BAULKHAM HILLS
HIGH
SCHOOL

2019

YEAR 11
YEARLY
EXAMINATIONS

Mathematics Extension

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- A reference sheet is provided at the back of this paper
- In Questions 11 – 14, show relevant mathematical reasoning and/or calculations
- Marks may be deducted for careless or badly arranged work

Total marks: 70

Section I – 10 marks (pages 2 – 5)
- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 6 – 10)
- Attempt Questions 11 – 14
- Allow about 1 hour 45 minutes for this section

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10

1 The population of bacteria after t hours is given by

$$P(t) = 5000e^{0.18t}$$

The rate of increase of the population, to the nearest unit, after 15 minutes is;

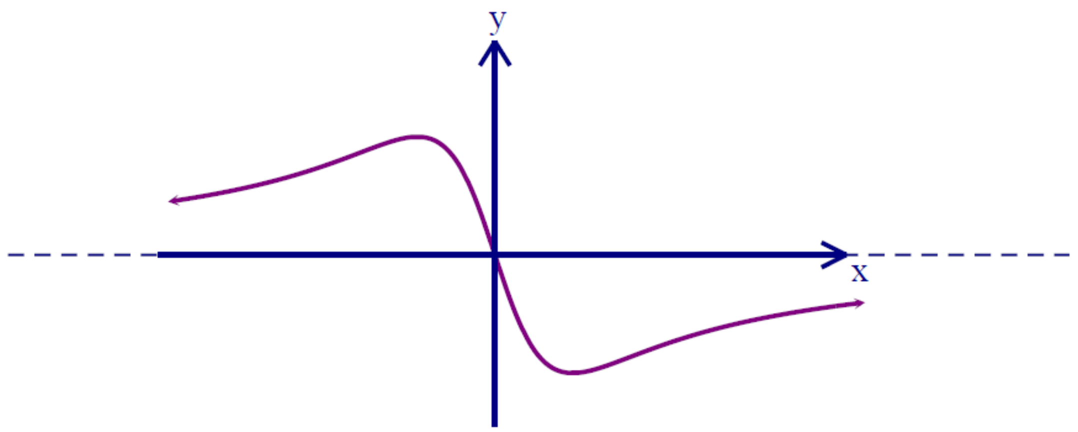
(A) 941 bacteria/hour

(B) 5230 bacteria/hour

(C) 13392 bacteria/hour

(D) 74399 bacteria/hour

2



Which of the following four equations describes the graph drawn above?

(A) $y = \dfrac{1}{1+x^2}$

(B) $y = 1 - \dfrac{1}{1+x^2}$

(C) $y = \dfrac{x}{1+x^2}$

(D) $y = \dfrac{-x}{1+x^2}$

3 Which of the following expressions is equivalent to $\cos x + \sqrt{3}\sin x$?

(A) $2\cos\left(x+\frac{\pi}{6}\right)$

(B) $2\cos\left(x-\frac{\pi}{6}\right)$

(C) $2\cos\left(x+\frac{\pi}{3}\right)$

(D) $2\cos\left(x-\frac{\pi}{3}\right)$

4 Find the number of ways all the letters of the word **EPSILON** can be arranged such that the three vowels are all next to each other.

(A) 7!

(B) 5!

(C) 4!5!

(D) 3!5!

5 Given $f(x) = 2\sec x$ for $0 \leq x \leq \frac{\pi}{2}$, then $f^{-1}(x) =$

(A) $2\cos^{-1}\left(\frac{1}{x}\right)$

(B) $2\cos^{-1} x$

(C) $\cos^{-1}\left(\frac{2}{x}\right)$

(D) $\cos^{-1}\left(\frac{1}{2x}\right)$

6 If $t = \tan\left(\dfrac{\theta}{2}\right)$, the correct expression for $\dfrac{\sec^2\theta}{\operatorname{cosec}^2\theta}$ is;

(A) $\dfrac{4t^2}{(1-t^2)^2}$

(B) $\dfrac{(1+t^2)^2}{(1-t^2)^2}$

(C) $\dfrac{1+t^2}{(1-t^2)^2}$

(D) $\dfrac{(1-t^2)^2}{4t^2}$

7 The parametric equations of a function are

$$x = 2t^2 \quad , \quad y = 4 - t$$

The Cartesian equation is;

(A) $x = 4(2-y)^2$

(B) $x = 4(y+2)^2$

(C) $x = 2(y+4)^2$

(D) $x = 2(4-y)^2$

8 Which of the following is the range of the function $f(x) = \sin^{-1}x + \tan^{-1}x$?

(A) $-\pi < y < \pi$

(B) $-\pi \leq y \leq \pi$

(C) $-\dfrac{3\pi}{4} \leq y \leq \dfrac{3\pi}{4}$

(D) $-\dfrac{\pi}{2} \leq y \leq \dfrac{\pi}{2}$

9 For $0° \leq \theta \leq 90°$, the least value of $\dfrac{30}{3\sin^2\theta + 2\sin^2(90° - \theta)}$ is;

(A) 5

(B) 6

(C) 10

(D) 15

10 By considering the binomial expansion of $(1+x)^{100} - (1-x)^{100}$, what is the value of

$$\binom{100}{3} + \binom{100}{5} + \ldots + \binom{100}{99}?$$

(A) $2^{100} - 100$

(B) $2^{99} - 100$

(C) 2^{100}

(D) 2^{99}

END OF SECTION I

Section II

60 marks
Attempt Questions 11 – 14
Allow about 1 hour 45 minutes for this section

Answer each question on the appropriate answer sheet. Each answer sheet must show your name. Extra paper is available.

In Questions 11 to 14, your responses should include relevant mathematical reasoning and/or calculations.

Marks

Question 11 ***(15 marks)*** Use a *separate* answer sheet

(a) Express $\frac{{}^{10}C_k}{{}^{9}C_k}$ in simplest form *2*

(b) Consider the expansion $\left(\frac{x^3}{2}+\frac{a}{x}\right)^8$. If the constant term is 5103, determine the possible value(s) of a. *3*

(c) (i) Find the linear factors of $x^3 - 5x^2 + 8x - 4$ *2*

(ii) Hence solve $x^3 - 5x^2 + 8x - 4 > 0$ *2*

(d) Solve $\frac{x}{3-2x} \le 1$ *3*

(e) Find the exact value of $\sin\left[\cos^{-1}\left(\frac{4}{5}\right) - \tan^{-1}\left(\frac{5}{12}\right)\right]$, showing all working. *3*

Marks

<u>**Question 12**</u> ***(15 marks)*** Use a *separate* answer sheet

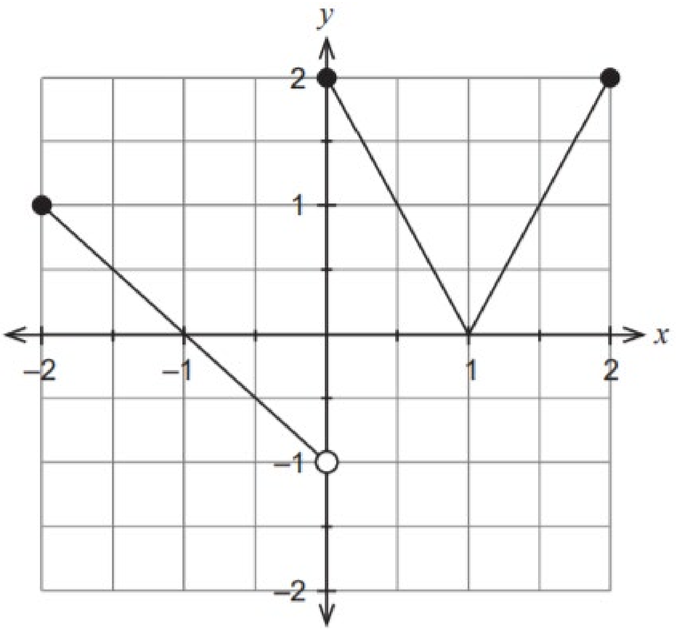
(a) (i) Sketch the graph of $f(x) = |3x + 3| + |x - 1|$ **2**

(ii) On the same number plane sketch $g(x) = 4x + 3$ **1**

(iii) Hence, or otherwise, solve $f(x) \leq g(x)$ **1**

(b) Write $\tan\left(\cos^{-1}\left(-\frac{1}{3}\right)\right)$ in the form $a\sqrt{b}$ where a and b are rational. **2**

(c) The graph of $y = f(x)$ is shown



(i) Sketch the graph of $y = \frac{1}{f(x)}$ **2**

(ii) Sketch the graph of $y = \sqrt{f(x)}$ **2**

(iii) Solve the equation $|f(x) - 1| = 1$ **2**

(d) Five digit numbers are to be formed using the digits 5, 6, 7, 8 and 9. How many odd numbers greater than 70000 can be formed if no digit is repeated? **3**

Marks

Question 13 ***(15 marks)*** Use a *separate* answer sheet

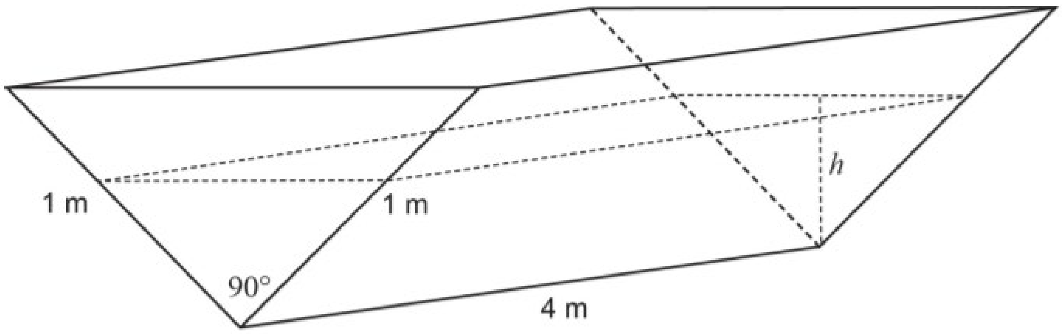
(a) Let the cubic polynomial $P(x) = x^3 - 3x^2 + kx + 12$ have roots α, β and γ .

(i) Find the value of $\alpha + \beta + \gamma$ *1*

(ii) Find the value of $\dfrac{1}{\alpha\beta} + \dfrac{1}{\beta\gamma} + \dfrac{1}{\alpha\gamma}$ *2*

(iii) Given that two of its roots have a sum of zero, find the value of k *2*

(b) A four metres long water tank, open at the top, is in the shape of a triangular prism. The triangular face is a right isosceles triangle with congruent sides of one metre length.



Initially the tank is completely full of water, but it develops a leak and loses water at a constant rate of 0.08 cubic metres per hour.

Let h = the depth of water, in metres, in the tank after t hours.

(i) Show that the volume, V, of water in the tank in cubic metres, is given by the expression *2*

$$V = 4h^2$$

(ii) Determine the rate of change of the depth, when the depth is 0.6 metres. *2*

Question 13 continues on page 9

Marks

Question 13 (continued)

(c) It is known that the function $y = 4 - x^2$ does not have an inverse function.

(i) Explain why $y = 4 - x^2$ does not have an inverse function. *1*

(ii) Suggest a suitable restriction to the domain so that the restricted function may have an inverse function, and the range of the function is unaffected. *1*

(iii) Find an expression for $f^{-1}(x)$ *2*

(iv) Calculate the point of intersection of the restricted function and its inverse. *2*

End of Question 13

Marks

<u>Question 14</u> ***(15 marks)*** Use a *separate* answer sheet

(a) The equation $8x^4 - 44x^3 + 54x^2 - 25x + 4 = 0$ has a triple root. *3*
Solve this equation.

(b) Show that *2*

$$\tan x \equiv \frac{\sin 2x}{1 + \cos 2x}$$

(c) When a body falls, the rate of change in velocity is given by

$$\frac{dV}{dt} = -k(V - 500)$$

where k is a constant.

(i) Show that $V = 500 + Ae^{-kt}$ is a solution of this differential equation. *1*

(ii) The body starts at rest and 5 seconds later it is travelling at 21 m/s. *2*
Find the values of A and k.

(iii) Find the body's velocity 20 seconds after it starts falling, to the nearest m/s. *1*

(iv) Find the maximum possible velocity this body could travel. *1*

(d) Using the fact that $(1+x)^4(1+x)^9 \equiv (1+x)^{13}$, show that *2*

$$\binom{4}{0}\binom{9}{4} + \binom{4}{1}\binom{9}{3} + \binom{4}{2}\binom{9}{2} + \binom{4}{3}\binom{9}{1} + \binom{4}{4}\binom{9}{0} = \binom{13}{4}$$

(e) In ΔABC, $\sin C = 2\sin A\cos B$. *3*
Prove that ΔABC is an isosceles triangle.

End of paper

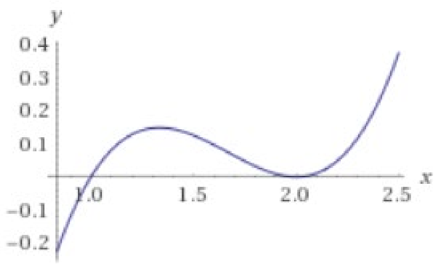
BAULKHAM HILLS HIGH SCHOOL
YEAR 11 EXTENSION YEARLY 2019 SOLUTIONS

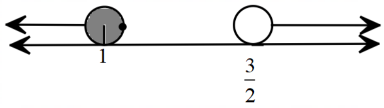
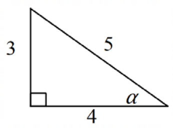
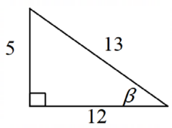
Solution	Marks	Comments
SECTION I		
1. A - $P = 5000e^{0.18t}$ $\frac{dP}{dt} = 900e^{0.18t}$ When $t = 0.25$, $\frac{dP}{dt} = 900e^{0.18 \times 0.25}$ $= 900e^{0.045}$ $= 941.4251...$ $= 941$ (to nearest unit)	**1**	
2. D – (A) does not pass through (0,0) (B) has range $0 \leq y \leq 1$ (C) $y < 0$ when $x < 0$ and $y > 0$ when $x > 0$ Thus it must be $y = \frac{-x}{1+x^2}$	**1**	
3. D – $2\cos\left(x - \frac{\pi}{3}\right) = 2\left(\cos x\cos\frac{\pi}{3} + \sin x\sin\frac{\pi}{3}\right)$ $= 2\left(\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x\right)$ $= \cos x + \sqrt{3}\sin x$	**1**	
4. D – three vowels grouped together = 3! Group of vowels plus four consonants = 5! $\therefore$ total = 3!5!	**1**	
5. C - $x = 2\sec y$ $\frac{x}{2} = \sec y$ $\frac{2}{x} = \cos y$ $y = \cos^{-1}\left(\frac{2}{x}\right)$	**1**	
6. A – $\frac{\sec^2\theta}{\operatorname{cosec}^2\theta} = \left(\frac{1+t^2}{1-t^2}\right)^2 \times \left(\frac{2t}{1+t^2}\right)^2$ $= \frac{4t^2}{(1-t^2)^2}$	**1**	
7. D - $y = 4 - t$ $t = 4 - y$ $x = 2(4-y)^2$	**1**	
8. C - domain $-1 \leq x \leq 1$ (as $\sin^{-1}x$ has a restricted domain) when $x = -1$, $\sin^{-1}(-1) + \tan^{-1}(-1)$ $= -\frac{\pi}{2} - \frac{\pi}{4}$ $= -\frac{3\pi}{4}$ when $x = 1$, $\sin^{-1}(1) + \tan^{-1}(1)$ $= \frac{\pi}{2} + \frac{\pi}{4}$ $= \frac{3\pi}{4}$ range: $-\frac{3\pi}{4} \leq y \leq \frac{3\pi}{4}$	**1**	
9. C – $\frac{30}{3\sin^2\theta + 2\sin^2(90° - \theta)} = \frac{30}{3\sin^2\theta + 2\cos^2\theta}$ $= \frac{30}{2 + \sin^2\theta}$ The largest $\sin^2\theta$ could be is 1, so therefore the smallest value is $\frac{30}{2+1}$ $= 10$	**1**	

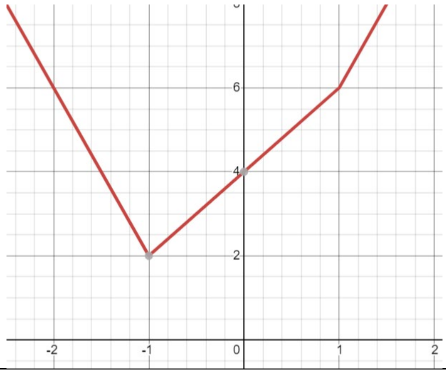
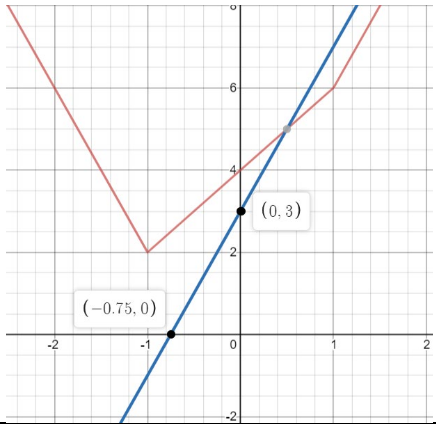
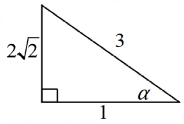
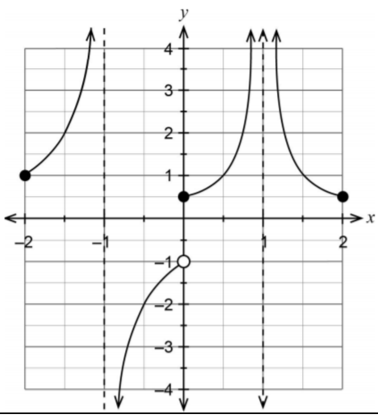
Solution	Marks	Comments
10. B – $(1+x)^{100}-(1-x)^{100}$ $= \binom{100}{0} + \binom{100}{1}x + \binom{100}{2}x^2 + \binom{100}{3}x^3 + \ldots + \binom{100}{99}x^{99} + \binom{100}{100}x^{100}$ $- \left(\binom{100}{0} + \binom{100}{1}x - \binom{100}{2}x^2 + \binom{100}{3}x^3 + \ldots + \binom{100}{99}x^{99} - \binom{100}{100}x^{100}\right)$ $= 2\binom{100}{1}x + 2\binom{100}{3}x^3 + \ldots + 2\binom{100}{99}x^{99}$ Let $x = 1$; $(1+1)^{100}-(1-1)^{100} = 2\binom{100}{1} + 2\binom{100}{3} + \ldots + 2\binom{100}{99}$ $2^{100} = 2\binom{100}{1} + 2\binom{100}{3} + \ldots + 2\binom{100}{99}$ $2^{100} - 2\binom{100}{1} = 2\left[\binom{100}{3} + \binom{100}{5} + \ldots + \binom{100}{99}\right]$ $2^{99} - 100 = \binom{100}{3} + \binom{100}{5} + \ldots + \binom{100}{9}$	**1**	

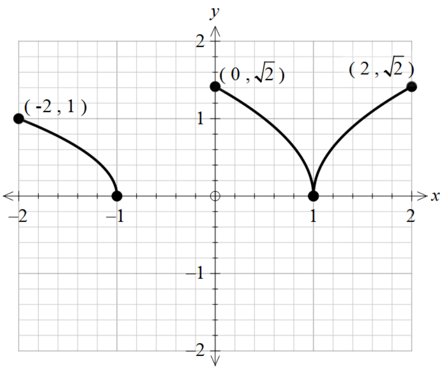
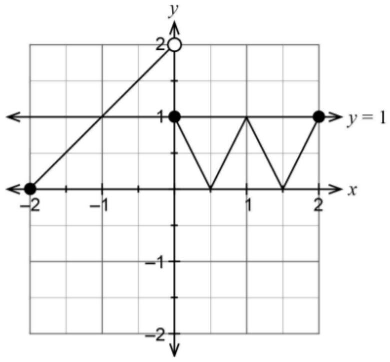
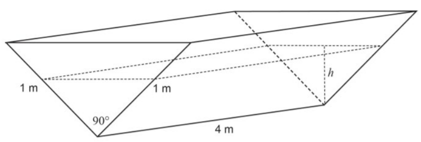
SECTION II

QUESTION 11

Solution	Marks	Comments
11(a) $\dfrac{{}^{10}\mathbf{C_k}}{{}^{9}\mathbf{C_k}} = \dfrac{10!}{\mathbf{k}!(10-\mathbf{k})!} \times \dfrac{\mathbf{k}!(9-\mathbf{k})!}{9!}$ $= \dfrac{10}{(10-\mathbf{k})}$	**2**	**2 marks** • Correct solution **1 mark** • Correctly converts one expression into factorial notation
11(b) $\left(\dfrac{x^3}{2}+\dfrac{a}{x}\right)^8$; $T_{k+1} = \binom{8}{k}\left(\dfrac{x^3}{2}\right)^{8-k}\left(\dfrac{a}{x}\right)^k$ $= \binom{8}{k}\dfrac{a^k}{2^{8-k}}x^{24-4k}$ constant term has $x^0 \Rightarrow 24-4k=0$ $k=6$ constant is $\binom{8}{6}\dfrac{a^6}{2^2}$, thus $\binom{8}{6}\dfrac{a^6}{2^2} = 5103$ $7a^6 = 5103$ $a^6 = 729$ $a = \pm 3$	**3**	**3 marks** • Correct solution **2 marks** • Finds the value of k that produces the constant term **1 mark** • Finds an expression for the general term • Recognises that the constant term is when the power of x is zero
11(c) (i) $x^3-5x^2+8x-4 = (x-2)^2(x-1)$ Thus the linear factors are $(x-2)$ and $(x-1)$	**2**	**2 marks** • Correct solution **1 mark** • Finds one linear factor
11(c) (ii) $x^3-5x^2+8x-4 > 0$ $1 < x < 2$ and $x > 2$  	**2**	**2 marks** • Correct solution **1 mark** • Includes $x = 2$ in solution i.e. $x > 1$

Solution	Marks	Comments
11d) $\frac{x}{3-2x} \le 1$ $3-2x \ne 0$ $x \ne \frac{3}{2}$ $\frac{x}{3-2x} = 1$ $x = 3-2x$ $3x = 3$ $x = 1$  $x \le 1$ or $x > \frac{3}{2}$	3	**3 marks** • Correct graphical solution on number line or algebraic solution, with correct working **2 marks** • Bald answer • Identifies the two correct critical points via a correct method • Correct conclusion to their critical points obtained using a correct method **1 mark** • Uses a correct method • Acknowledges a problem with the denominator. **0 marks** • Solves like a normal equation , with no consideration of the denominator.
11 (e) Let $\alpha = \cos^{-1}\left(\frac{4}{5}\right)$ Let $\beta = \tan^{-1}\left(\frac{5}{12}\right)$   $\sin\left[\cos^{-1}\left(\frac{4}{5}\right) - \tan^{-1}\left(\frac{5}{12}\right)\right] = \sin(\alpha-\beta)$ $= \sin\alpha\cos\beta - \cos\alpha\sin\beta$ $= \left(\frac{3}{5}\right)\left(\frac{12}{13}\right) - \left(\frac{4}{5}\right)\left(\frac{5}{13}\right)$ $= \frac{16}{65}$	2	**3 marks** • Correct solution **2 marks** • Correctly substututes into $\sin(\alpha-\beta)$ **1 mark** • Recognises that the inverse trig functions represents angles and finds other trig ratios to use in the substitution **0 marks** • Bald answer

Solution	Marks	Comments
QUESTION 12		
12(a) (i) $x < -1$; $f(x) = -(3x+3) - (x-1)$ $f(x) = -4x - 2$ $-1 \le x \le 1$; $f(x) = (3x+3) - (x-1)$ $f(x) = 2x + 4$ $x > 1$; $f(x) = (3x+3) + (x-1)$ $f(x) = 4x + 2$ 	2	**2 marks** • Correct graph **1 mark** • Correctly identifies at least two of the branches of the graph • Correct shape with equations
12(a) (ii) 	1	**1 mark** • Correct line
12(a) (iii) point of intersection is when line meets the middle branch: $4x + 3 = 2x + 4$ $2x = 1$ $x = \frac{1}{2}$ As the line is parallel to the third branch (both have a slope of 4), they do not meet again $f(x) \le g(x)$ $x \ge \frac{1}{2}$	1	**1 mark** • Correct solution
12 (b) Let $\alpha = \cos^{-1}\left(-\frac{1}{3}\right) \Rightarrow \frac{\pi}{2} < \alpha < \pi$ $\therefore \tan\alpha < 0$ Thus $\tan\left(\cos^{-1}\left(-\frac{1}{3}\right)\right) = -2\sqrt{2}$ 	2	**2 marks** • Correct solution **1 mark** • $\tan\alpha = 2\sqrt{2}$ • $\tan\alpha < 0$
12(c) (i) 	2	**2 marks** • Correct graph **1 mark** • At least two of the key features ○ Asymptotes at $x = -1$, $x = 1$ ○ Correct behaviour as $x \to -1$ $x \to 1$ ○ Locates "endpoints" correctly

Solution	Marks	Comments
12(c) (ii)  	2	**2 marks** • Correct graph **1 mark** • Locates "endpoints" correctly • Cusp at $x = 1$ & vertical tangent at $x = -1$ • Locates "endpoints" correctly
12(c) (iii)  $x = -1, 0, 1$ and 2	2	**2 marks** • Correct solution **1 mark** • Provides a correct sketch of $y = \lvert f(x) - 1\rvert$ • Finds the x coordinates of the points of intersection of $y = 1$ and their graph obtained via a reflection in the x axis • Obtains $f(x) = 0$ or 2
12(d) restrictions * first digit must be 7, 8 or 9 * last number must be 5, 7 or 9 **Case 1: first digit 7 or 9** Ways = $2 \times 2 \times 3! = 24$ 2 ways of first number Leaving 2 numbers for last spot Arrange the 3 numbers in the middle **Case 2: first digit is 8** Ways = $1 \times 3 \times 3! = 18$ 1 way of first number Leaving 3 numbers for last spot Arrange the 3 numbers in the middle Total odd numbers > 70000 $= 24 + 18$ $= 42$	3	**3 marks** • Correct solution **2 marks** • Finds one case correctly **1 mark** • Recognises that there are two cases to find • Provides logic for one case, that should lead to a correct answer
QUESTION 13		
13(a) (i) $\alpha + \beta + \gamma = 3$	1	**1 mark** • Correct answer
13(a) (ii) $\dfrac{1}{\alpha\beta} + \dfrac{1}{\beta\gamma} + \dfrac{1}{\alpha\gamma} = \dfrac{\alpha + \beta + \gamma}{\alpha\beta\gamma}$ $= \dfrac{3}{-12}$ $= -\dfrac{1}{4}$	2	**2 marks** • Correct solution **1 mark** • Finds $\alpha\beta\gamma$
13(a) (iii) Let the roots be $\alpha, -\alpha$ and β $\alpha - \alpha + \beta = 3$ $\beta = 3$ $P(3) = 0$ $(3)^3 - 3(3)^2 + 3k + 12 = 0$ $3k = -12$ $k = -4$	2	**2 marks** • Correct solution **1 mark** • Finds the third root
13(b) (i)  Triangular base is half a square (or rhombus), whose diagonals are $2h$ metres long $A = \dfrac{1}{2}\left(\dfrac{1}{2} \times 2h \times 2h\right)$ $= h^2$ $\therefore V = Ah$ $= h^2 \times 4$ $= 4h^2$	2	**2 marks** • Correct solution **1 mark** • Forms the area of the triangular base in terms of h

Solution	Marks	Comments
13(b) (ii) $\frac{dh}{dt}=\frac{dV}{dt}\times\frac{dh}{dV}$ $V=4h^2$, $\frac{dV}{dh}=8h$ $=-0.08\times\frac{1}{8h}$ $=-\frac{1}{100h}$ When $h=0.6$, $\frac{dh}{dt}=-\frac{1}{100\times 0.6}$ $=-\frac{1}{60}$ metres/hour	2	**2 marks** • Correct solution **1 mark** • Uses the chain rule to find $\frac{dh}{dt}$ in terms of h
13(c) (i) Reasons could include; • It is not a one-to-one function • It is not continually increasing or decreasing over its domain • It fails the horizontal line test	1	**1 mark** • Correct explanation
13(c) (ii) A suitable restriction would be $x\leq 0$ or $x\geq 0$	1	**1 mark** • Correct restriction
13(c) (iii) $x=4-y^2$ $y^2=4-x$ $y=\pm\sqrt{4-x}$ If restriction is $x\leq 0$ then $y=-\sqrt{4-x}$ If restriction is $x\geq 0$ then $y=\sqrt{4-x}$	2	**2 marks** • Correct solution **1 mark** • Exchanges x and y in the original function
13(c) (iv) The restricted function and its inverse will meet on line $y=x$ $x=4-x^2$ $x^2+x-4=0$ $x=\frac{-1\pm\sqrt{17}}{2}$ If restriction is $x\leq 0$ then intersection is $\left(\frac{-1-\sqrt{17}}{2},\frac{-1-\sqrt{17}}{2}\right)$ If restriction is $x\geq 0$ then intersection is $\left(\frac{-1+\sqrt{17}}{2}\ \frac{-1+\sqrt{17}}{2}\right)$	2	**2 marks** • Correct solution **1 mark** • Recognises that the point of intersection lies on $y=x$
QUESTION 14		
14(a) $P(x)=8x^4-44x^3+54x^2-25x+4$ $P'(x)=32x^3-132x^2+108x-25$ $P''(x)=96x^2-264x+108$ $=12(8x^2-22x+9)$ $=12(2x-1)(4x-9)$ Triple root is either $x=\frac{1}{2}$ or $x=\frac{9}{4}$ Since 4 is NOT divisible by 9, $\frac{1}{2}$ is the logical choice for triple root $P\left(\frac{1}{2}\right)=P'\left(\frac{1}{2}\right)=P''\left(\frac{1}{2}\right)=0$ Thus $\frac{1}{2}$ is the triple root $8x^4-44x^3+54x^2-25x+4=0$ $(2x-1)^3(x-4)=0$ $x=\frac{1}{2}$ or $x=4$	3	**3 marks** • Correct solution **2 marks** • Finds the triple root, and justifies it selection • Solves the equation without justifying the selection of the triple root • Factorises $P(x)$ **1 mark** • Recognises that the derivative is used to find multiple roots. • Finds a root

Solution	Marks	Comments
14(b) $\frac{\sin 2x}{1+\cos 2x} = \frac{2\sin x\cos x}{1+2\cos^2 x-1}$ $= \frac{2\sin x\cos x}{2\cos^2 x}$ $= \frac{\sin x}{\cos x}$ $= \tan x$	**2**	**2 marks** • Correct solution **1 mark** • Correct use of a double angle result
14(c) (i) $V = 500 + Ae^{-kt}$ $\frac{dV}{dt} = -kAe^{-kt}$ $= -k(500 + Ae^{-kt} - 500)$ $= -k(V-500)$	**1**	**1 mark** • Correct solution
14(c) (ii) when $t=0$, $V=0$ $\Rightarrow$ $A=-500$ when $t=5$, $V=21$ m/s $\Rightarrow$ $21 = 500-500e^{-5k}$ $-479 = -500e^{-5k}$ $\frac{479}{500} = e^{-5k}$ $-5\boldsymbol{k} = \ln\left(\frac{479}{500}\right)$ $\boldsymbol{k} = -\frac{1}{5}\ln\left(\frac{479}{500}\right)$ $= \frac{1}{5}\ln\left(\frac{500}{479}\right)$	**2**	**2 marks** • Correct solution **1 mark** • Finding either A or k
14(c) (iii) when $t=20$, $\Rightarrow$ $V = 500-500e^{-20k}$ $= 500-500e^{4\ln\left(\frac{479}{500}\right)}$ $= 500-500\times\left(\frac{479}{500}\right)^4$ $= 78.85462015\ldots$ The body is falling at 79 m/s	**1**	**1 mark** • Correct solution
14(c) (iv) The maximum possible velocity (limiting velocity) is 500 m/s	**1**	**1 mark** • Correct answer
14(d) By equating the coefficients of x^4 on both sides of the identity LHS: $(1+x)^4(1+x)^9 = \left[\binom{4}{0}+\binom{4}{1}x+\ldots+\binom{4}{4}x^4\right]\left[\binom{9}{0}+\binom{9}{1}x+\ldots+\binom{9}{9}x^9\right]$ Term with $x^4 = \binom{4}{0}\times\binom{9}{4}x^4+\binom{4}{1}x\times\binom{9}{3}x^3+\binom{4}{2}x^2\times\binom{9}{2}x^2+\binom{4}{3}x^3\times\binom{9}{1}x+\binom{4}{4}x^4\times\binom{9}{0}$ $= \left[\binom{4}{0}\binom{9}{4}+\binom{4}{1}\binom{9}{3}+\binom{4}{2}\binom{9}{2}+\binom{4}{3}\binom{9}{1}+\binom{4}{4}\binom{9}{0}\right]x^4$ RHS: $(1+x)^{13} = \binom{13}{0}+\binom{13}{1}x+\ldots+\binom{13}{4}x^4+\ldots\binom{13}{13}x^{13}$ Term with $x^4 = \binom{13}{4}$ Since $(1+x)^4(1+x)^9 \equiv (1+x)^{13}$ the coefficients of x^4 must be equal i.e. $\binom{4}{0}\binom{9}{4}+\binom{4}{1}\binom{9}{3}+\binom{4}{2}\binom{9}{2}+\binom{4}{3}\binom{9}{1}+\binom{4}{4}\binom{9}{0}=\binom{13}{4}$	**2**	**2 marks** • Correct solution **1 mark** • Realises that the problem involves finding the coefficients of x^4

Solution	Marks	Comments
14(e) $2\sin A\cos B = \sin(A+B) + \sin(A-B)$ $A + B + C = 180°$ $A + B = 180° - C$ $\therefore \quad \sin C = \sin(180 - C) + \sin(A - B)$ $\sin C = \sin C + \sin(A - B)$ $\sin(A - B) = 0$ $A - B = 0°, 180°, \ldots$ But A and B are both less than $180°$ Since $\therefore A - B = 0$ $A = B$ i.e. ΔABC is isosceles	3	**3 marks** • Correct solution **2 marks** • obtains $\sin(A - B) = 0$ **1 mark** • Recognises that $\sin(A + B) = \sin C$ • Uses the product to sum formula