Blacktown Boys' High School

2023

Year 11 Assessment Task 3

Yearly Examination

Mathematics Extension 1

General Instructions

- Reading time – 10 minutes
- Working time – 90 minutes
- Write using black pen
- NESA approved calculators may be used
- All diagrams are not drawn to scale
- In Questions 11 – 13, show relevant mathematical reasoning and/or calculations

Total marks: 55

Section I – 10 marks (pages 3 – 6)
- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 45 marks (pages 7 – 11)
- Attempt Questions 11 – 13
- Allow about 75 minutes for this section

Assessor: X. Chirgwin

Student Name: _____

Teacher Name: _____

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Section I

10 marks
Attempt Questions 1–10

Use the multiple choice answer sheet provided for Questions 1–10.

Q1. What is the remainder when $x^4 - 7x + 3$ is divided by $x + 2$?

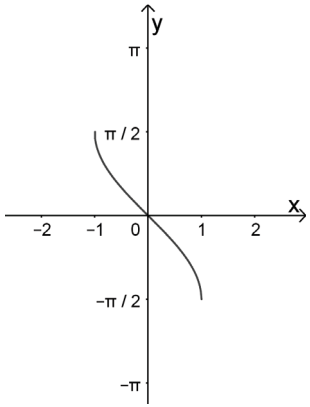
A. 1

B. 5

C. 33

D. 42

Q2. Which of the following functions represents the graph shown?



A. $y = \cos^{-1} x$

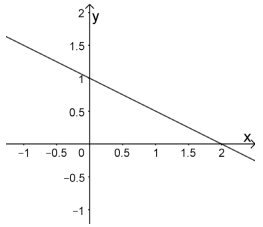
B. $y = -\cos^{-1} x$

C. $y = \sin^{-1} x$

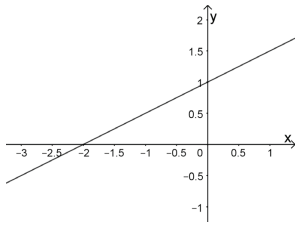
D. $y = -\sin^{-1} x$

Q3. The graph of the function which has parametric equations $x = -2t, y = 1 + t$ is

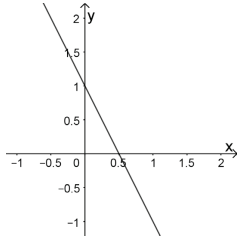
A.



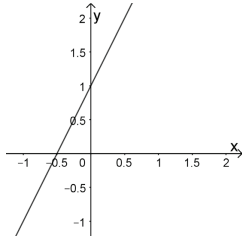
B.



C.



D.



Q4. It is given that $\sin\left(\frac{7\pi}{12}\right) = \frac{1+\sqrt{3}}{2\sqrt{2}}$.

Which of the following is the value of $\sin^{-1}\left(\frac{1+\sqrt{3}}{2\sqrt{2}}\right)$?

A. $\frac{5\pi}{12}$

B. $\frac{7\pi}{12}$

C. $\frac{17\pi}{12}$

D. $\frac{19\pi}{12}$

Q5. Three students are running for the School Captain position. There are 802 students who vote. The student with the most votes will gain the School Captain position. What is the minimum number of votes needed to determine the new School Captain?

A. 402

B. 401

C. 268

D. 267

Q6. The angles α and β satisfies $\cos\alpha = \frac{12}{13}$ and $\cos\beta = -\frac{3}{4}$.

Given that $0 \leq \alpha \leq \pi$ and $0 \leq \beta \leq \pi$, what is the value of $\cos(\alpha + \beta)$?

A. $\frac{-36 + 5\sqrt{7}}{52}$

B. $\frac{-36 - 5\sqrt{7}}{52}$

C. $\frac{-12\sqrt{7} + 15}{52}$

D. $\frac{-12\sqrt{7} - 15}{52}$

Q7. A polynomial $P(x)$ has a triple root at $x = 3$. Which statement is true?

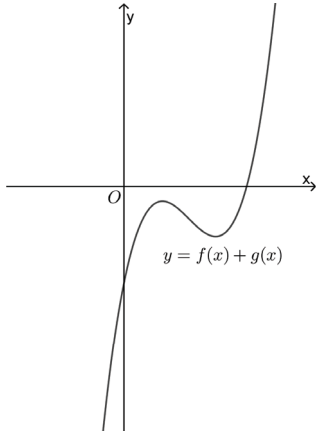
A. $(x+3)^3$ is a factor of $P'(x)$

B. $(x-3)^3$ is a factor of $P'(x)$

C. $(x+3)^2$ is a factor of $P'(x)$

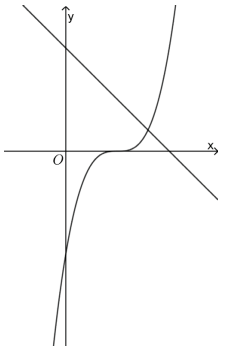
D. $(x-3)^2$ is a factor of $P'(x)$

Q8. The diagram shows the graph of the sum of the functions $f(x)$ and $g(x)$.

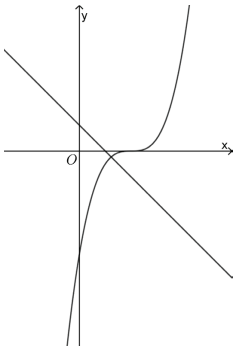


Which of the following best represents the graphs of both $f(x)$ and $g(x)$?

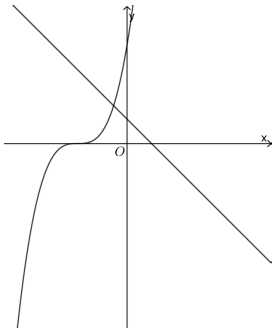
A.



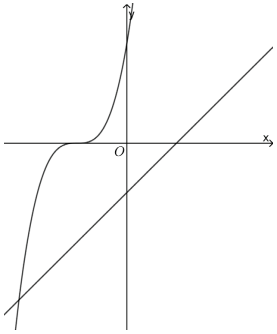
B.



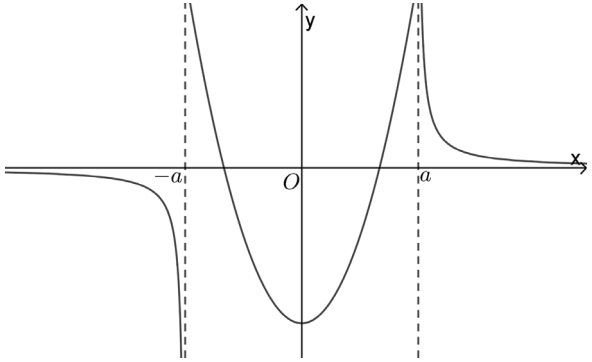
C.



D.



Q9. Consider the function $f(x)$ given in the graph below.



Which domain of the function $f(x)$ above is valid for the inverse function to exist?

A. $-a < x < 0$

B. $-a < x < a$

C. $x < 0$

D. $x > 0$

Q10. By considering the binomial expansion of $(1+x)^{10}$, what is the value of

$$5 \times {}^{10}C_1 + 5^2 \times {}^{10}C_2 + 5^3 \times {}^{10}C_3 + \cdots + 5^{10} \times {}^{10}C_{10} \text{ ?}$$

A. 4^{10}

B. 6^{10}

C. $4^{10} - 1$

D. $6^{10} - 1$

End of Section I

Section II

45 Marks
Attempt Questions 11–13

Answer each question on a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11–13, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) Let $P(x) = x^3 + 2x^2 - 7x - 12$.

(i) Show that $P(-3) = 0$. **1**

(ii) Hence, factor the polynomial $P(x)$ as $A(x)B(x)$, where $B(x)$ is a quadratic polynomial. **2**

(b) Show through expansion that $\left(3 - \sqrt{5}\right)^4 = 376 - 168\sqrt{5}$. **2**

(c) How many arrangements of the letters of the word SENTENCES are possible? **2**

(d) Solve $\dfrac{x - 14}{3x + 2} \geq -1$. **3**

(e) A function $f(x)$ is defined by $f(x) = \log_e(10 - 2x)$.

(i) What is the equation of the inverse function $f^{-1}(x)$? **2**

(ii) What is the range of the inverse function? **1**

(iii) Sketch the graph of $y = f^{-1}(x)$, showing all key features including the intercepts. **2**

End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) From a group of 9 girls and 5 boys, they are chosen to form different patrol groups of 5 to monitor a busy pedestrian crossing.

(i) If there are no restrictions, how many different groups of 5 are possible? **1**

(ii) What is the probability of choosing a group with 3 girls and 2 boys? **2**

(iii) If a group of 3 girls and 2 boys stand in a line, what is the probability that the boys stand together? **2**

(b) (i) Use $t = \tan\frac{\theta}{2}$, show that $\frac{2-2\cos\theta}{3\sin\theta} = \frac{2}{3}\tan\frac{\theta}{2}$. **2**

(ii) Hence show that $2\tan 15° = 4 - 2\sqrt{3}$. **1**

(c) For the function $f(x) = 2\tan^{-1}x + \pi$.

(i) Find the values of $f(-1)$ and $f(1)$. **1**

(ii) Sketch the function $y = f(x)$, showing clearly the two points from part (i) and all other key features. **2**

(d) Find the constant term in the expansion of $\left(\frac{6}{x^2} + \frac{x}{3}\right)^{12}$. **2**

(e) Sketch the graph of $f(x) = -|x|^2 + 4|x| - 3$, showing all key features. **2**

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet.

(a) A group of twelve people arrive at a restaurant. The only seating available for them is at two circular tables, one that seats eight people and another that seats four. How many different seating arrangements are there for this group of twelve people? **2**

(b) The rate of increase of a population P of sandflies at a particular location is proportional to the excess of the population over 1800. This can be expressed as $\frac{dP}{dt} = k(P - 1800)$ where k is a constant and t represents time in weeks.

(i) Show that $P = 1800 + Ae^{kt}$, where A is a constant, satisfies the differential equation $\frac{dP}{dt} = k(P - 1800)$. **1**

(ii) Initially the population is 2200 and 2 weeks later it is 2700. Find the exact value of A and k. **2**

(iii) Find how many weeks it takes for the population to double, round your answer to 1 decimal place. **2**

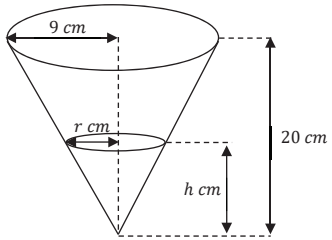
(c) (i) Show that $\cos 3A = 4\cos^3 A - 3\cos A$ **2**

(ii) Hence show that if $2\cos A = x + \frac{1}{x}$, then $2\cos 3A = x^3 + \frac{1}{x^3}$. **1**

Question 13 continues on next page

Question 13 (continued)

(d) Soft serve ice cream is served into a jumbo sized right circular cone as shown in the diagram below. The cone has height $20\ cm$ and radius $9\ cm$.



Ice cream is leaking through a hole of negligble size at the bottom of the cone at a constant rate of $15\ cm^3$ per minute.

(i) Use similar triangles to express the radius r in terms of the height h. **1**

(ii) Show that when the depth of the ice cream in the cone is $h\ cm$, the volume V of the ice cream is $V = \dfrac{27\pi h^3}{400}$. **1**

(iii) At what rate is the depth of the ice cream in the cone decreasing when $h = 12\ cm$? **2**

End of Paper

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Student Name:_____

Multiple Choice Answer Sheet

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

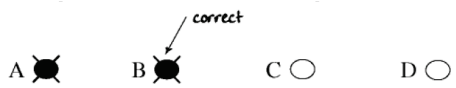
Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word 'correct' and drawing an arrow as follows.



Start Here →

1.	A ○	B ○	C ○	D ○
2.	A ○	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ○
5.	A ○	B ○	C ○	D ○
6.	A ○	B ○	C ○	D ○
7.	A ○	B ○	C ○	D ○
8.	A ○	B ○	C ○	D ○
9.	A ○	B ○	C ○	D ○
10.	A ○	B ○	C ○	D ○

2023 Year 11 Mathematics Extension 1 AT3 Yearly Solutions

Section 1

Q1. — 1 Mark

C

$P(x) = x^4 - 7x + 3$

$P(-2) = (-2)^4 - 7 \times (-2) + 3 = 33$

Q2. — 1 Mark

D

$y = -\sin^{-1} x$

Q3. — 1 Mark

A

$x = -2t \rightarrow t = -\frac{x}{2}$

$y = 1 + \left(-\frac{x}{2}\right)$

$y = 1 - \frac{x}{2}$

The linear function with gradient $-\frac{1}{2}$ and y intercept of 1.

Q4. — 1 Mark

A

$\sin\left(\frac{7\pi}{12}\right) = \frac{1+\sqrt{3}}{2\sqrt{2}}$

$-\frac{\pi}{2} \leq \sin^{-1}\left(\frac{1+\sqrt{3}}{2\sqrt{2}}\right) \leq \frac{\pi}{2}$

$\frac{7\pi}{12} > \frac{\pi}{2}$

$\pi - \frac{7\pi}{12} = \frac{5\pi}{12}$

$\therefore \sin^{-1}\left(\frac{1+\sqrt{3}}{2\sqrt{2}}\right) = \frac{5\pi}{12}$

Q5. — 1 Mark

C

$\frac{802}{3} = 267.333\ldots$

To ensure a win, at least 268 votes is required. Someone could win with a 267-267-268 split.

Q6. — 1 Mark

B

$\cos\alpha = \frac{12}{13}, \quad \sin\alpha = \frac{5}{13}$

$\cos\beta = -\frac{3}{4}, \quad \sin\beta = \frac{\sqrt{7}}{4}$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\cos(\alpha+\beta) = \frac{12}{13} \times -\frac{3}{4} - \frac{5}{13} \times \frac{\sqrt{7}}{4}$

$\cos(\alpha+\beta) = \frac{-36-5\sqrt{7}}{52}$

Q7. — 1 Mark

D

$P(x) = (x-3)^3 Q(x)$

$P'(x) = 3(x-3)^2 Q(x) + (x-3)^3 Q'(x)$

$P'(x) = (x-3)^2[3Q(x) + (x-3)Q'(x)]$

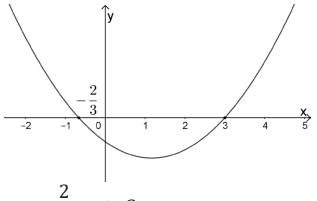
$\therefore (x-3)^2$ is a factor of $P'(x)$

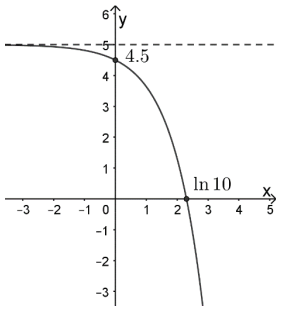
Q8. — 1 Mark

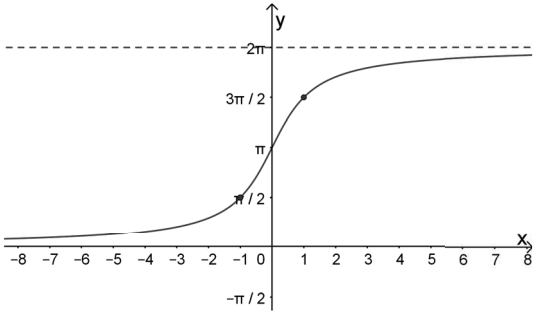
B

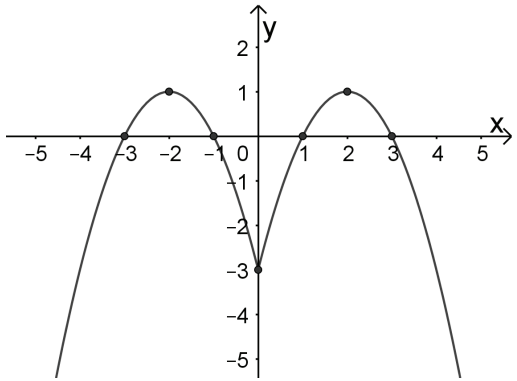
The y-intercept of $f(x) + g(x)$ is negative. The only option that results in in a negative y-intercept is B.

Q9.	**A** The only domain of $f(x)$ that passes through the horizontal line test is $-a < x < 0$	1 Mark
Q10.	**D** $(1+x)^{10} = {}^{10}C_0 + x \times {}^{10}C_1 + x^2 \times {}^{10}C_2 + x^3 \times {}^{10}C_3 + \cdots + x^{10} \times {}^{10}C_{10}$ Let $x = 5$, $(1+5)^{10} = {}^{10}C_0 + 5 \times {}^{10}C_1 + 5^2 \times {}^{10}C_2 + 5^3 \times {}^{10}C_3 + \cdots + 5^{10} \times {}^{10}C_{10}$ $6^{10} - 1 = 5 \times {}^{10}C_1 + 5^2 \times {}^{10}C_2 + 5^3 \times {}^{10}C_3 + \cdots + 5^{10} \times {}^{10}C_{10}$	1 Mark

Section 2		
Q11ai	$P(x) = x^3 + 2x^2 - 7x - 12$ $P(-3) = (-3)^3 + 2(-3)^2 - 7(-3) - 12$ $P(-3) = -27 + 18 + 21 - 12$ $P(-3) = 0$	1 Mark Correct solution
Q11aii	$x^2 - x - 4$ $x+3 \;\big)\; x^3 + 2x^2 - 7x - 12$ $x^3 + 3x^2$ $-x^2 - 7x$ $-x^2 - 3x$ $-4x - 12$ $-4x - 12$ 0 Or $x^3 + 2x^2 - 7x - 12 = (x+3)(ax^2 + bx + c)$ $ax^3 = x^3$ $\quad 3c = -12$ $\quad 3bx + cx = -7x$ $a = 1$ $\quad c = -4$ $\quad 3b - 4 = -7$ $b = -1$ $\therefore P(x) = (x+3)(x^2 - x - 4)$	2 Marks Correct solution 1 Mark Attempts to divide by $x+3$ and makes significant progress
Q11b	$\left(3 - \sqrt{5}\right)^4$ $= 3^4 + 4 \times 3^3 \times \left(-\sqrt{5}\right) + 6 \times 3^2 \times \left(-\sqrt{5}\right)^2 + 4 \times 3 \times \left(-\sqrt{5}\right)^3 + \left(-\sqrt{5}\right)^4$ $= 81 - 108\sqrt{5} + 270 - 60\sqrt{5} + 25$ $= 376 - 168\sqrt{5}$	2 Marks Correct solution 1 Mark Correct expansion
Q11c	SENTENCES has 9 letters altogether with S occurring twice, E occurring 3 times, N occurring twice. $\frac{9!}{2! \times 3! \times 2!} = 15120$	2 Marks Correct solution 1 Mark Makes significant progress
Q11d	$\frac{x-14}{3x+2} \geq -1 \qquad x \neq -\frac{2}{3}$ $(x-14)(3x+2) \geq -(3x+2)^2$ $(x-14)(3x+2) + (3x+2)^2 \geq 0$ $(3x+2)\big((x-14) + (3x+2)\big) \geq 0$ $(3x+2)\big((4x-12)\big) \geq 0$  $x < -\frac{2}{3},\ x \geq 3 \qquad or \qquad \left(-\infty, -\frac{2}{3}\right) \cup [3, \infty)$	3 Marks Correct solution 2 Marks Makes significant progress and identifies critical values 1 Mark Correctly multiplies both sides by the square of the denominator

Q11ei	$y = \log_e(10 - 2x)$ $x = \log_e(10 - 2y)$ $e^x = 10 - 2y$ $2y = 10 - e^x$ $y = \frac{1}{2}(10 - e^x)$ $\therefore f^{-1}(x) = \frac{1}{2}(10 - e^x)$	2 Marks Correct solution 1 Mark Swaps x and y and attempts to make y the subject
Q11eii	Domain of $f(x)$ is $x < 5$ Range of $f^{-1}(x)$ is $y < 5$ or $(-\infty, 5)$	1 Mark Correct solution
Q11eiii	$y = \frac{1}{2}(10 - e^x)$ y-intercept, $x = 0$ $y = \frac{1}{2}(10 - e^0)$ $y = \frac{1}{2}(10 - 1) = \frac{9}{2}$ x-intercept, $y = 0$ $\frac{1}{2}(10 - e^x) = 0$ $e^x = 10$ $x = \ln 10$ 	2 Marks Correct solution 1 Mark Correct graph showing some key features
Q12ai	There are 14 people altogether, choosing 5 people. ${}^{14}C_5 = 2002$	1 Mark Correct solution
Q12aii	Choosing 3 girls from 9 girls and 2 boys from 5 boys. ${}^9C_3 \times {}^5C_2 = 840$ $Probability = \frac{840}{2002} = \frac{60}{143}$	2 Marks Correct solution 1 Mark Finds ${}^9C_3 \times {}^5C_2$
Q12aiii	2 boys and 3 girls arranged in a line where the boys stand together: $2! \times 4! = 48$ No restriction of 5 people arranged in a line: $5! = 120$ $Probability = \frac{48}{120} = \frac{2}{5}$	2 Marks Correct solution 1 Mark Finds $2! \times 4!$
Q12bi	RTP: $\frac{2 - 2\cos\theta}{3\sin\theta} = \frac{2}{3}\tan\frac{\theta}{2}$ $t = \tan\frac{\theta}{2}$ $LHS = \frac{2 - 2\left(\frac{1 - t^2}{1 + t^2}\right)}{3 \times \frac{2t}{1 + t^2}}$ $LHS = \frac{2(1 + t^2) - 2(1 - t^2)}{1 + t^2} \times \frac{1 + t^2}{6t}$ $LHS = \frac{4t^2}{1 + t^2} \times \frac{1 + t^2}{6t}$ $LHS = \frac{2}{3}t$ $LHS = \frac{2}{3}\tan\frac{\theta}{2}$ $LHS = RHS$	2 Marks Correct solution 1 Mark Correct substitution and progresses to $\frac{4t^2}{1 + t^2} \times \frac{1 + t^2}{6t}$

Q12bii	$Let \tan\frac{\theta}{2} = \tan 15°$ $\theta = 30°$ $2\tan 15° = 3 \times \frac{2 - 2\cos 30°}{3\sin 30°}$ $2\tan 15° = \frac{2 - 2 \times \frac{\sqrt{3}}{2}}{\frac{1}{2}}$ $2\tan 15° = \left(2 - \sqrt{3}\right) \times 2$ $2\tan 15° = 4 - 2\sqrt{3}$	1 Mark Correct solution
Q12ci	$f(-1) = 2\tan^{-1}(-1) + \pi$ $f(-1) = -\frac{\pi}{2} + \pi$ $f(-1) = \frac{\pi}{2}$ $f(1) = 2\tan^{-1}(1) + \pi$ $f(1) = \frac{\pi}{2} + \pi$ $f(1) = \frac{3\pi}{2}$	1 Mark Correct solution
Q12cii	$f(x) = 2\tan^{-1}x + \pi$ 	2 Marks Correct solution 1 Mark Correct graph with some key features shown
Q12d	$\left(\frac{6}{x^2} + \frac{x}{3}\right)^{12}$ $= \sum_{k=0}^{12} \binom{12}{k} \left(\frac{6}{x^2}\right)^{12-k} \left(\frac{x}{3}\right)^k$ $= \sum_{k=0}^{12} \binom{12}{k} 6^{12-k} \times x^{-24+2k} \times 3^{-k} \times x^k$ Constant term occurs when x^0 $x^{-24+2k} \times x^k = x^0$ $-24 + 2k + k = 0$ $3k = 24$ $k = 8$ Constant term is: $\binom{12}{8} 6^{12-8} \times 3^{-8}$ $= \frac{880}{9}$	2 Marks Correct solution 1 Mark Finds the correct values of k

Q12e	$f(x) = -\|x\|^2 + 4\|x\| - 3$  	2 Marks Correct solution 1 Mark Correct graph with some key features shown
Q13a	For every group of 8 at the large table, there is a corresponding group of 4 at the small table. The number of arrangements: $^{12}C_8 \times 7! \times 3!$ $= 14\,968\,800$	2 Marks Correct solution 1 Mark Obtains $7! \times 3!$
Q13bi	$P = 1800 + Ae^{kt}$ $\frac{dP}{dt} = k \times Ae^{kt}$ $\frac{dP}{dt} = k(P - 1800)$	1 Mark Correct solution
Q13bii	$t = 0, P = 2200$ $2200 = 1800 + Ae^0$ $A = 400$ $t = 2, P = 2700$ $2700 = 1800 + 400e^{2k}$ $900 = 400e^{2k}$ $\frac{9}{4} = e^{2k}$ $2k = \ln\frac{9}{4}$ $k = \frac{1}{2}\ln\frac{9}{4}$ $k = \ln\frac{3}{2}$	2 Marks Correct solution 1 Mark Find A
Q13biii	Population doubled is 4400 $4400 = 1800 + 400e^{t \times \ln\frac{3}{2}}$ $2600 = 400e^{t \times \ln\frac{3}{2}}$ $\frac{2600}{400} = e^{t \times \ln\frac{3}{2}}$ $t \times \ln\frac{3}{2} = \ln\frac{13}{2}$ $t = \ln\frac{13}{2} \div \ln\frac{3}{2}$ $t \approx 4.6$ It takes approximately 4.6 weeks for the population to double.	2 Marks Correct solution 1 Mark Finds $e^{t \times \ln\frac{3}{2}} = \frac{2600}{400}$

Q13ci	$\cos 3A$ $= \cos(2A + A)$ $= \cos 2A \cos A - \sin 2A \sin A$ $= (2\cos^2 A - 1)\cos A - 2\sin A \cos A \sin A$ $= 2\cos^3 A - \cos A - 2\sin^2 A \cos A$ $= 2\cos^3 A - \cos A - 2(1 - \cos^2 A)\cos A$ $= 2\cos^3 A - \cos A - 2\cos A + 2\cos^3 A$ $= 4\cos^3 A - 3\cos A$	2 Marks Correct solution 1 Mark Correct expansion of
Q13cii	$2\cos 3A$ $= 2(4\cos^3 A - 3\cos A)$ $= 8\cos^3 A - 6\cos A$ $= (2\cos A)^3 - 3(2\cos A)$ $= \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right)$ $= x^3 + 3x^2 \times \frac{1}{x} + 3x \times \frac{1}{x^2} + \frac{1}{x^3} - 3x - \frac{3}{x}$ $= x^3 + 3x + \frac{3}{x} + \frac{1}{x^3} - 3x - \frac{3}{x}$ $= x^3 + \frac{1}{x^3}$	1 Mark Correct solution
Q13di	$\frac{r}{9} = \frac{h}{16}$ $r = \frac{9h}{20}$	1 Mark Correct solution
Q13dii	$V = \frac{1}{3}\pi r^2 h$ $V = \frac{1}{3} \times \pi \times \left(\frac{9h}{20}\right)^2 \times h$ $V = \frac{27\pi h^3}{400}$	1 Mark Correct solution
Q13diii	$\frac{dV}{dh} = \frac{81\pi}{400}h^2$ $\frac{dV}{dt} = -15\ cm^3/min$ $\frac{dh}{dt} = \frac{dV}{dt} \times \frac{dh}{dV}$ $\frac{dh}{dt} = -15 \times \frac{400}{81\pi h^2}$ $\frac{dh}{dt} = -\frac{2000}{27\pi h^2}$ $At\ h = 12\ cm$ $\frac{dh}{dt} = -\frac{2000}{27\pi \times 12^2}$ $\frac{dh}{dt} = -\frac{125}{243\pi}\ cm/min$ ∴The depth of the ice cream is decreasing at a rate of $\frac{125}{243\pi}\ cm/min$ when $h = 12\ cm$.	3 Marks Correct solution 2 Marks Finds $\frac{dh}{dt} = -\frac{2000}{27\pi h^2}$ 1 Marks Correct $\frac{dV}{dh}$