Student Number

Barker College

2023 YEAR 11 PRELIMINARY EXAMINATION

Mathematics Extension

Staff Involved:

- JZT
- JAI
- GPF
- ARM
- DXC*
- AXD

Wednesday September 13
1:30pm

105 COPIES

TOPICS COVERED:		
Further Graphs	Polynomials	Discrete Probability
Further Trigonometry	Combinatorics	

General Instructions:

- Reading time - 5 minutes
- Working time - 2 hours
- Write your Student Number on every answer page
- Write using black pen
- Calculators approved by NESA may be used
- A separate reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and / or calculations
- Diagrams are not to scale unless specifically stated
- Marks may not be awarded for careless or poorly arranged working
- Detach the Question 13 answer page

Total marks: 70

Section I - 10 marks (pages 2 - 5)

- Attempt Questions 1 - 10
- Allow about 20 minutes for this section

Section II - 60 marks (pages 6 - 12)

- Attempt Questions 11 - 15
- Show all necessary working
- Allow about 1 hour and 40 minutes for this section

Section I

10 marks
Attempt Questions 1 – 10
Allow about 20 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10.

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1 The polynomial $P(x) = x^3 - 4x^2 - x + 4$ has distinct integer roots. Which of the following is **NOT** a factor of the polynomial?

A. $(x+1)$
B. $(x-1)$
C. $(x+4)$
D. $(x-4)$

2 In how many ways can the letters of the word ROAMING be arranged if the new word must end in a vowel?

A. 720
B. 2160
C. 4320
D. 5040

3 Which represents the solution to the inequality $4x^2 \leq 3 - 11x$?

A. $(-\infty, -3) \cup \left(\frac{1}{4}, \infty\right)$
B. $(-\infty, -3] \cup \left[\frac{1}{4}, \infty\right)$
C. $\left(-3, \frac{1}{4}\right)$
D. $\left[-3, \frac{1}{4}\right]$

4 How many different arrangements of the letters of the word EXTENSION are possible?

A. 15 120
B. 90 720
C. 181 440
D. 362 880

5 The number of possums appearing in my backyard on any given night is the discrete random variable X represented by the following probability distribution table.

x	0	1	2	3
$P(X = x)$	0.4	$7p$	$6p$	0.08

What is the expected number of possums that would appear in my backyard on any given night?

A. 0

B. 1

C. 2

D. 3

6 It is given that $\sin\alpha = \frac{15}{17}$ and $\cos\beta = \frac{3}{5}$ where $0 < \alpha < \frac{\pi}{2}$ and $\frac{3\pi}{2} < \beta < 2\pi$.
What is the value of $\tan(\alpha - \beta)$?

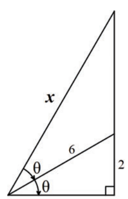
A. $-\frac{77}{36}$

B. $-\frac{11}{12}$

C. $\frac{11}{12}$

D. $\frac{77}{36}$

7 What is the exact value of x in the diagram?



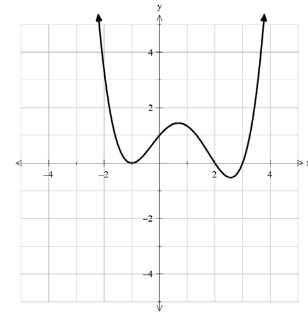
A. $6\sqrt{3}$

B. 9

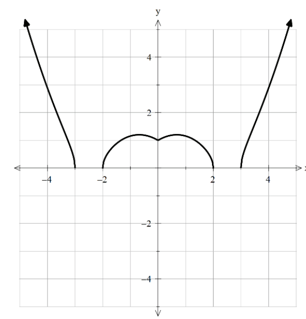
C. $\frac{36\sqrt{2}}{7}$

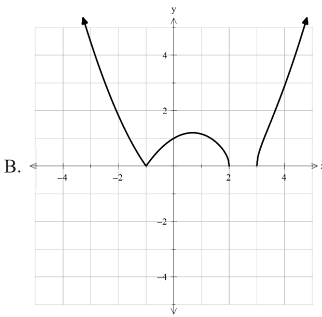
D. $\frac{9}{\sqrt{2}}$

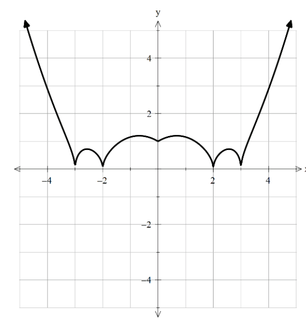
8 The following graph shows $y = f(x)$.

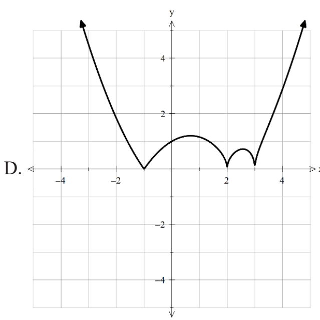


Which graph could represent $y = \sqrt{f(|x|)}$?

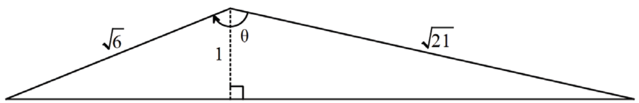
A.


B.


C.


D.


9 What is the exact value of $\sin\theta$ in the following diagram?



A. $\frac{\sqrt{5}}{3\sqrt{14}}$

B. $\frac{\sqrt{5}}{\sqrt{14}}$

C. $\frac{3}{\sqrt{14}}$

D. $\frac{11}{3\sqrt{14}}$

10 25 students audition for a play. There are 10 different named roles in the main cast of the play. Of the 15 students not given a role in the main cast, 9 students are assigned to be unnamed extras and 6 students are assigned to be backstage assistants.

Andrew has been told that he has been assigned one of the roles in the main cast, however, he does not know which character he will play. How many ways are there of assigning roles for the 25 students involved in the play?

A. $\frac{25!}{10!9!6!}$

B. $\frac{25!}{9!6!}$

C. $10 \times \frac{24!}{9!9!6!}$

D. $10 \times \frac{24!}{9!6!}$

End of Section I

Section II

60 marks
Attempt Questions 11 - 15
Allow about 1 hour and 40 minutes for this section

Answer each question in a NEW writing booklet apart from Question 13(c).

In Questions 11 - 15, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (12 marks) **USE THE QUESTION 11 WRITING BOOKLET**

(a) Solve the inequality $\frac{3x-5}{x-1} \geq 1$. **3**

(b) Simplify $\tan(x + 45°)$ in terms of $\tan x$. **2**

(c) Fully factorise the polynomial $P(x) = x^3 - 4x^2 + x + 6$ **2**

(d) Prove that $\sin\left(\frac{\pi}{2} - \theta\right)\cos\left(\frac{\pi}{2} - \theta\right) = \frac{\sin 2\theta}{2}$. **2**

(e) The roots of $2x^3 - 5x + 3 = 0$ are α, β, γ. **1**

Find the value of $\alpha\beta + \beta\gamma + \alpha\gamma$.

(f) Consider the following function. **2**

$$f(x) = \frac{3x + 4}{2x - 3}$$

Show that $f(x)$ is its own inverse function.

Question 12 (12 marks) **USE THE QUESTION 12 WRITING BOOKLET**

(a) Allison has a win rate of 55% in the game scissors-paper-rock. She plays two games in a row and records her results. Let the number of games that Allison wins be the discrete random variable X.

(i) Copy and complete the table in your booklet. You may wish to use a tree diagram to assist. **2**

x	0	1	2
$P(X = x)$			

(ii) Hence calculate the variance and standard deviation, correct to 3 decimal places. **2**

(b) (i) Express $\cos 2x$ in terms of $\sin x$. **1**

(ii) Hence or otherwise, find the exact value of $\sin\frac{\pi}{8}$. **2**

(c) A three-letter code is to be created by using each of the 5 letters of the word SLEEP at most once each.

(i) How many three-letter codes can be made if the code must contain both Es? **1**

(ii) How many three-letter codes can be made if each letter must be distinct? **1**

(iii) Hence, find the total number of three-letter codes that can be made. **1**

(d) Prove that ${}^{2n}C_2 - 2 \times {}^{n}C_2 = n^2$ **2**

Question 13 (12 marks) **USE THE QUESTION 13 WRITING BOOKLET**

(a) The polynomial $P(x) = x^4 + 4x^3 - 3x^2 + ax + b$ has remainder $R(x) = 32x + 44$ when divided by $(x^2 - 2x - 3)$. Find the values of a and b. **3**

(b) 10 friends go to the cinemas to watch a movie.

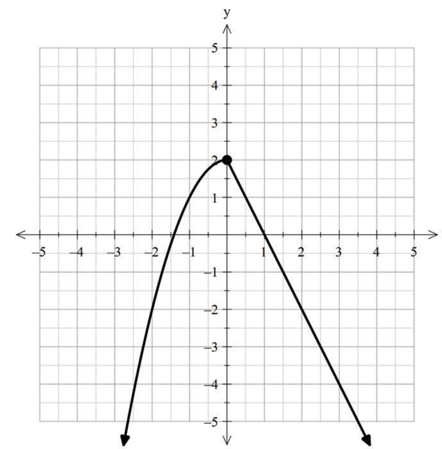
They book two rows of seats to sit together. The first row of seats contains six seats and the second row of seats contains four seats.

How many ways can the friends sit:

(i) without any restrictions? **1**

(ii) if Daniel, Greg, and Jodi must sit next to each other in the same row? **3**

(c) Consider the function $y = f(x)$ shown below.



Answer Question 13(c) on the separate page provided. Note that this is a double-sided page. Hand this page in with your booklet for Question 13.

Sketch the following curves showing any intercepts and asymptotes and clearly show any critical points.

(i) $y = [f(x)]^2$ **2**

(ii) $y = \frac{1}{f(x)}$ **3**

Question 14 (12 marks) **USE THE QUESTION 14 WRITING BOOKLET**

(a) The discrete random variable X is given by the probability distribution table below.

x	0	b	5	10
$P(X=x)$	0.69	0.2	a	a^2

It is known that $E(X) = 1$

Evaluate a and b. **3**

(b) Solve the following equation for $0 \le x \le 2\pi$. Give your answers in exact form. **4**

$$1 - \frac{\sec^2 x}{2} = \sqrt{3}\tan x$$

(c) The polynomial $P(x) = 4x^3 - 8kx^2 + 29x - 4k$ has three distinct zeroes, with $k > 0$. It is known that k is one of the zeroes of the polynomial.

(i) Show that the other zeroes of the polynomial are reciprocals of each other. **1**

(ii) Show that k is equal to the sum of the other zeroes. **1**

(iii) Hence, or otherwise, find the zeroes of the polynomial. **3**

Question 15 (12 marks) **USE THE QUESTION 15 WRITING BOOKLET**

(a) A curve is defined parametrically by the equations $x = 3t^2 - 1$ and $y = 2t^4 - 6t^2$. Find the gradient of the tangent to the curve when $t = 2$. **2**

(b) In the television game show Letters & Numbers, contestants must choose six numbered tiles from a set of twenty-four shuffled tiles. The tiles are arranged into two groups: *large numbers* and *small numbers*.

There are four tiles in the *large numbers* set: {25, 50, 75, 100}

There are twenty tiles in the *small numbers* set, two each of the numbers from 1 to 10.

Contestants may choose to select any number of large number tiles or small number tiles to make up their set of six numbers in total. The order the tiles are chosen is not important.

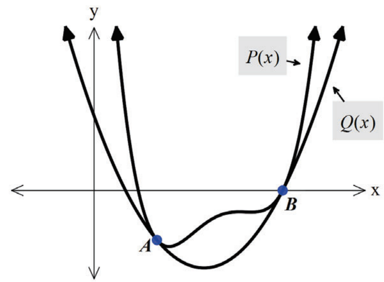
How many ways are there of:

(i) choosing four large numbers and two equal small numbers? **1**

(ii) choosing four large numbers and two distinct small numbers? **1**

(iii) choosing two large numbers and four small numbers? **2**

(c) The polynomial $P(x) = 2x^4 - 16x^3 + 46x^2 - 55x + 21$ is tangent to the quadratic function $Q(x) = 2x^2 - 7x + 3$ at two distinct points ***A*** and ***B***.



The point ***B*** lies on the x-axis.

(i) Find the coordinates of the points ***A*** and ***B***. **3**

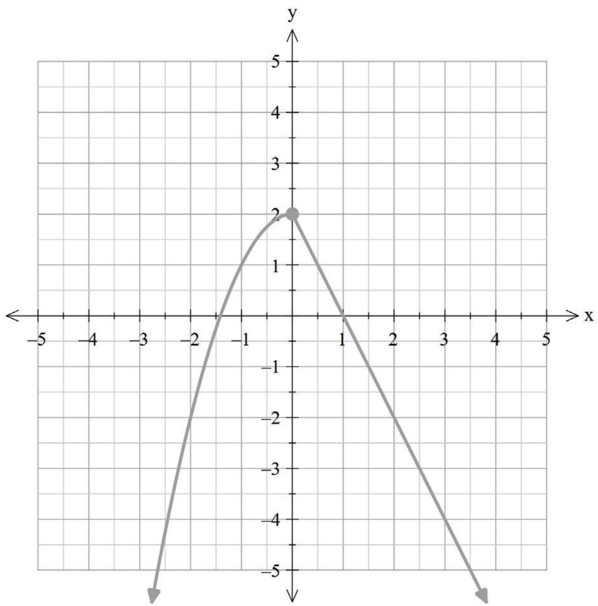
(ii) Hence, show that the line passing through ***A*** and ***B*** is tangent to $P(x)$ at the midpoint of ***AB***. **3**

End of Paper

Additional Answer page for Question 13(c)

Answer Question 13(c) (i), (ii) here and hand in with Question 13 booklet.

13(c) (i) Given the function $y = f(x)$ shown below, sketch $y = [f(x)]^2$.
Show any intercepts and asymptotes and clearly show any critical points. **2**



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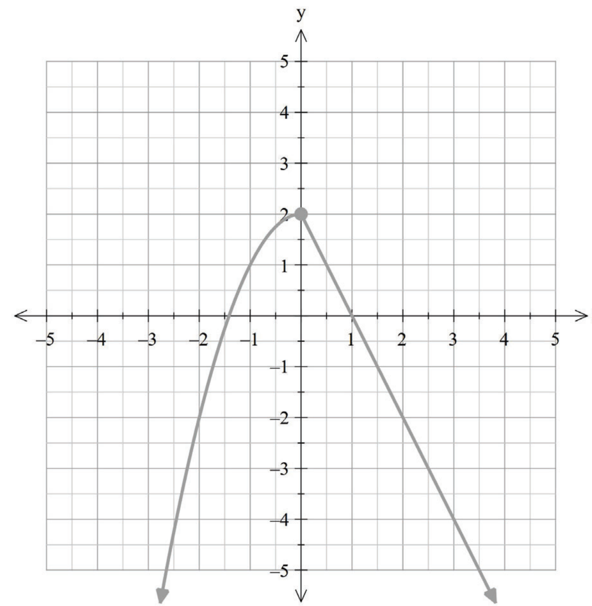
13(c) (ii) Given the function $y = f(x)$ shown below, sketch $y = \frac{1}{f(x)}$.
Show any intercepts and asymptotes and clearly show any critical points. **3**



2023 Year 11 Mathematics Extension Solutions

1. $P(x) = x^3 - 4x^2 - x + 4$

$P(-4) = -64 - 64 + 4 + 4$

$= -120$

$\therefore (x+4)$ is NOT a factor. $\therefore$ C

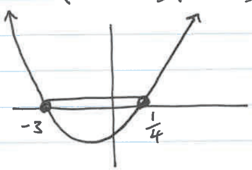
2. $\underline{6} \times \underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} \times \underline{3}$

$\therefore 6! \times 3 = 2160$ $\therefore$ B

3. $4x^2 \le 3 - 11x$

$4x^2 + 11x - 3 \le 0$

$(4x-1)(x+3) \le 0$



$\therefore -3 \le x \le \frac{1}{4}$ or $[-3, \frac{1}{4}]$

$\therefore$ D

4. $\frac{9!}{2!2!} = 90720$

$\therefore$ B

5. $0.48 + 13p = 1$

$13p = 0.52$

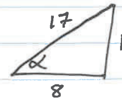
$p = 0.04$

$\therefore 1 \times 0.28 + 2 \times 0.24 + 3 \times 0.08$

$= 1$

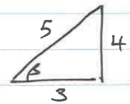
$\therefore E(X) = 1$ $\therefore$ B

6. $\tan\alpha = \frac{15}{8}$



$\tan\beta = -\frac{4}{3}$

(as $\frac{3\pi}{2} < \beta < 2\pi$)

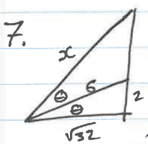


$\therefore \tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

$= \frac{\frac{15}{8} + \frac{4}{3}}{1 - \frac{15}{8} \times \frac{4}{3}} = -\frac{77}{36}$

$\therefore$ A

7.



$\sin\theta = \frac{1}{3}$

$\cos\theta = \frac{\sqrt{32}}{6} = \frac{4\sqrt{2}}{6} = \frac{2\sqrt{2}}{3}$

$\therefore \cos 2\theta = \frac{\sqrt{32}}{x}$

$\cos^2\theta - \sin^2\theta = \frac{\sqrt{32}}{x}$

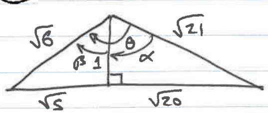
$\frac{8}{9} - \frac{1}{9} = \frac{\sqrt{32}}{x}$

$\therefore x = \frac{9 \times 4\sqrt{2}}{7} = \frac{36\sqrt{2}}{7}$

$\therefore$ C

8. A

9.



$\sin\alpha = \frac{\sqrt{20}}{\sqrt{21}}$ $\sin\beta = \frac{\sqrt{5}}{\sqrt{6}}$

$\cos\alpha = \frac{1}{\sqrt{21}}$ $\cos\beta = \frac{1}{\sqrt{6}}$

$\therefore \sin\theta = \sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta = \frac{\sqrt{5}}{\sqrt{14}}$

$\therefore$ B

10. $10 \times {}^{24}P_9 \times {}^{15}C_9 = 10 \times \frac{24!}{\cancel{15!}} \times \frac{\cancel{15!}}{9!6!}$

$\therefore$ D

Question 11.

(a) $\frac{3x-5}{x-1} \ge 1$

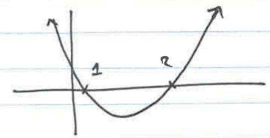
$(3x-5)(x-1) \ge (x-1)^2$

$3x^2 - 8x + 5 \ge x^2 - 2x + 1$

$2x^2 - 6x + 4 \ge 0$

$2(x^2 - 3x + 2) \ge 0$

$2(x-1)(x-2) \ge 0$



Note $x \ne 1$

$\therefore x < 1$ or $x \ge 2$

(b) $\tan(x + 45^\circ) = \frac{\tan x + \tan 45}{1 - \tan x \times \tan 45}$

$= \frac{\tan x + 1}{1 - \tan x}$

(c) $P(x) = x^3 - 4x^2 + x + 6$

$P(-1) = -1 - 4 - 1 + 6$

$= 0$

$\therefore (x+1)$ is a factor.

$\therefore P(x) = (x+1)(x^2 - 5x + 6)$

$= (x+1)(x-2)(x-3)$

(d) $\sin(\frac{\pi}{2} - \theta)\cos(\frac{\pi}{2} - \theta) = \frac{\sin 2\theta}{2}$

LHS: $\sin(\frac{\pi}{2} - \theta)\cos(\frac{\pi}{2} - \theta)$

$= \cos\theta \sin\theta$

$= \frac{1}{2} \times 2\sin\theta\cos\theta$

$= \frac{1}{2}\sin 2\theta$

$= \frac{\sin 2\theta}{2} = RHS$

(e) $2x^3 - 5x + 3 = 0$

$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a}$

$\therefore \alpha\beta + \beta\gamma + \alpha\gamma = \frac{-5}{2}$

(f) Let $y = \frac{3x+4}{2x-3}$

$\therefore x = \frac{3y+4}{2y-3}$

$x(2y-3) = 3y + 4$

$2xy - 3x = 3y + 4$

$2xy - 3y = 3x + 4$

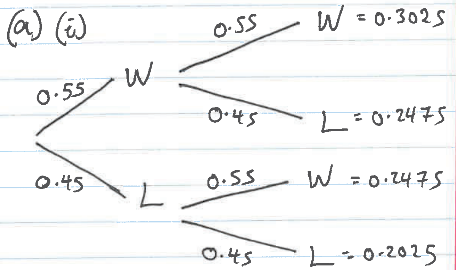
$y(2x-3) = 3x + 4$

$y = \frac{3x+4}{2x-3}$

As $f(x)$ is a hyperbola, it is one-to-one. Therefore its inverse is also a function.

Question 12.

(a) (i)



x	0	1	2
$P(X=x)$	0.2025	0.495	0.3025

(ii) $Var(X) = 0.495$

$\sigma \doteqdot 0.704$ (3d.p)

(b) (i) $\cos 2x = \cos^2 x - \sin^2 x$

$= 1 - \sin^2 x - \sin^2 x$

$= 1 - 2\sin^2 x$

(ii) $\cos 2x = 1 - 2\sin^2 x$

Let $x = \frac{\pi}{8}$

$\therefore \cos\left(2\times\frac{\pi}{8}\right) = 1 - 2\sin^2\left(\frac{\pi}{8}\right)$

$\cos\frac{\pi}{4} = 1 - 2\sin^2\frac{\pi}{8}$

$\frac{1}{\sqrt{2}} = 1 - 2\sin^2\frac{\pi}{8}$

$\frac{1}{\sqrt{2}} - 1 = -2\sin^2\frac{\pi}{8}$

$\frac{1}{2} - \frac{1}{2\sqrt{2}} = \sin^2\frac{\pi}{8}$

$\frac{\sqrt{2}-1}{2\sqrt{2}} = \sin^2\frac{\pi}{8}$

$\frac{2-\sqrt{2}}{4} = \sin^2\frac{\pi}{8}$

$\therefore \sin\frac{\pi}{8} = \pm\frac{\sqrt{2-\sqrt{2}}}{2}$

$\sin\frac{\pi}{8} > 0 \quad \therefore \sin\frac{\pi}{8} = \frac{\sqrt{2-\sqrt{2}}}{2}$

(c) (i) $\frac{3!}{2!} \times 3 = 9$

(ii) ${}^4P_3 = 24$

(iii) $9 + 24 = 33$

(d) ${}^{2n}C_2 - 2\times{}^nC_2 = n^2$

$LHS = {}^{2n}C_2 - 2\times{}^nC_2$

$= \frac{(2n)!}{2!(2n-2)!} - \cancel{2}\times\frac{n!}{\cancel{2}!(n-2)!}$

$= \frac{\cancel{2}n(2n-1)\cancel{(2n-2)!}}{\cancel{2}\cancel{(2n-2)!}} - \frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}}$

$= 2n(2n-1) - n(n-1)$

$= 2n^2 - n - n^2 + n$

$= n^2 = RHS$

Question 13.

(a) $x^4 + 4x^3 - 3x^2 + ax + b = (x^2 - 2x - 3)Q(x) + 32x + 44$

$x^2 - 2x - 3 = (x-3)(x+1)$

$\therefore P(x) = (x-3)(x+1)Q(x) + 32x + 44$

$\therefore P(-1) = 0 - 32 + 44 = 12$

$\therefore 1 - 4 - 3 - a + b = 12$

$-a + b = 18$ — ①

$P(3) = 0 + 96 + 44 = 140$

$\therefore 81 + 108 - 27 + 3a + b = 140$

$3a + b = -22$ — ②

$\therefore$ ② − ①:

$4a = -40$

$a = -10$

Sub $a = -10$ into ①:

$10 + b = 18$

$\therefore b = 8$

(b) (i) $10! = 3628800$

(ii) In the six-seater:

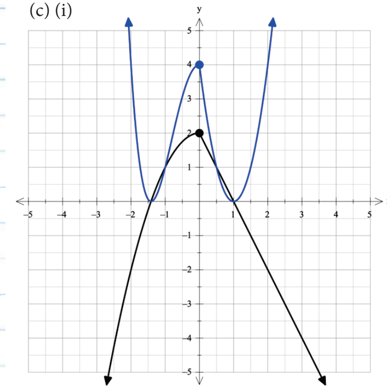
$4 \times 3! \times 7! = 120960$

In the four-seater:

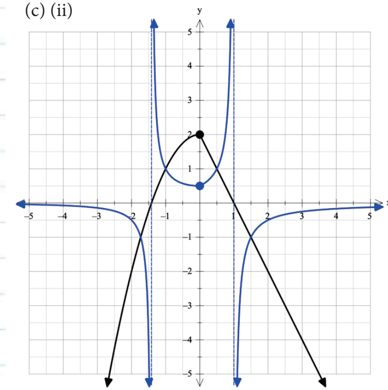
$2 \times 3! \times 7! = 60480$

$\therefore 4\times3!\times7! + 2\times3!\times7! = 181440$

(c) (i)



(c) (ii)



Question 14.

(a) $0.69 + 0.2 + a + a^2 = 1$

$a^2 + a - 0.11 = 0$

$100a^2 + 100a - 11 = 0$

$(10a - 1)(10a + 11) = 0$

$\therefore a = \frac{1}{10}$ or $a = -\frac{11}{10}$

$a > 0 \quad \therefore a = 0.1$

$\therefore E(X) = 1 = 0 + 0.2b + 0.5 + 0.1$

$1 = 0.2b + 0.6$

$0.2b = 0.4$

$b = 2$

$\therefore a = 0.1$ & $b = 2$

(b) $1 - \frac{\sec^2 x}{2} = \sqrt{3}\tan x$

$2 - \sec^2 x = 2\sqrt{3}\tan x$

$2 - 1 - \tan^2 x = 2\sqrt{3}\tan x$

$1 - \tan^2 x = 2\sqrt{3}\tan x$

$1 = \frac{2\sqrt{3}\tan x}{1 - \tan^2 x}$

$\therefore \frac{2\tan x}{1 - \tan^2 x} = \frac{1}{\sqrt{3}}$

$\tan 2x = \frac{1}{\sqrt{3}}$

$\therefore 2x = \frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}, \frac{19\pi}{6}$ $\quad 0 \le 2x \le 4\pi$

$x = \frac{\pi}{12}, \frac{7\pi}{12}, \frac{13\pi}{12}, \frac{19\pi}{12}$

OR $1 - \tan^2 x = 2\sqrt{3}\tan x$

$\therefore \tan^2 x + 2\sqrt{3}\tan x - 1 = 0$

$\tan x = \frac{-2\sqrt{3} \pm \sqrt{12 + 4}}{2}$

$= \frac{-2\sqrt{3} \pm 4}{2}$

$\therefore \tan x = -\sqrt{3} \pm 2$

$\therefore$ If $\tan x = -\sqrt{3} + 2 > 0$

$x = \frac{\pi}{12}, \frac{13\pi}{12}$

If $\tan x = -\sqrt{3} - 2 < 0$

$x = \frac{7\pi}{12}, \frac{19\pi}{12}$

$\therefore x = \frac{\pi}{12}, \frac{7\pi}{12}, \frac{13\pi}{12}, \frac{19\pi}{12}$

(c)(i) $\sum \alpha\beta\gamma = \frac{-4k}{-4} = k$

$\therefore$ Let α & β be the other roots.

$\therefore k\alpha\beta = k$

$\alpha\beta = 1$

$\alpha = \frac{1}{\beta}$

$\therefore$ the other roots are reciprocals

(ii) $\sum \alpha = \frac{8k}{4} = 2k$

$\therefore k + \alpha + \beta = 2k$

$\alpha + \beta = k$

$\therefore k$ is equal to the sum of the other roots.

Question 14.

(c)(iii) Let k, α & $\frac{1}{\alpha}$ be the roots.

$\sum \alpha\beta = \frac{29}{4}$

$\therefore k\alpha + \frac{k}{\alpha} + 1 = \frac{29}{4}$

$k\left(\alpha + \frac{1}{\alpha}\right) = \frac{25}{4}$

Since $\alpha + \frac{1}{\alpha} = k$ from (ii):

$\therefore k^2 = \frac{25}{4}$

$\therefore k = \pm\frac{5}{2}$

However, $k > 0$. $\therefore k = \frac{5}{2}$

$\therefore \frac{5\alpha}{2} + \frac{5}{2\alpha} + 1 = \frac{29}{4}$

$\therefore 10\alpha^2 + 10 + 4\alpha = 29\alpha$

$10\alpha^2 - 25\alpha + 10 = 0$

$5(2\alpha^2 - 5\alpha + 2) = 0$

$5(2\alpha - 1)(\alpha - 2) = 0$

$\therefore \alpha = \frac{1}{2}$ or $\alpha = 2$

$\therefore$ The roots are $\frac{5}{2}, \frac{1}{2}, 2$

Question 15.

(a) $x = 3t^2 - 1 \qquad y = 2t^4 - 6t^2$

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$

$\therefore \frac{dx}{dt} = 6t \qquad \frac{dy}{dt} = 8t^3 - 12t$

$\therefore \frac{dy}{dx} = \frac{8t^3 - 12t}{6t} = \frac{4t^2 - 6}{3}$

$\therefore$ At $t = 2$, $m = \frac{16 - 6}{3} = \frac{10}{3}$

$\therefore$ Gradient of tangent is $\frac{10}{3}$.

(b) (i) ${}^4C_4 \times {}^{10}C_1 = 10$

(ii) ${}^4C_4 \times {}^{10}C_2 = 45$

(iii) Three cases:

① Two pairs of equal small numbers: ${}^4C_2 \times {}^{10}C_2 = 270$

② One pair of equal small numbers & two distinct small numbers: ${}^4C_2 \times {}^{10}C_1 \times {}^9C_2 = 2160$

③ Four distinct small numbers: ${}^4C_2 \times {}^{10}C_4 = 1260$

$\therefore$ Total ways $= 270 + 2160 + 1260$

$= 3690$

Question 15

(c) (i) $P(x) = 2x^4 - 16x^3 + 46x^2 - 55x + 21$

$Q(x) = 2x^2 - 7x + 3$

Since B is on the x-axis, this means B is a root of $Q(x)$.

$\therefore 0 = 2x^2 - 7x + 3$

$0 = (2x-1)(x-3)$

$\therefore x = \frac{1}{2}$ or $x = 3$

$P(\frac{1}{2}) = \frac{25}{8}$ $\quad P(3) = 0$

$\therefore B = (3, 0)$

$P'(x) = 8x^3 - 48x^2 + 92x - 55$

$Q'(x) = 4x - 7$

$\therefore P'(x) - Q'(x) = 8x^3 - 48x^2 + 88x - 48$

$$
\begin{array}{r|l}
 & 8x^2 - 24x + 16 \\
\hline
x-3 & 8x^3 - 48x^2 + 88x - 48 \\
 & 8x^3 - 24x^2 \\
\hline
 & -24x^2 + 88x \\
 & -24x^2 + 72x \\
\hline
 & 16x - 48 \\
 & 16x - 48 \\
\hline
 & 0
\end{array}
$$

$\therefore P'(x) - Q'(x) = (x-3)(8x^2 - 24x + 16)$

$= 8(x-3)(x^2 - 3x + 2)$

$= 8(x-3)(x-1)(x-2)$

$P(1) = -2, \quad Q(1) = -2$

$\therefore A = (1, -2)$

& $B = (3, 0)$

(c)(ii)

Midpoint of AB:

$M = \left(\frac{1+3}{2}, \frac{-2+0}{2}\right) = (2, -1)$

Gradient ~~M~~ $m_{AB} = \frac{0+2}{3-1} = \frac{2}{2} = 1$

Check: $P(2) = 2(2)^4 - 16(2)^3 + 46(2)^2 - 55(2) + 21$

$= -1$

$\therefore (2, -1)$ is on $P(x)$

$P'(2) = 8(2)^3 - 48(2)^2 + 92(2) - 55$

$= 1$

$= m_{AB}$

$\therefore P(x)$ intersects AB at M with the same gradient, so AB is tangent to $P(x)$ at its midpoint.