Blacktown Boys' High School

2020

Year 11 Assessment Task 2

Yearly Examination

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 90 minutes
- Write using black pen
- NESA approved calculators may be used
- All diagrams are not drawn to scale
- In Questions 11 – 13, show relevant mathematical reasoning and/or calculations

Total marks: 55

Section I – 10 marks (pages 3 – 6)
- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 45 marks (pages 7 – 11)
- Attempt Questions 11 – 13
- Allow about 75 minutes for this section

Assessor: X. Chirgwin

Student Name: _____

Teacher Name: _____

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Section I

10 marks
Attempt Questions 1–10

Use the multiple choice answer sheet provided for Questions 1–10.

Q1. Which expression is equivalent to $\cos 5x - \cos 7x$?

A. $2\cos 6x \cos x$

B. $2\sin 6x \sin x$

C. $2\cos 6x \sin x$

D. $2\sin 6x \cos x$

Q2. How many ten-letter arrangements can be made using letters of the word STATISTICS?

A. $5!$

B. $10!$

C. $\dfrac{10!}{3! \times 2!}$

D. $\dfrac{10!}{3! \times 3! \times 2!}$

Q3. What is the remainder when $4x^3 - x^2 + 7$ is divided by $x + 1$?

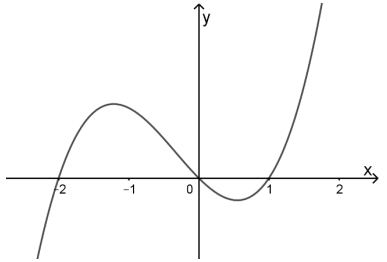
A. 2

B. 4

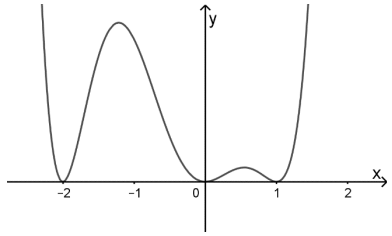
C. 10

D. 12

Q4. The graph of the function $y = f(x)$ is shown.



A second graph is obtained from the function $y = f(x)$.



Which equation best represents the second graph?

A. $y = \sqrt{f(x)}$

B. $y = |f(x)|$

C. $y = f(|x|)$

D. $y = \big(f(x)\big)^2$

Q5. What is the minimum number of people required in a room to guarantee that 3 of them were born in the same month?

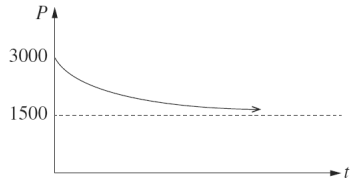
A. 3

B. 13

C. 25

D. 37

Q6. The diagram shows the number of pelicans, $P(t)$, on an island at time t.



Which equation best represents this graph?

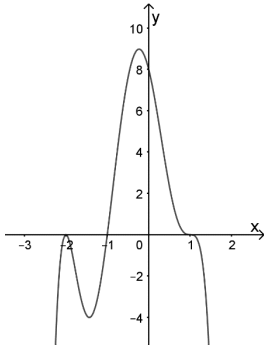
A. $P(t) = 1500e^{-kt}$

B. $P(t) = 1500 + 1500e^{-kt}$

C. $P(t) = 3000 - 1500e^{-kt}$

D. $P(t) = 4500 - 1500e^{-kt}$

Q7. The diagram below shows the graph of $y = a(x+b)(x+c)^2(x+d)^3$.



What are possible values of a, b, c, and d?

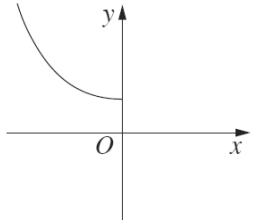
A. $a = -2, b = 1, c = 2, d = -1$

B. $a = -2, b = -1, c = -2, d = 1$

C. $a = 8, b = 1, c = 2, d = -1$

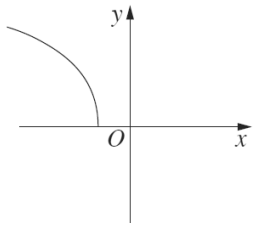
D. $a = 8, b = -1, c = -2, d = 1$

Q8. The diagram shows the graph $y = f(x)$.

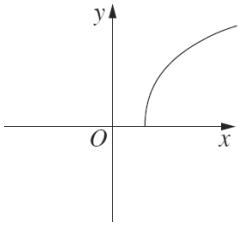


Which diagram shows the graph $y = f^{-1}(x)$?

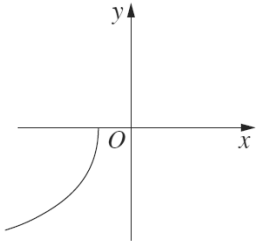
A.



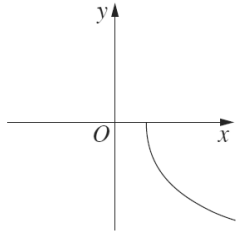
B.



C.



D.



Q9. When the polynomial $P(x)$ is divided by $(x-1)(x+4)$, the remainder is $2x + 11$.

What is the remainder when $P(x)$ is divided by $x + 4$?

A. 3

B. 4

C. 11

D. 13

Q10. How many solutions does the equation $x^{\frac{1}{2}} = |x-1| - 1$ have?

A. 0

B. 1

C. 2

D. 3

End of Section I

Section II

45 Marks
Attempt Questions 11–13

Answer each question on a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11–13, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

a) Consider the function $f(x) = 2\sin^{-1}\left(\frac{x}{3}\right)$.

i) Evaluate $f(3)$. **1**

ii) Sketch the graph of $y = f(x)$, showing the intercept and the coordinates of the end points. **2**

iii) State the domain and range of $y = f(x)$ using the interval notation. **2**

b) Solve $\frac{3}{2x+5} \leq 1$. **3**

c) Consider the polynomial $P(x) = 4x^3 - 13x^2 + 4x + 12$.

i) Show that $P(x)$ has a double root at $x = 2$. **2**

ii) Hence, factorise $P(x)$ completely. **1**

d) Find the Cartesian equation of the parabola given that $x = 2t - 5$ and $y = 12t^2 + 1$. **1**

e) Show that $\frac{\cos 2x - \cos x}{\sin 2x + \sin x} = \cot x - \operatorname{cosec} x$ **3**

End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet.

a) From a group of 7 cats and 8 dogs, 10 are chosen at random as a pet package to go to a nursing home.

i) In how many ways can this pet package be chosen? **1**

ii) In how many ways can this pet package be chosen if there are exactly 6 cats and 4 dog? **1**

iii) What is the probability that this pet package will consist of at least 6 cats? **2**

b) i) How many odd 4 digit numbers can be made from the digits $2, 3, 4, 5, 6$, if none of the digits are repeated? **1**

ii) What is the probability of an odd number being selected if the digits may be repeated? **2**

c) A table has 4 boys and 4 girls sitting around it and the seating is arranged at random.

i) Find the number of ways of sitting around the table so that the boys and girls alternate. **1**

ii) Find the number of ways of sitting around the table if 2 particular people will sit apart. **2**

d) Find the constant term of $\left(4x^3 + \frac{3}{x}\right)^8$. **2**

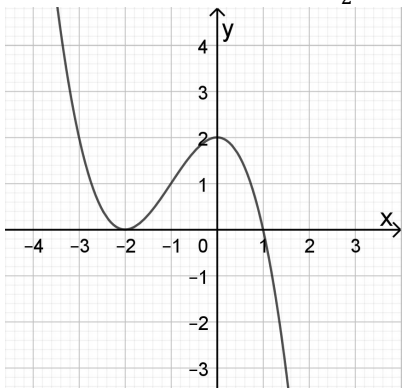
e) If $\alpha = \sin^{-1}\left(\frac{3}{5}\right)$ and $\beta = \sin^{-1}\left(\frac{4}{7}\right)$, find the exact value of **3**

$\sin(\alpha - \beta)$.

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet.

a) The diagram shows the graph of the function $f(x) = \frac{1}{2}(1-x)(x+2)^2$.



Sketch each of the following functions on separate set of axes, showing all key features.

i) $y = |f(x)|$ **1**

ii) $y = \frac{1}{f(x)}$ **2**

b) N is the number of finches in a certain population at time t years. The population size N satisfies the equation $\frac{dN}{dt} = -k(N - 1500)$, for some constant k.

i) Verify that $N = 1500 + Ae^{-kt}$ (where A is a constant) is a solution of the equation, $\frac{dN}{dt} = -k(N - 1500)$. **1**

ii) Initially there are 3500 finches but after 2 years there are only 2800 left. Find the exact values of A and k. **2**

iii) How long will it take for the population to drop to 2100, correct to one decimal place? **1**

Question 13 continues on next page

Question 13 (continued)

c) $P(x) = x^3 - 6x^2 + ax - 4$ where $a > 0$. Given that all the roots of $P(x)$ are real and positive, and that one of the roots is the product of the other two roots, find the value of a. **3**

d) A balloon in the shape of a cylinder, with height h and radius r, expands so that h is always proportional to r, that is $h = kr$ for some constant k. When $r = 5\ cm$, the volume of expanding at the rate of $0.25\ cm^3/s$.

i) Show that when $r = 5\ cm$ the rate of change of the radius is given by $\frac{dr}{dt} = \frac{1}{300\pi k}$. **2**

ii) If the surface area of the balloon is expanding at the rate of $0.15\ cm^2/s$ when $r = 5\ cm$, find the constant of proportionality k. **3**

End of Paper

BBHS 2020 Preliminary Mathematics Extension 1 Assessment Task 2 Yearly Examination

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Student Name:_____

Multiple Choice Answer Sheet

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

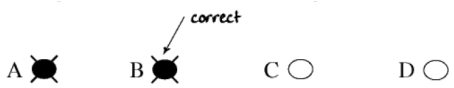
Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word 'correct' and drawing an arrow as follows.

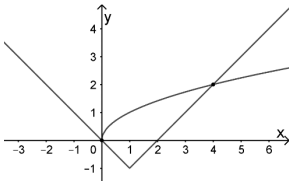


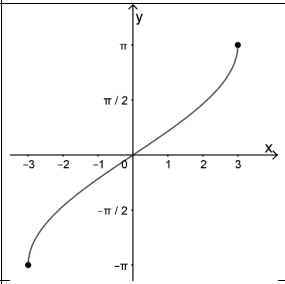
Start Here →

1.	A ○	B ○	C ○	D ○
2.	A ○	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ○
5.	A ○	B ○	C ○	D ○
6.	A ○	B ○	C ○	D ○
7.	A ○	B ○	C ○	D ○
8.	A ○	B ○	C ○	D ○
9.	A ○	B ○	C ○	D ○
10.	A ○	B ○	C ○	D ○

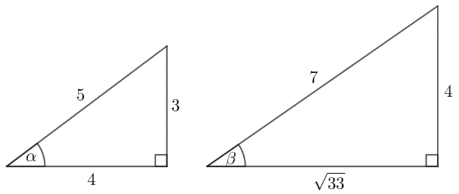
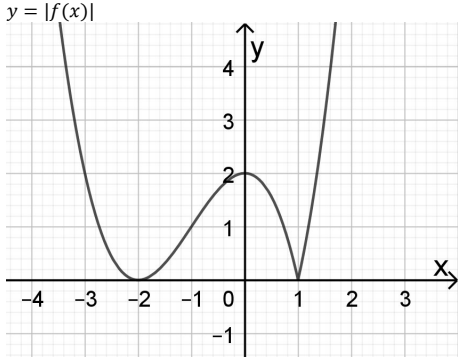
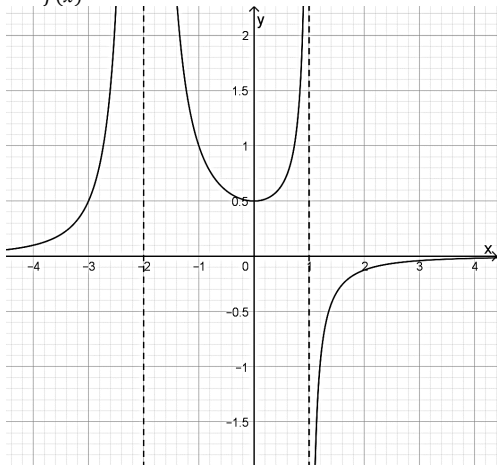
2020 Year 11 Mathematics Extension 1 AT2 Yearly Solutions

Section 1

Q1.	**B** $2\sin 6x \sin x = \cos(6x - x) - \cos(6x + x)$ $2\sin 6x \sin x = \cos 5x - \cos 7x$	1 Mark
Q2.	**D** STATISTICS 10 letters with 3S, 3T, 2I $\frac{10!}{3! \times 3! \times 2!}$	1 Mark
Q3.	**A** $P(x) = 4x^3 - x^2 + 7$ $P(-1) = 4 \times (-1)^3 - (-1)^2 + 7$ $P(-1) = 2$	1 Mark
Q4.	**D** All single roots in the first graph became double roots in the second graph. $y = \big(f(x)\big)^2$	1 Mark
Q5.	**C** $\frac{25}{12} = 2\frac{1}{12}$ A minimum of 25 people is required to guarantee that 3 of them were born in the same month.	1 Mark
Q6.	**B** At $t = 0$ $P(0) = 1500 + 1500 = 3000$ As $t \to \infty, P(t) \to 1500$ $P(t) = 1500 + 1500e^{-kt}$	1 Mark
Q7.	**A** Single root at $x = -1$, double root at $x = -2$, triple root at $x = 1$ y-intercept at 8, $-2 \times 1 \times 2^2 \times (-1)^3 = 8$ $y = -2(x + 1)(x + 2)^2(x - 1)^3$	1 Mark
Q8.	**D** Reflect the graph along the line $y = x$.	1 Mark
Q9.	**A** $P(x) = (x - 1)(x + 4)Q(x) + 2x + 11$ $P(-4) = 0 + 2 \times -4 + 11$ $P(-4) = 3$	1 Mark
Q10.	**C**  2 points of intersection, so 2 solution2 for $x^{\frac{1}{2}} = \lvert x - 1\rvert - 1$	1 Mark

Section 2		
Q11ai	$f(x) = 2\sin^{-1}\left(\frac{x}{3}\right)$ $f(3) = 2\sin^{-1}\left(\frac{3}{3}\right)$ $f(3) = 2\sin^{-1} 1$ $f(3) = \pi$	1 Mark Correct solution
Q11aii		2 Marks Correct solution 1 Mark Correct graph with most features shown
Q11aiii	Domain: $-3 \le x \le 3$ $[-3, 3]$ Range: $-\pi \le x \le \pi$ $[-\pi, \pi]$	2 Marks Correct solution 1 Mark Correct domain or range in interval notation
Q11b	$\frac{3}{2x+5} \le 1 \quad x \ne -\frac{5}{2}$ $3(2x+5) \le (2x+5)^2$ $3(2x+5) - (2x+5)^2 \le 0$ $(2x+5)\big(3-(2x+5)\big) \le 0$ $(2x+5)(-2x-2) \le 0$ $(2x+5)(x+1) \ge 0$ $x < -\frac{5}{2}, x \ge -1$	3 Marks Correct solution 2 Marks Correctly identifies $-\frac{5}{2}$ and -1 as important 1 Mark Correctly multiply by square of denominator
Q11ci	$P(x) = 4x^3 - 13x^2 + 4x + 12$ $P(2) = 4 \times 2^3 - 13 \times 2^2 + 4 \times 2 + 12$ $P(2) = 0$ $P'(x) = 12x^2 - 26x + 4$ $P'(2) = 12 \times 2^2 - 26 \times 2 + 4$ $P'(2) = 0$	2 Marks Correct solution 1 Mark Correct $P'(x)$ and $P'(2)$ or $P(2)$
Q11cii	$P(x) = 4x^3 - 13x^2 + 4x + 12$ $P(x) = (x-2)^2(ax+b)$ Compare coefficient of x^3, $a = 4$ Compare constant term, $b = 3$ $P(x) = (x-2)^2(4x+3)$	1 Mark Correct solution
Q11d	$x = 2t - 5$ $t = \frac{x+5}{2}$ $y = 12t^2 + 1$ $y = 12\left(\frac{x+5}{2}\right)^2 + 1$ $y = 3x^2 + 30x + 75 + 1$ $y = 3x^2 + 30x + 76$	1 Mark Correct solution

Q11e	$RTP: \frac{\cos 2x - \cos x}{\sin 2x + \sin x} = \cot x - \operatorname{cosec} x$ $LHS = \frac{\cos 2x - \cos x}{\sin 2x + \sin x}$ $LHS = \frac{2\cos^2 x - 1 - \cos x}{2\sin x \cos x + \sin x}$ $LHS = \frac{(2\cos x + 1)(\cos x - 1)}{\sin x\,(2\cos x + 1)}$ $LHS = \frac{\cos x - 1}{\sin x}$ $LHS = \frac{\cos x}{\sin x} - \frac{1}{\sin x}$ $LHS = \cot x - \operatorname{cosec} x$ $LHS = RHS$	3 Marks Correct solution 2 Marks Makes significant progress 1 Mark Correct expansion of $\cos 2x$ or $\sin 2x$
Q12ai	${}^{15}C_{10} = 3003$	1 Mark Correct solution
Q12aii	${}^{7}C_6 \times {}^{8}C_4 = 490$	1 Mark Correct solution
Q12aiii	At least 6 cats → 6 cats or 7 cats 6 cats → ${}^{7}C_6 \times {}^{8}C_4 = 490$ 7 cats → ${}^{7}C_7 \times {}^{8}C_3 = 56$ Probability of at least 6 cats: $\frac{490+56}{3003} = \frac{2}{11}$	2 Marks Correct solution 1 Mark Correct combination of at least 6 cats
Q12bi	____, ____, ____, ____ Last digit can only be 3 or 5 if the number is odd. $2 \times 4 \times 3 \times 2 = 48$	1 Mark Correct solution
Q12bii	Digits may be repeated, the total number of arrangments is 5^4 Last digit can only be 3 or 5 if the number is odd is 2×5^3 Probability of an odd number is: $\frac{2 \times 5^3}{5^4} = \frac{2}{5}$	2 Marks Correct solution 1 Mark Finds total no. or odd
Q12ci	$3! \times 4! = 144$	1 Mark Correct solution
Q12cii	2 people sit apart = Total – 2 people sit together $7! - 2! \times 6! = 3600$	2 Marks Correct solution 1 Mark Finds no. of ways two people sit together
Q12d	$\left(4x^3 + \frac{3}{x}\right)^8$ General term: ${}^{8}C_k (4x^3)^{8-k} (3x^{-1})^k$ $= {}^{8}C_k 4^{8-k} \times x^{24-3k} \times 3^k \times x^{-k}$ $= {}^{8}C_k 4^{8-k} \times 3^k \times x^{24-4k}$ For constant term, $24 - 4k = 0$ $k = 6$ Constant term is ${}^{8}C_6 \times 4^2 \times 3^6 = 326592$	2 Marks Correct solution 1 Mark Finds the simplified general term

Q12e	$\alpha = \sin^{-1}\left(\frac{3}{5}\right) \rightarrow \sin\alpha = \frac{3}{5}, \cos\alpha = \frac{4}{5}$ $\beta = \sin^{-1}\left(\frac{4}{7}\right) \rightarrow \sin\beta = \frac{4}{7}, \cos\beta = \frac{\sqrt{33}}{7}$  $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$ $\sin(\alpha-\beta) = \frac{3}{5}\times\frac{\sqrt{33}}{7} - \frac{4}{5}\times\frac{4}{7}$ $\sin(\alpha-\beta) = \frac{3\sqrt{33}-16}{35}$	3 Marks Correct solution 2 Marks Makes significant progress 1 Mark Correct expansion of $\sin(\alpha-\beta)$ and obtains the exact value of $\cos\alpha$ and $\cos\beta$
Q13ai	$y = \lvert f(x)\rvert$ 	1 Mark Correct solution
Q13aii	$y = \frac{1}{f(x)}$ 	2 Marks Correct solution 1 Mark Correct graph with most key features shown

Q13bi	$N = 1500 + Ae^{-kt}$ $N - 1500 = Ae^{-kt}$ $\frac{dN}{dt} = -kAe^{-kt}$ $\frac{dN}{dt} = -k(N-1500)$	1 Mark Correct solution
Q13bii	When $t = 0, N = 3500$ $3500 = 1500 + Ae^{0}$ $A = 2000$ When $t = 2, N = 2800$ $2800 = 1500 + 2000e^{-2k}$ $\frac{1300}{2000} = e^{-2k}$ $\ln\left(\frac{13}{20}\right) = -2k$ $k = \frac{\ln\left(\frac{13}{20}\right)}{-2}$	2 Marks Correct solution 1 Mark Finds correct value of A
Q13biii	$N = 2100, t = ?$ $2100 = 1500 + 2000e^{-kt}$ $2100 = 1500 + 2000e^{-kt}$ $\frac{600}{2000} = e^{-kt}$ $\ln\left(\frac{3}{10}\right) = \frac{\ln\left(\frac{13}{20}\right)}{2}t$ $t = \frac{\ln\left(\frac{3}{10}\right)}{\frac{\ln\left(\frac{13}{20}\right)}{2}}$ $t = 5.589\ldots$ $t = 5.6$ years	1 Mark Correct solution
Q13c	$P(x) = x^3 - 6x^2 + ax - 4$ Let the roots be $\alpha, \beta, \alpha\beta$ Sum of roots one at a time: $\alpha + \beta + \alpha\beta = 6$ (1) Sum of roots two at a time: $\alpha\beta + \alpha^2\beta + \alpha\beta^2 = a$ (2) Product of roots: $\alpha^2\beta^2 = 4$ (3) From (3) $\alpha^2\beta^2 = 4$ $\alpha\beta = \pm 2$ $\alpha\beta = 2$ (since all roots are positive) Substitute into (1) $\alpha + \beta + 2 = 6$ $\alpha + \beta = 4$ Substitute into (2) $\alpha\beta + \alpha^2\beta + \alpha\beta^2 = a$ $\alpha\beta(1 + \alpha + \beta) = a$ $2(1+4) = a$ $a = 10$	3 Marks Correct solution 2 Marks Progresses to find $\alpha + \beta = 4$ 1 Mark Establishes all three sums and product of roots

Q13di	$h = kr$ $V = \pi r^2 h$ $V = \pi r^2 \times kr$ $V = \pi k r^3$ $\frac{dV}{dr} = 3\pi k r^2$ $\frac{dr}{dV} = \frac{1}{3\pi k r^2}, \quad \frac{dV}{dt} = 0.25\ cm^3 s^{-1}, \quad r = 5\ cm$ $\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt}$ $\frac{dr}{dt} = \frac{1}{3\pi k \times 5^2} \times 0.25$ $\frac{dr}{dt} = \frac{1}{300\pi k}$	2 Marks Correct solution 1 Mark Finds $\frac{dV}{dr}$
Q13dii	$A = 2\pi r h + 2\pi r^2$ $A = 2\pi k r^2 + 2\pi r^2$ $\frac{dA}{dr} = 4\pi k r + 4\pi r$ $\frac{dr}{dA} = \frac{1}{4\pi k r + 4\pi r}, \quad \frac{dA}{dt} = 0.15\ cm^2 s^{-1}, \quad r = 5\ cm$ $\frac{dr}{dt} = \frac{dr}{dA} \times \frac{dA}{dt}$ $\frac{dr}{dt} = \frac{1}{4\pi k \times 5 + 4\pi \times 5} \times 0.15$ $\frac{dr}{dt} = \frac{1}{20\pi(k+1)} \times 0.15$ $\frac{dr}{dt} = \frac{3}{400\pi(k+1)}$ $\frac{3}{400\pi(k+1)} = \frac{1}{300\pi k}$ $9k = 4(k+1)$ $5k = 4$ $k = \frac{4}{5}$	3 Marks Correct solution 2 Marks Finds $\frac{dr}{dt}$ using $\frac{dr}{dA}$ 1 Mark Finds $\frac{dA}{dr}$