $3\tan^3\theta + 3\tan^2\theta - \tan\theta - 1 = 0$ for $0° \leq \theta \leq 180°$. **3**

(b) Find $\lim_{x \to \infty} \frac{x^2}{(x-1)(x-2)}$. **2**

(c) Is the function $y = x^7 - 3x^5$ odd, even or neither? Show all working to justify your answer. **3**

(d) From a lighthouse, *L*, a ship, *S*, bears $053°T$ and is at a distance of 8 nautical miles from the base of the lighthouse. From the lighthouse a boat, *B*, is on a bearing of $293°T$ and is at a distance of 6 nautical miles.

(i) Find the exact distance of the ship from the boat, leaving your answer in simplest surd form. **3**

(ii) Find the bearing of the ship from the boat, giving your answer to the nearest degree. **3**

End of Question 14
End of Section II
End of Examination

Trinity Grammar School
Mathematics Department
2016
Year 11 Mathematics Extension 1
HALF-YEARLY EXAMINATION

Preliminary Assessment Task 3
Solutions

SOLUTIONS

1	●	B	C	D
2	●	B	C	D
3	A	B	●	D
4	A	●	C	D
5	A	B	●	D
6	A	B	●	D
7	A	●	C	D
8	A	●	C	D
9	A	B	●	D
10	A	B	C	●

Question 11

(a)
$$\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$$
$$\frac{1}{2\cdot 34}=\frac{1}{u}+\frac{1}{7\cdot 61}$$
$$u = 3\cdot 37901...$$
$$= 3\cdot 38$$
1 mark

(b)
$$\frac{2^{4n}\times 3^{2n}}{8^n\times 6^n}+3^n$$
$$=\frac{2^{4n}\times 3^{2n}}{2^{3n}\times 2^n\times 3^n}+3^n$$ 1 mark
$$=\frac{\cancel{2^{4n}}\times 3^{2n}}{\cancel{2^{4n}}\times 3^n}+3^n$$ 1 mark
$$= 3^n + 3^n$$
$$= 2.\ 3^n$$ 1 mark

(c)
$$3u + v - 4w = -4 \quad (1)$$
$$u - 2v + 7w = -7 \quad (2)$$
$$4u + 3v - w = 9 \quad (3)$$

Multiply (1) by 2
$$6u + 2v - 8w = -8 \quad (4)$$

Multiply (1) by -3
$$-9u - 3v + 12w = 12 \quad (5)$$

Add (4) and (2)
$$7u - w = -15 \quad (6)$$
Add (5) and (3)
$$-5u + 11w = 21 \quad (7)$$

Rearrange (6)
$$w = 7u + 15$$

Substitute into (7)
$$-5u + 11(7u + 15) = 21$$
$$-5u + 77u + 165 = 21$$
$$72u = -144$$
$$u = -2$$

$\therefore$
$$w = 7(-2) + 15$$
$$= 1$$

$$3(-2) + v - 4(1) = -4$$
$$v = 6$$

2 progress marks
2 mark total u, v, w

(d) $\dfrac{5}{6-\sqrt{3}} - \dfrac{3}{5+\sqrt{3}}$

$= \dfrac{5}{6-\sqrt{3}} \times \dfrac{6+\sqrt{3}}{6+\sqrt{3}} - \dfrac{3}{5+\sqrt{3}} \times \dfrac{5-\sqrt{3}}{5-\sqrt{3}}$ 1 mark for rationalising

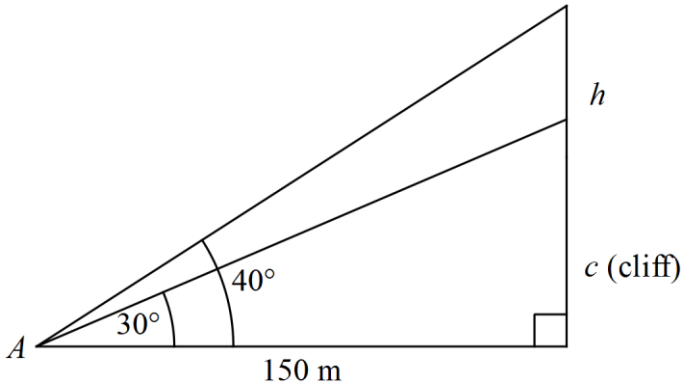
$= \dfrac{30+5\sqrt{3}}{36-3} - \dfrac{15-3\sqrt{3}}{25-3}$

$= \dfrac{2(30+5\sqrt{3}) - 3(15-3\sqrt{3})}{66}$ 1 mark for common denominator

$= \dfrac{15+19\sqrt{3}}{66}$ 1 mark

(e)

(i) 1 mark for diagram with angles and base length labelled.



(ii) $\tan 30 = \dfrac{c}{150}$

$c = 150\tan 30$ 1 mark

$\tan 40 = \dfrac{h+c}{150}$

$h + c = 150\tan 40$ 1 mark

$h = 150\tan 40 - c$

$= 150\tan 40 - 150\tan 30$

$= 39{\cdot}26...$

$= 39m$ 1 mark. *MUST* be to nearest m, *ROUNDING QUESTION*

End of Question 11

Question 12

(a)

$$\frac{8x^2+14x+3}{8x^2-10x+3}\times\frac{12x^2-6x}{4x^2+5x+1}\div\frac{18x^2-6x}{4x^2+x-3}$$

$$=\frac{(4x+1)(2x+3)}{(2x-1)(4x-3)}\times\frac{6x(2x-1)}{(4x+1)(x+1)}\times\frac{(x+1)(4x-3)}{6x(3x-1)}$$

1 mark each fraction factorised (3 marks)

$$=\frac{2x+3}{3x-1}$$

1 mark

(b)

(i) $|2x|=|x+3|$

$2x=x+3$ or $-2x=x+3$

$x=3$ $-3x=3$

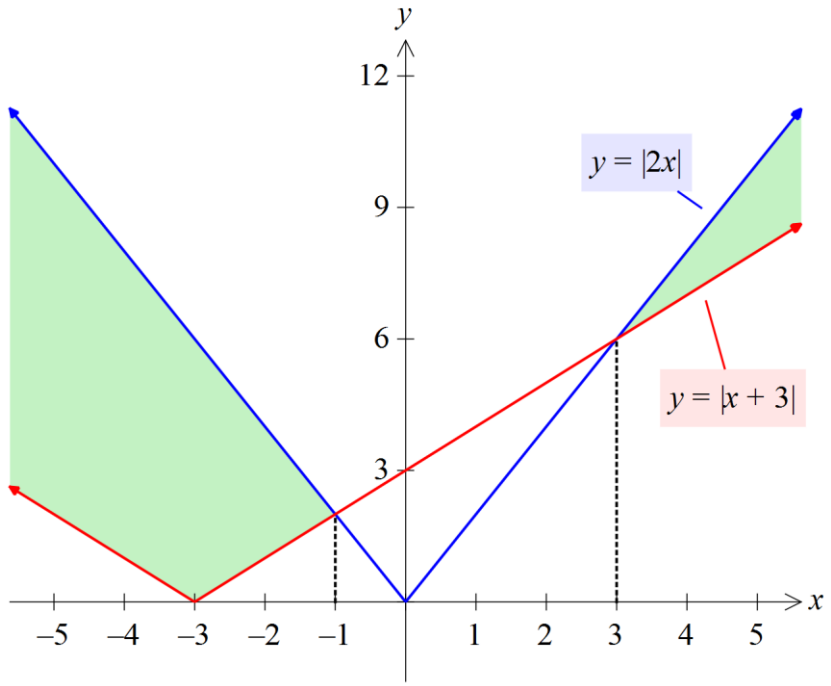
$x=-1$

1 mark each value of x

(ii) 1 mark correct lines

1 mark for labels of intercepts AND intersections

(iii) 1 mark correct shading



SOLUTIONS

(c)

$$\frac{\sin^3 x + \cos^3 x}{\sin x + \cos x} \equiv 1 - \sin x \cos x$$

$$\text{LHS} = \frac{\sin^3 x + \cos^3 x}{\sin x + \cos x}$$

$$= \frac{(\cancel{\sin x + \cos x})(\sin^2 x - \sin x \cos x + \cos^2 x)}{\cancel{\sin x + \cos x}}$$

1 mark cubes factorisation

$$= \sin^2 x + \cos^2 x - \sin x \cos x$$

1 mark for division

$$= 1 - \sin x \cos x$$

1 mark for $\sin^2 x + \cos^2 x = 1$

$$= RHS$$

(d) $x - \frac{1}{x} \leq 0$

Solve

$$x - \frac{1}{x} = 0$$

$$x^2 - 1 = 0$$

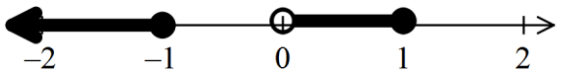
$$x = \pm 1$$

$$x \neq 0$$

1 mark

$x \leq -1$ or $0 < x \leq 1$

1 mark including inequality sign on 0

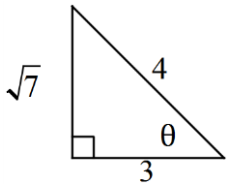


1 mark

End of Question 12

Question 13

(a) $\cos\theta = -\frac{3}{4}$ and $\sin\theta > 0$, therefore 2nd quadrant.



$\therefore \cot\theta = \frac{-3}{\sqrt{7}}$ 1 mark for fraction, 1 mark for sign

(b) $\sqrt{3}\sec 2\theta = 2 \qquad 0° \leq \theta \leq 360°$

$\sec 2\theta = \frac{2}{\sqrt{3}} \qquad 0° \leq 2\theta \leq 720°$ 1 mark for domain change

$2\theta = 30, 360 - 30, 360 + 30, 720 - 30$

$= 30°, 330°, 390°, 690°$ 1 mark for 30° and 330°

$\therefore \quad \theta = 15°, 165°, 195°, 345°$ 1 mark for halving

(c)

(i) $f(-1) + f\left(2\frac{1}{2}\right)$.

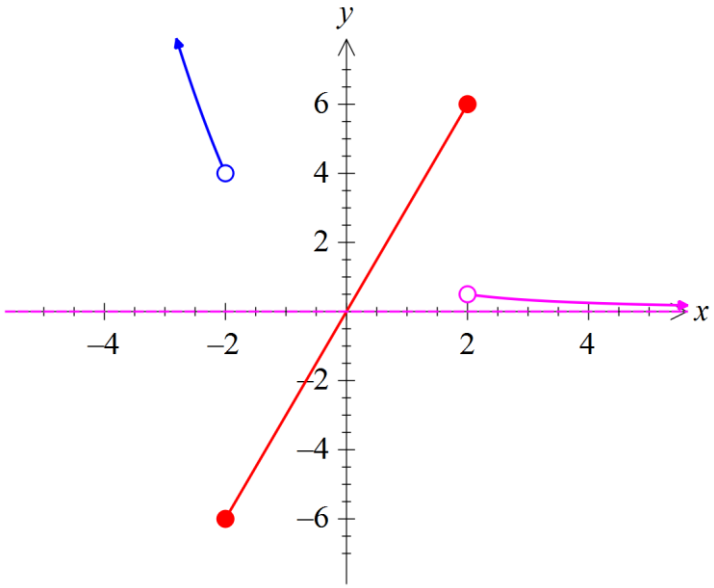
$= 3 \times (-1) + \frac{1}{\frac{5}{2}}$ 1 mark

$= -2\frac{3}{5}$ 1 mark

(ii) $y \geq -6$ 1 mark

(iii) 1 mark for lines and shape

1 mark for boundary points



(d)

(i) $\angle QSR = 180 - (49 + 67)$

$= 64°$ 1 mark

In ΔQRS

$$\frac{QS}{\sin 49} = \frac{1200}{\sin 64}$$ 1 mark

$$QS = \frac{1200\sin 49°}{\sin 64°}$$

(ii) $\tan 8 = \frac{h}{QS}$

$h = QS \tan 8$

$= 141{\cdot}613\ldots$

$= 142\ m$ 1 mark

(iii) $$\frac{RS}{\sin 67} = \frac{1200}{\sin 64}$$

$$RS = \frac{1200\sin 67}{\sin 64}$$ 1 mark

$\tan(\angle PRS) = \frac{h}{RS}$

$= 6{\cdot}57\ldots$

$= 7°$ 1 mark

End of Question 12

Question 14 (15 marks) Use a SEPARATE writing booklet

(a) (i) $3x^3 + 3x^2 - x - 1$

$= 3x^2(x + 1) - (x + 1)$

$= (x + 1)(3x^2 - 1)$ 1 mark

(ii) $3\tan^3\theta + 3\tan^2\theta - \tan\theta - 1 = 0 \quad 0 \leq \theta \leq 180$

$(\tan\theta + 1)(3\tan^2\theta - 1) = 0$ 1 mark

$\tan\theta + 1 = 0$

$\tan\theta = -1$

$\theta = 180 - 45$

$= 135°$ 1 mark

OR

$3\tan^2\theta - 1 = 0$

$\tan^2\theta = \frac{1}{3}$

$\tan\theta = \frac{\pm 1}{\sqrt{3}}$

$\theta = 30, 180 - 30$

$= 30°, 150°$ 1 mark

(b) $\lim_{x \to \infty} \frac{x^2}{(x-1)(x-2)}$

$= \lim_{x \to \infty} \frac{x^2}{x^2 - 3x + 2}$

$= \lim_{x \to \infty} \frac{\frac{x^2}{x^2}}{\frac{x^2}{x^2} - \frac{3x}{x^2} + \frac{2}{x^2}}$ 1 mark for dividing

$= \lim_{x \to \infty} \frac{1}{1 - \frac{3}{x} + \frac{2}{x^2}}$

$= \frac{1}{1 - 0 + 0} = 1$ 1 mark

(c) For odd, $f(-x) = -f(x)$

$f(x) = x^7 - 3x^5$

$f(-x) = (-x)^7 - 3(-x)^5$ 1 mark

$= -x^7 + 3x^5$

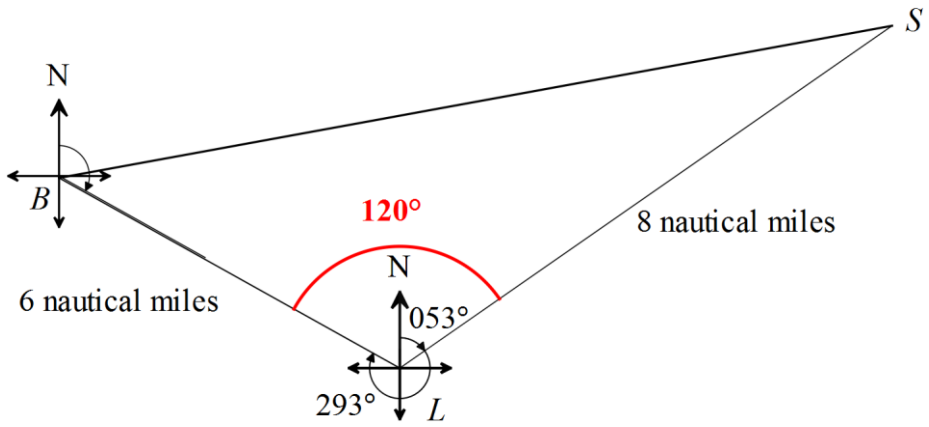
$= -(x^7 - 3x^5)$ 1 mark

$= -f(x)$

$\therefore$ the function is odd as $f(-x) = -f(x)$ 1 mark

SOLUTIONS

(d)



(i)

$BS^2 = 6^2 + 8^2 - 2.\ 6.\ 8\cos 120$ 1 mark for 120°; 1 mark for substitution

$= 36 + 64 - 96.\left(\frac{-1}{2}\right)$

$= 148$

$BS = \sqrt{148}$

$= 2\sqrt{37}$ 1 mark

(ii)

$$\frac{\sin \angle SBL}{8} = \frac{\sin 120}{2\sqrt{37}}$$

$$\sin \angle SBL = \frac{8\sin 120}{2\sqrt{37}}$$

$\angle SBL = 34{\cdot}7\ldots$

$= 35°$ 1 mark

$\angle NBL = 180 - 67$

$= 113°$ 1 mark

$\therefore \quad \angle NBS = 78°$

$\therefore$ The bearing is $078°T$. 1 mark

End of Question 14
End of Section II
End of Examination