Student Name: ……………………………………

Teacher's Initials: ……………………

Barker College

2017
YEAR 11
EXAMINATION
TERM 3

Mathematics Extension 1

Staff Involved:

- **AXD**
- **WMD**
- **HG**
- **RMH**
- **KJL**
- **ESP***

PM WEDNESDAY 13TH SEPTEMBER

110 copies

- **Attempt Questions 1 to 6**
- **Answer in the booklets provided.**
- **Start each question in a new booklet.**
- **Write using blue or black pen for working.**
- **Show all necessary working. Marks may not be awarded for careless or poorly arranged working.**
- **Write your name and teacher's initials on each booklet.**
- **Board-approved calculators may be used.**

Total marks – 75

Working time – 90 minutes

Topics Covered:

- **All topics in 2 Unit Mathematics exam**
- **Geometric Applications of the Derivative**
- **Quadratic Functions**
- **Locus and the Parabola**
- **Circle Geometry**
- **3 Dimensional Trigonometry**
- **Trigonometric Sums and Difference and the Angle Between Two Lines**
- **Harder Inequalities**
- **Division of an Interval in a Given Ratio**

QUESTION 1 (13 Marks) **[START A NEW BOOKLET]** **Marks**

(a) Show that if $f(x) = 2x(1+4x)^5$ then $f'(x) = 2(24x+1)(1+4x)^4$ **2**

(b) Solve $\dfrac{7}{x-2} \geq 3$ **3**

(c) Factorise $9a^3 - 72b^3$ **2**

(d) Solve $\tan^2(2\theta) = 3$ for $-180° \leq \theta \leq 180°$ **3**

(e) Find the equation of the locus of a point P(x,y) that moves such that it is always equidistant from (3,1) and (-1,-2) **3**

End of Question 1

QUESTION 2 (13 Marks) **[START A NEW BOOKLET]** **Marks**

(a) For the curve $y = x^3 - 3x + 3$,

(i) Find the coordinates of any stationary points and determine their nature. **3**

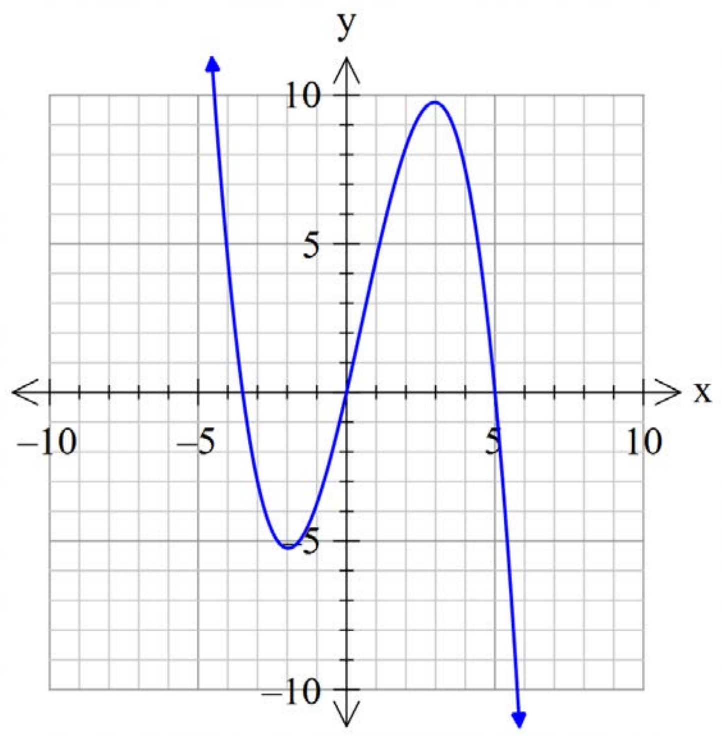
(ii) Find any points of inflexion. **2**

(iii) Sketch the curve showing the above information.
Your graph should take about 1/3 of the page. **2**

(iv) Find the equation of the tangent at the point of inflexion **2**

(b) Find the coordinates of the focus and the equation of the directrix of $y^2 = 8x$ **2**

(c) Sketch the gradient function of the following graph. Label any x intercepts on your graph. **2**



End of Question 2

QUESTION 3 (13 Marks) **[START A NEW BOOKLET]** **Marks**

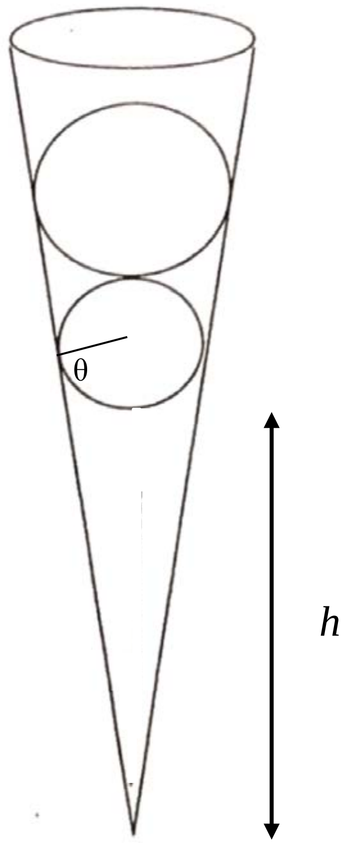
(a) If α and β are the zeros of the function $y = x^2 + 2x - 1$, find the value of $\alpha^2 + \beta^2$ **3**

(b) Find the obtuse angle between the lines $4x - y + 6 = 0$ and $x + 3y - 7 = 0$.
Give your answer to the nearest minute. **3**

(c) (i) Give the expression for $\tan(2x)$ in terms of $\tan x$ **1**

(ii) Hence show if $f(x) = x \cot x$ then $f(2x) = (1 - \tan^2 x) f(x)$ **3**

(d)



Two marbles of radius 3cm and 5cm are placed in an inverted cone so that the marbles touch each other and the sides of the cone.

(i) Find θ, the angle the smaller marble makes with the side of the cone at the point where they touch. Give a reason for your answer. **1**

(ii) Using similar triangles, find the distance h from the vertex of the cone to the smallest marble. **2**

End of Question 3

QUESTION 4 (12 Marks) **[START A NEW BOOKLET]** **Marks**

(a) Find the domain and range of $y = \sqrt{x^2 - 16}$ **2**

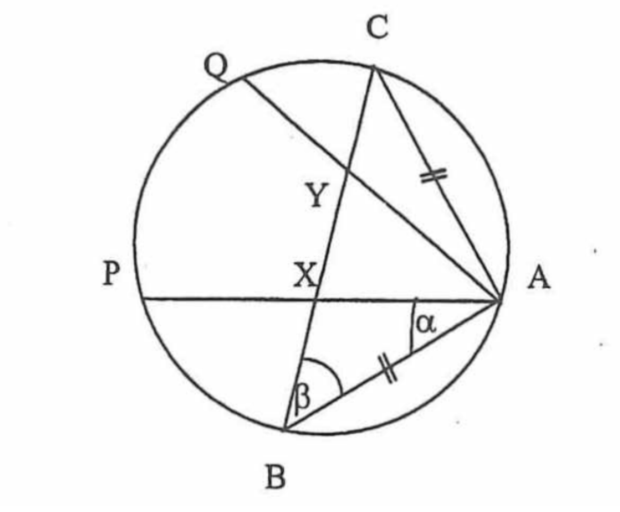
(b) Point A has coordinates (- 3, 1) and point B has coordinates (b, - 4).
The point P (11, - 8) divides the interval AB externally in the ratio 9:4. Find b. **2**

(c) Two houses, P and Q, lie in the same plane as S. S is the foot of a hill RS.
The height of the hill is 200m and the angle of elevation from P to the top of the hill is 14°.
P and Q subtend an angle of 75° at S and Q is 500m from S.

(i) Draw a diagram to show the above information. **1**

(ii) Find the distance between the two houses, to the nearest metre. **3**

(d)



Let $ABPQC$ be a circle with AB = AC.
AP intersects BC at X and AQ intersects BC at Y. Let $\angle PAB = \alpha$ and $\angle ABC = \beta$.
Trace or draw the diagram in your answer booklet.

(i) Explain why $\angle AXC = \alpha + \beta$ **1**

(ii) Explain why $\angle PQB = \alpha$ **1**

(iii) Prove $XYQP$ is a cyclic quadrilateral **2**

End of Question 4

QUESTION 5 (12 Marks) **[START A NEW BOOKLET]** **Marks**

(a) Find the values of m for which $-x^2 + (m-2)x - 25$ has no real roots. **3**

(b) A piece of wire 84cm long is to be cut into two pieces.
One piece is used to form a square and the other a rectangle 3 times as long as it is wide.

(i) Let x be the side length of the square and y be the width of the rectangle.
Show the sum of the areas of the square and rectangle is $A = 7(y^2 - 12y + 63)$ **3**

(ii) Find the lengths of both pieces if the sum of the areas is to be a minimum. **3**

(c) (i) Simplify $(\cos x + \sin x)(\cos x - \sin x)$ **1**

(ii) Hence, or otherwise, prove $\dfrac{\cos x + \sin x}{\cos x - \sin x} = \dfrac{\sin 2x + 1}{\cos 2x}$ **2**

End of Question 5

QUESTION 6 (12 Marks) **[START A NEW BOOKLET]** **Marks**

(a) Consider the function $f(x) = \dfrac{2x^2}{1-x^2}$

(i) Show $f(x)$ is an even function. **1**

(ii) Find any vertical asymptotes of $f(x)$. **1**

(iii) Find the turning point and determine its nature. **3**

(iv) Write the following sentence in your answer booklet and fill in the blank:
As x approaches $\pm\infty$, *y approaches* ________ **1**

(v) Sketch $f(x)$ **2**

(b) (i) Sketch $y = \dfrac{5}{|x-1|}$ **2**

(ii) Hence, or otherwise, find where $y < 3$ **2**

End of Question 6
End of Paper

Year 11 Ext 1 Yearly Exam T3 2017 Student Solutions

Q1 a) $f'(x) = 2x \times 5 \times 4(1+4x)^4 + 2(1+4x)^5$

$= 40x(1+4x)^4 + 2(1+4x)^5$

$= 2(1+4x)^4[20x + 1 + 4x]$

$= 2(24x+1)(1+4x)^4$

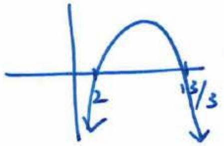
b) $x \neq 2$ $\quad \dfrac{7}{x-2} - 3 \geq 0$

$\dfrac{7}{x-2} - \dfrac{3(x-2)}{x-2} \geq 0$

$\dfrac{13-3x}{x-2} \geq 0$

$(x-2)^2\dfrac{(13-3x)}{(x-2)} \geq 0$

$(x-2)(13-3x) \geq 0$



$2 < x \leq 13/3$

c) $9(a-2b)(a^2+2ab+4b^2)$

d) $\tan 2\theta = \pm\sqrt{3}$ $\quad -360° \leq 2\theta \leq 360°$

$2\theta = \pm 60°, \pm 120°, \pm 240°, \pm 300°$

$\theta = \pm 30°, \pm 60°, \pm 120°, \pm 150°$

e) $P(x,y)$ let $A(3,1)$ $B(-1,-2)$

Now $PA^2 = PB^2$

$(x-3)^2 + (y-1)^2 = (x+1)^2 + (y+2)^2$

$x^2 - 6x + 9 + y^2 - 2y + 1 = x^2 + 2x + 1 + y^2 + 4y + 4$

$-8x - 6y + 5 = 0$

$8x + 6y - 5 = 0$

Q2 a)(i) $y' = 3x^2 - 3$

$y'' = 6x$ for stat pts $y' = 0$

$0 = 3x^2 - 3$

$0 = x^2 - 1$

$0 = (x+1)(x-1)$

$\therefore x = 1 \quad x = -1$

$y = 1 \quad y = 5$

when $x = 1$ $y'' = 6 > 0$

$\therefore (1,1)$ min t.p.

when $x = -1$ $y'' = -6 < 0$

$\therefore (-1,5)$ max t.p.

(ii) for inflexions $y'' = 0$

$6x = 0$

$x = 0$

$y = 3$

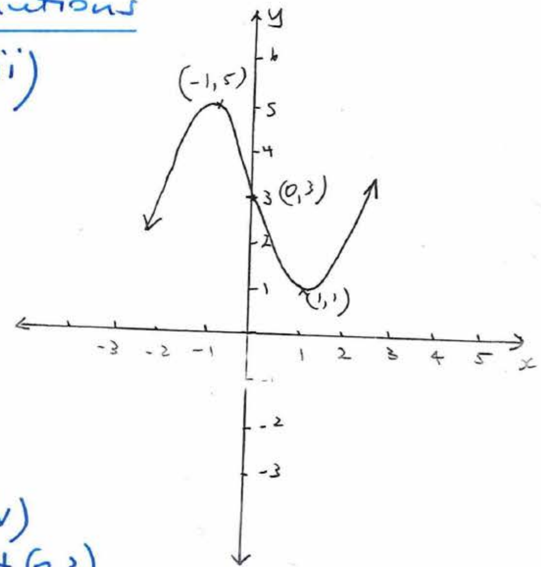
$\therefore (0,3)$ possible POI

x	-1	0	1
y''	-6	0	6

$\therefore$ change in concavity

$\therefore (0,3)$ POI.

(iii)



(iv) at $(0,3)$

$y' = 3x^2 - 3$

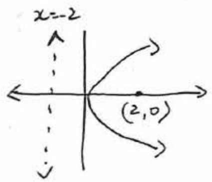
$m = -3$

$y - 3 = -3(x - 0)$

$y = -3x + 3$ or $3x + y - 3 = 0$

b) $y^2 = 8x$ $\quad$ vertex $(0,0)$

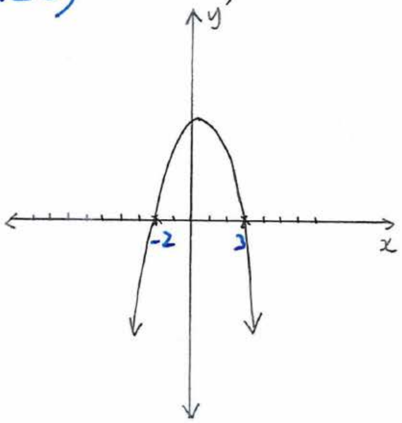
$a = 2$



focus $(2,0)$

directrix $x = -2$

Q2 c)



Q3 a) $\alpha+\beta=-2 \qquad \alpha\beta=-1$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=(-2)^2-2(-1)=6$

b) $4x-y+6=0 \qquad y=4x+6$

$m_1=4$

$x+3y-7=0 \qquad y=-\frac{1}{3}x+7/3$

$m_2=-\frac{1}{3}$

$\tan\theta=\left|\dfrac{4+\frac{1}{3}}{1-4/3}\right|=13$

$\theta=85°36'$

obtuse $\theta=180-85°36'$

$=94°23'55.34''$

$=94°24'$ (nearest min)

c)(i) $\tan 2x=\dfrac{2\tan x}{1-\tan^2 x}$

c)(ii) $f(2x)=2x\cot 2x$

$=\dfrac{2x}{\tan 2x}$

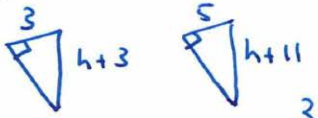
$=2x \times \dfrac{(1-\tan^2 x)}{2\tan x}$

$=x\cot x(1-\tan^2 x)$

$=(1-\tan^2 x)f(x).$

d)(i) $\theta=90°$ the angle between the radius and tangent to a circle is $\underline{h}$.

(ii)



$\dfrac{3}{5}=\dfrac{h+3}{h+11}$

$3h+33=5h+15$

$h=9$

Q4 a) $x^2-16\geq 0 \qquad (x-4)(x+4)\geq 0$

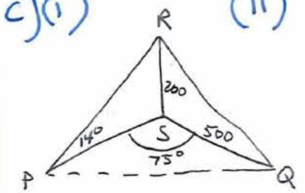
Domain
$x\geq 4$
$x\leq -4$

Range $y\geq 0$

b) $11,-8=\left(\dfrac{9b+12}{5},\dfrac{-36-4}{5}\right)$

$\dfrac{9b+12}{5}=11 \qquad b=43/9$

c)(i)

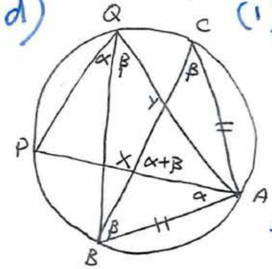


(ii) $PS=\dfrac{200}{\tan 14°}$

$PQ^2=\left(\dfrac{200}{\tan 14°}\right)^2+500^2-2\left(\dfrac{200}{\tan 14°}\right)\times 500\cos 75°$

$PQ=828.16=828$ m (nearest m)

d)



(i) $\angle AXC=\alpha+\beta$ exterior angle of Δ is equal to sum of opp. interior $\angle$'s.

(ii) $\angle PQB=\angle PAB=\alpha$ angles at the circum. standing on the same arc.

(iii) In ΔABC $\angle ACB=\beta$ (base angles in isos. Δ)

Now $\angle BQA=\beta$ (angles at the circum. standing on the same arc.)

From (i) $\angle AXB=\alpha+\beta$

$\therefore \angle YXP=180-(\alpha+\beta)$ (straight line)

$\angle PQA=\alpha+\beta$ on diagram

Now $\angle PQA+\angle YXP=180°$

$(\alpha+\beta)+(180-\alpha+\beta)=180°$

$\therefore$ XYQP is a cyclic quadrilateral since opp $\angle$'s supplementary.

Q5 a) $\Delta=(m-2)^2-4\times-1\times-25$

$=m^2-4m-96$

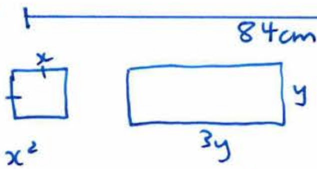
for no real roots $\Delta<0$

$m^2-4m-96<0$

$(m-12)(m+8)<0$

$-8<m<12$

5b)



$A = 3y^2$
$P = 8y$

$A = x^2$
$P = 4x$

$4x + 8y = 84$
$x = 21 - 2y$

(i) Sum Areas $= x^2 + 3y^2$
$= (21 - 2y)^2 + 3y^2$
$= 7y^2 - 84y + 441$
$= 7(y^2 - 12y + 63)$

(ii) $A' = 14y - 84$ for min $A' = 0$
$0 = 14y - 84$
$y = 6$
$A'' = 14 > 0 \quad \therefore y = 6$ is a min
$x = 21 - 2(6)$
$x = 9$
length $P_1 = 4x = 4(9) = 36$cm
length $P_2 = 8y = 8(6) = 48$cm
Check $36 + 48 = 84$cm

5c)(i) $\cos^2 x - \sin^2 x$

(ii) RHS $= \dfrac{2\sin x \cos x + 1}{\cos^2 x - \sin^2 x}$

LHS $= \dfrac{\cos x + \sin x}{\cos x - \sin x} \times \dfrac{\cos x + \sin x}{\cos x + \sin x}$

$= \dfrac{\cos^2 x + 2\sin x \cos x + \sin^2 x}{\cos^2 x - \sin^2 x}$

$= \dfrac{1 + 2\sin x \cos x}{\cos^2 x - \sin^2 x} =$ RHS

Q6 (i) $f(-x) = \dfrac{2(-x)^2}{1-(-x)^2} = \dfrac{2x^2}{1-x^2} = f(x)$

$\therefore$ even function

(ii) $1 - x^2 = 0$
$(1-x)(1+x) = 0$
$\therefore x = 1$ or $x = -1$ are vertical asymptotes

(iii) $f'(x) = \dfrac{(1-x^2)(4x) - 2x^2(-2x)}{(1-x^2)^2}$

$= \dfrac{4x}{(1-x^2)^2}$ for t.p. $f'(x) = 0$

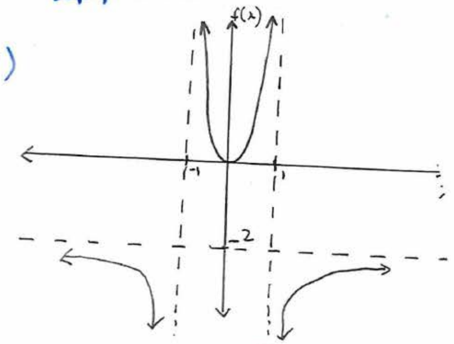
$0 = \dfrac{4x}{1-x^2} \quad \therefore x = 0, y = 0$

x	$-\frac{1}{2}$	0	$\frac{1}{2}$
$f'(x)$	-3.5	0	3.5
shape	\	—	/

$\therefore (0,0)$ min t.p.

(iv) as x approaches $\pm\infty$, y approaches -2

(v)



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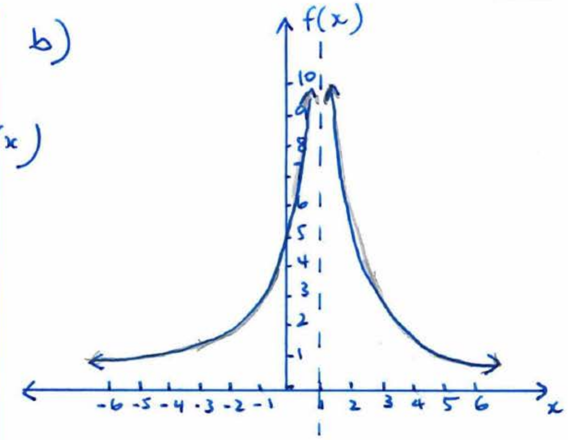
b)



(ii) $3 = \dfrac{5}{x-1}$
$3x - 3 = 5$
$x = 8/3$

$3 = \dfrac{-5}{x-1}$
$3(x-1) = -5$
$x = -\dfrac{2}{3}$

or draw $y < 3$ on graph above

$\therefore x < -\dfrac{2}{3}$ or $x > \dfrac{8}{3}$