NAME: ______________________

Blacktown Boys' High School

2022 Year 11

Yearly Examination

Mathematics Extension 1

General Instructions

- Reading time – 10 minutes.
- Working time – 90 minutes.
- Write using blue or black pen
- Calculators approved by NESA may be used
- A reference sheet is provided for this paper
- In Questions in Section II, show all relevant mathematical reasoning and/or calculations
- All diagrams are not drawn to scale

Total marks: 55

Section I – 10 marks (pages 3-7)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II- 45 marks (pages 8-11)

- Attempt Questions 11 – 13
- Allow about 1 hour and 15 minutes for this section

Assessor: Villanueva

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Section I

10 marks
Attempt Questions 1–10
Allow about 10 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1 $^{n}P_{r}$ is equal to:

A. $\dfrac{^{n}C_{r}}{(n-r)!}$

B. $^{n}C_{r}\,(n-r)!$

C. $^{n}C_{r}\,r!$

D. $\dfrac{^{n}C_{r}}{r!}$

2 Which of the following gives the correct expression for $3\sin\theta - 2\cos\theta$ if $t = \tan\dfrac{\theta}{2}$?

A. $\dfrac{6t-2-2t^2}{1+t^2}$

B. $\dfrac{6t-2+2t^2}{1+t^2}$

C. $\dfrac{6t-2-2t^2}{1-t^2}$

D. $\dfrac{6t+2-2t^2}{1+t^2}$

3 Which expression is equivalent to $\sin 5x + \sin 3x$?

A. $2\sin 4x\cos x$

B. $2\sin 4x\sin x$

C. $2\cos 4x\sin x$

D. $2\cos 4x\cos x$

4 Seven pairs of different coloured socks are placed in a drawer. Individual socks are taken out one at a time. What is the least number of individual socks that must be taken out of the drawer to ensure at least one pair of matching colours is formed?

A. 14

B. 8

C. 7

D. 1

5 In how many ways can 7 boys be arranged in a line if Peter, Jeffery and Kevin have to stand together at the beginning of the line?

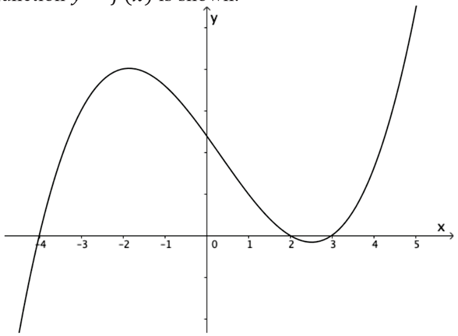
A. 4!

B. 7!

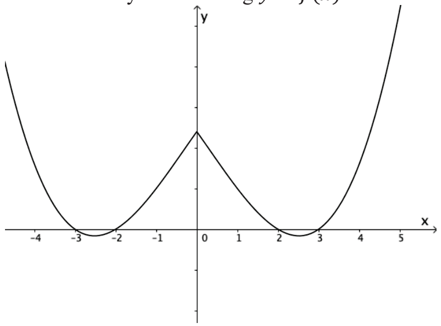
C. $4! \times 3!$

D. $7! \times 3!$

6 The graph of a function $y = f(x)$ is shown.



A second function is obtained by transforming $y = f(x)$ as shown below.



Which equation best represents the second graph?

A. $y = \sqrt{f(x)}$

B. $y = |f(x)|$

C. $y = f(|x|)$

D. $y = \left(f(x)\right)^2$

7 What is the Cartesian equation of the parabola given that $x = 3t - 4$ and $y = 18t^2 - 2$?

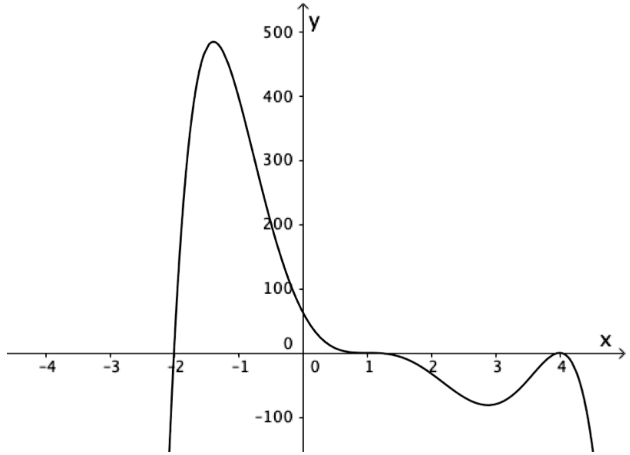
A. $y = 2x^2 + 16x + 30$

B. $y = 2x^2 + 30$

C. $y = x^2 + 8x + 15$

D. $y = 3x^2 + 24x + 46$

8 The diagram below shows a graph of $y = a(x + b)(x + c)^2(x + d)^3$. What are the possible values of a, b, c and d?



A. $a = 2, b = -2, c = 4, d = -1$

B. $a = 2, b = 2, c = -4, d = 1$

C. $a = -2, b = -2, c = 4, d = -1$

D. $a = -2, b = 2, c = -4, d = -1$

9 What is the expansion to $(3x-y)^4$?

A. $81x^4 - 27x^3y + 9x^2y^2 - 3xy^3 + y^4$

B. $81x^4 - 108x^3y + 54x^2y^2 - 12xy^3 + y^4$

C. $x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4$

D. $81x^4 + y^4$

10 The roots of the equation $2x^3 - 5x^2 + 8x + 2 = 0$ are α, β and γ. Find the value of $\alpha(\beta^2+\gamma^2) + \beta(\alpha^2+\gamma^2) + \gamma(\alpha^2+\beta^2)$.

A. -4

B. $\frac{13}{2}$

C. 7

D. 13

End of Section I

Section II

45 Marks
Attempts Questions 11-13
Answer each question on a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11-13, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

a) Consider the function $f(x) = 3\cos^{-1}\left(\frac{x}{2}\right)$

i) Evaluate $f(2)$. **1**

ii) Sketch the graph of $y = f(x)$, showing the intercepts and the coordinates of the end points. **2**

iii) State the domain and range of $y = f(x)$ using the interval notation. **2**

b) Solve $\frac{2}{x-5} \geq -5$. **3**

c) Consider the polynomial $P(x) = 2x^3 - 19x^2 + 60x - 63$.

i) Show that $P(x)$ has a double root at $x = 3$. **2**

ii) Hence, factorise $P(x)$ completely. **1**

d) A five-person team is to be chosen at random from eight women and nine men.

i) In how many ways can this team be chosen? **1**

ii) What is the probability that the team will consist of five men? **1**

e) Use binomial theorem to find an expression for the constant term in the expansion of $\left(3x^2 - \frac{1}{x}\right)^{15}$. **2**

Question 12 (15 marks) Use a SEPARATE writing booklet.

a) The letters of the word ‘DIFFERENTIATE’ are arranged in a straight line.

i) Find the number of unique arrangements if there are no restrictions. **1**

ii) Find the number of unique arrangements if the Ts occupy the end positions. **1**

iii) Find the probability that the letters E are together. **2**

b) A table at a wedding has 16 people sitting around it and the seating is arranged at random.

i) Find the number of ways of sitting around the table if there is no restriction. **1**

ii) Find the number of ways of sitting around the table if there are 8 couples and the members of each couple would like to sit together. **1**

iii) For a group dance, 7 people from the group of 16 are chosen and then arranged in a circle. Find the number of ways this can be done. **2**

c) Find the coefficient of x^8 in the expansion $(3 - 4x)(1 + x)^{10}$. **2**

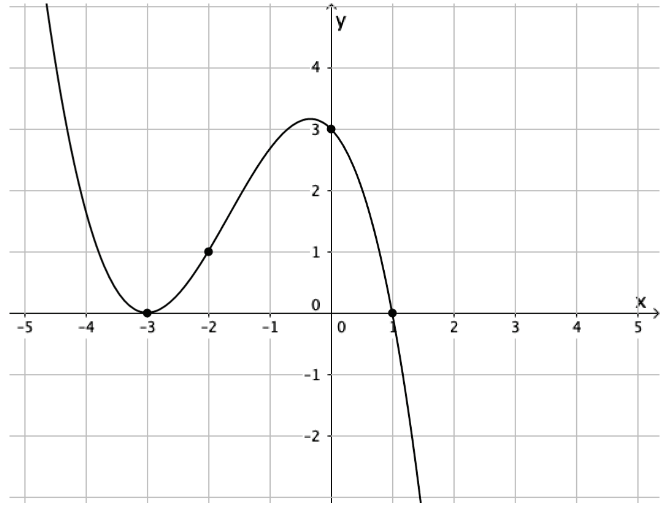
d) If $\alpha = \sin^{-1}\left(\frac{12}{13}\right)$ and $\beta = \sin^{-1}\left(\frac{2}{5}\right)$, find the exact value of $\cos(\alpha - \beta)$. **2**

e) i) Prove that $\dfrac{\sin 2a}{1 - \cos 2a} = \cot A$ **1**

ii) Hence, find the values of a and b if $\cot\dfrac{3\pi}{8} = a + \sqrt{b}$ for integers a and b. **2**

Question 13 (15 marks) Use a SEPARATE writing booklet.

a) The diagram shows the graph of the function $f(x)$. Sketch each of the following functions on a separate set of axes, showing all key features.



i) $y = |f(x)|$ **1**

ii) $y = \dfrac{1}{f(x)}$ **2**

iii) $y^2 = f(x)$ **2**

b) A sculptor is rolling modelling clay on a board. The clay is in the shape of a cylinder and has a constant volume $30\pi\ cm^3$. As he rolls it into a thinner cylinder, the radius decreases at a constant rate of $2\ cm$ per minute. Find the rate of increase of the height h of the cylinder with respect to time, at the moment when the radius is $5\ cm$. **3**

Question 13 continues on the next page

c) Water is being heated in a kettle. At time t seconds, the temperature of the water is T °C. The rate of increase of the temperature of the water at any time after the kettle is switched on is modelled by the equation $\frac{dT}{dt} = k(122 - T)$, where k is a positive constant. The temperature of the water is at 22°C when the kettle is switched on.

i) Show that $T = 122 - 100e^{-kt}$ is both a solution to the differential equation and satisfies the initial conditions. **2**

ii) When the temperature of the water reaches 100°C, the kettle switches off. If it takes 8 seconds for the temperature to increase 10°C once the kettle is switched on, find how long it takes for the kettle to switch off, to the nearest second. **2**

d) i) Show that $\frac{1+\tan x}{1-\tan x} - \frac{1-\tan x}{1+\tan x} = 2\tan 2x$ **2**

ii) Hence, find the exact value of $\frac{1+\tan\frac{7\pi}{12}}{1-\tan\frac{7\pi}{12}} - \frac{1-\tan\frac{7\pi}{12}}{1+\tan\frac{7\pi}{12}}$ **1**

End of Examination

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Student Name: ____________________

Multiple Choice Answer Sheet

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word 'correct' and drawing an arrow as follows.

correct

A ⊗ B ⊗ C ○ D ○

Start Here →

1.	A ○	B ○	C ○	D ○
2.	A ○	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ○
5.	A ○	B ○	C ○	D ○
6.	A ○	B ○	C ○	D ○
7.	A ○	B ○	C ○	D ○
8.	A ○	B ○	C ○	D ○
9.	A ○	B ○	C ○	D ○
10.	A ○	B ○	C ○	D ○

2022 Yr11 Maths Ext 1 Trial Exam

SECTION I

① ${}^nP_r$ is equal to:

a) $\dfrac{{}^nC_r}{(n-r)!}$

b) ${}^nC_r\,(n-r)!$

c) ${}^nC_r\, r!$ ✓

d) $\dfrac{{}^nC_r}{r!}$

${}^nP_r = \dfrac{n!}{(n-r)!}$

${}^nC_r = \dfrac{n!}{r!(n-r)!}$

② Which of the following gives the correct expression for $3\sin\theta - 2\cos\theta$ if $t = \tan\frac{\theta}{2}$?

a) $\dfrac{6t - 2 - 2t^2}{1+t^2}$

b) $\dfrac{6t - 2 + 2t^2}{1+t^2}$ ✓

c) $\dfrac{6t - 2 + 2t^2}{1-t^2}$

d) $\dfrac{6t + 2 - 2t^2}{1+t^2}$

$\sin\theta = \dfrac{2t}{1+t^2}$ $\cos\theta = \dfrac{1-t^2}{1+t^2}$

$= \dfrac{3(2t)}{1+t^2} - \dfrac{2(1-t^2)}{1+t^2}$

$= \dfrac{6t - 2 + 2t^2}{1+t^2}$

③ Which expression is equivalent to $\sin 5x + \sin 3x$?

a) $2\sin 4x \cos x$ ✓

b) $2\sin 4x \sin x$

c) $2\cos 4x \sin x$

d) $2\cos 4x \cos x$

$2\sin A \cos B = \sin(A+B) + \sin(A-B)$

$2\sin 4x \cos x = \sin(4x+x) + \sin(4x-x)$

$= \sin 5x + \sin 3x$

④ Seven pairs of different coloured socks are placed in a drawer. Individual socks are taken out one at a time. What is the least number of individual socks that must be taken out to ensure a pair of matching colours is found?

a) 14

b) 8 ✓

c) 7

d) 1

⑤ In how many ways can 7 boys be arranged in a line if Peter, Jeffery and Kevin have to stand together at the beginning of the line?

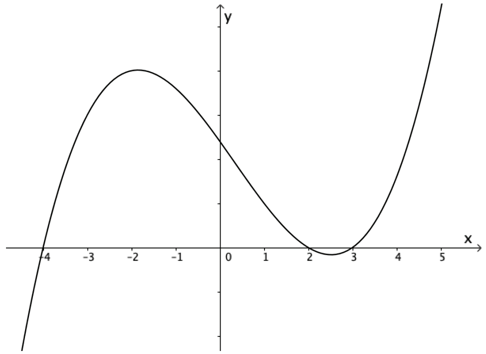
a) $4!$

b) $7!$

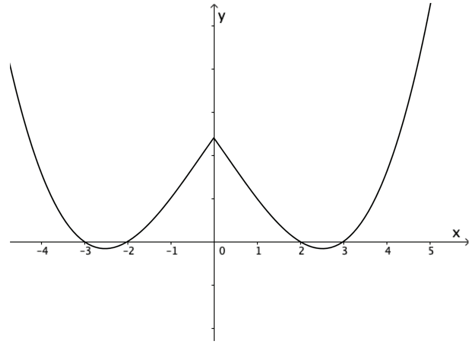
c) $4! \times 3!$ ✓

d) $7! \times 3!$

⑥ The graph of a function $y = f(x)$ is shown.



A second graph is obtained from the function $y = f(x)$



Which equation best represents the second graph?

a) $y = \sqrt{f(x)}$

b) $y = |f(x)|$

c) $y = f(|x|)$ ✓

d) $y = (f(x))^2$

⑦ What is the Cartesian equation of the parabola given that

$x = 3t - 4$ and $y = 18t^2 - 2$?

a) $y = 2x^2 + 16x + 30$ ✓

b) $y = 2x^2 + 30$

c) $y = x^2 + 8x + 15$

d) $y = 3x^2 + 24x + 46$

$x = 3t - 4$

$t = \dfrac{x+4}{3}$

$y = 18\left(\dfrac{x+4}{3}\right)^2 - 2$

$= 18\left(\dfrac{x^2+8x+16}{9}\right) - 2$

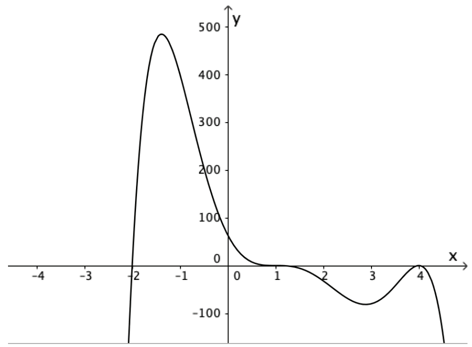
$= 2(x^2+8x+16) - 2$

$= 2x^2 + 16x + 32 - 2$

$= 2x^2 + 16x + 30$

⑧ The diagram below shows a graph of $y = a(x+b)(x+c)^2(x+d)^3$.

What are the possible values of a, b, c and d?



a) $a = 2$, $b = -2$, $c = 4$, $d = -1$

b) $a = 2$, $b = 2$, $c = -4$, $d = 1$

c) $a = -2$, $b = -2$, $c = 4$, $d = -1$

d) $a = -2$, $b = 2$, $c = -4$, $d = -1$ ✓

$-2(x+2)(x-4)^2(x-1)^3$

(9) what is the expansion to $(3x-y)^4$?

a) $81x^4 - 27x^3y + 9x^2y^2 - 3xy^3 + y^4$

b) $81x^4 - 108x^3y + 54x^2y^2 - 12xy^3 + y^4$ ✓

c) $x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4$

d) $81x^4 + y^4$

$$(3x-y)^4 = (3x)^4 + {}^4C_1(3x)^3(-y) + {}^4C_2(3x)^2(-y)^2 + {}^4C_3(3x)(-y)^3 + (-y)^4$$

$$= 81x^4 - 4\cdot27x^3y + 6\cdot9x^2y^2 - 4\cdot3xy^3 + y^4$$

$$= 81x^4 - 108x^3y + 54x^2y^2 - 12xy^3 + y^4$$

(10) The roots of the equation $2x^3 - 5x^2 + 8x + 2 = 0$ are α, β and γ

Find the value of $\alpha(\beta^2+\gamma^2) + \beta(\alpha^2+\gamma^2) + \gamma(\alpha^2+\beta^2)$.

a) -4

b) $\frac{13}{2}$

c) 7

d) 13 ✓

$\alpha\beta\gamma = -d/a$

$= -1$

$\alpha+\beta+\gamma = -b/a$

$= 5/2$

$\alpha\beta + \alpha\gamma + \beta\gamma = c/a$

$= 4$

$$= \alpha\beta^2 + \alpha\beta^3 + \beta\alpha^2 + \beta\gamma^2 + \gamma\alpha^2 + \gamma\beta^2$$

$$= \alpha\beta^2 + \beta\alpha^2 + \alpha\gamma^2 + \gamma\alpha^2 + \beta\gamma^2 + \gamma\beta^2$$

$$= \alpha\beta(\beta+\alpha) + \alpha\gamma(\gamma+\alpha) + \beta\gamma(\gamma+\beta)$$

$$= \alpha\beta(\beta+\alpha+\gamma) - \alpha\beta\gamma + \alpha\gamma(\gamma+\alpha+\beta) - \alpha\beta\gamma + \beta\gamma(\gamma+\beta+\alpha) - \alpha\beta\gamma$$

$$= (\alpha\beta+\alpha\gamma+\beta\gamma)(\alpha+\beta+\gamma) - 3(\alpha\beta\gamma)$$

$$= 4(5/2) - 3(-1)$$

$$= 13$$

SECTION II

(11) a) Consider the function $f(x) = 3\cos^{-1}\left(\frac{x}{2}\right)$

i) Evaluate $f(2)$ (1 mark)

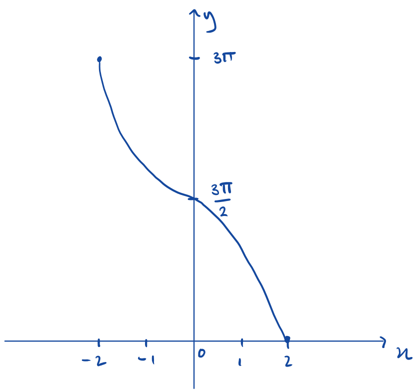
ii) Sketch the graph of $y = f(x)$, showing the intercept and the coordinates of the end points. (2 marks)

iii) State the domain and range of $y = f(x)$ using the interval notation. (2 marks)

i) $f(2) = 3\cos^{-1}\left(\frac{2}{2}\right)$

$= 0$

ii)



iii) Domain: $[-2, 2]$

Range: $[0, 3\pi]$

b) Solve : $\dfrac{2}{x-5} \geq -5$ (3 marks)

$x \neq 5$ as denominator cannot be zero.

$2(x-5) \geq -5(x-5)^2$

$2(x-5) + 5(x-5)^2 \geq 0$

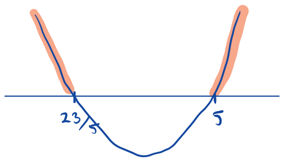
$(x-5)(2+5(x-5)) \geq 0$

$(x-5)(2+5x-25) \geq 0$

$(x-5)(5x-23) \geq 0$

$5 \quad\quad {}^{23}/_{5}$

$x \leq {}^{23}/_{5}$, $x > 5$



c) Consider the polynomial $P(x) = 2x^3 - 19x^2 + 60x - 63$.

i) Show that $P(x)$ has a double root at $x = 3$. (2 marks)

ii) Hence, factorise $P(x)$ completely. (1 mark)

i) $P(3) = 2(3)^3 - 19(3)^2 + 60(3) - 63$

$= 0$

$P'(x) = 6x^2 - 38x + 60$

$P'(3) = 6(3)^2 - 38(3) + 60$

$= 0$

ii) $P(x) = 2x^3 - 19x^2 + 60x - 63$

$P(x) = (x-3)^2(ax+b)$

Compare coefficients of x^3, $a = 2$

Compare constant term, $b = -7$

$P(x) = (x-3)^2(2x-7)$

d) A five-person team is to be chosen at random from eight women and nine men.

i) In how many ways can this team be chosen? (1 mark)

ii) What is the probability that the team will consist of five men? (1 mark)

i) ${}^{17}C_5 = 6188$

ii) $\dfrac{{}^{9}C_5}{6188} = \dfrac{126}{6188}$

$= \dfrac{9}{442}$

e) Use the binomial theorem to find an expression for the constant term in the expansion of $\left(3x^2 - \frac{1}{x}\right)^{15}$. (2 marks)

General term: ${}^{15}C_k \,(3x^2)^{15-k} \left(-\frac{1}{x}\right)^k$

${}^{15}C_k \; 3^{15-k} \; x^{30-2k} \; (-1)^k \; x^{-k}$

${}^{15}C_k \; 3^{15-k} \; x^{30-3k} \; (-1)^k$

Constant term occurs when $30 - 3k = 0$

$30 = 3k$

$k = 10$

$\therefore$ Constant term $= {}^{15}C_{10} \times 3^{15-10} \times (-1)^{10}$

$= 3003 \times 3^5 \times (-1)^{10}$

$= 729729$

(12) a) The letters of the word 'DIFFERENTIATE' are arranged in a straight line.

i) Find the number of unique arrangements if there are no restrictions. (1)

ii) Find the number of unique arrangements where the T's occupy the end positions. (1)

iii) Find the probability that the letters E are together. (2)

i) $\dfrac{13!}{2! \times 2! \times 3! \times 2!}$ I - 2, F - 2, E - 3, T - 2

$= 129729600$

ii) $T \underbrace{\_\,\_\,\_\,\_\,\_\,\_\,\_\,\_\,\_\,\_\,\_}_{\frac{11!}{2! \times 2! \times 3!}} T = 1663200$

iii) $\underbrace{\_\,\_\,\_}_{E's}\,\_\,\_\,\_\,\_\,\_\,\_\,\_\,\_\,\_\,\_$

arrangements with E's together $= \dfrac{11!}{2! \times 2! \times 2!}$

$= 4989600$

Probability $= \dfrac{4989600}{129729600}$

$= 1/26$

b) a table at a wedding has 16 people sitting around it and the seating is arranged at random.

i) Find the number of ways of sitting around the table if there is no restriction. (1)

ii) Find the number of ways of sitting around the table if there are 8 couples and the members of each couple would like to sit together. (1)

iii) For a group dance, 7 people from the group of 16 are chosen and then arranged in a circle. Find the number of ways this can be done. (2)

i) $15!$

i) $7! \times 2!^8 = 1290240$

iii) ${}^{16}C_7 \times 6! = 8236800$

c) Find the coefficient of x^8 in the expansion $(3-4x)(1+x)^{10}$. (2 marks)

General term for $(1+x)^{10} = {}^{10}C_k \, 1^{10-k} \, x^k$

$= {}^{10}C_k \, x^k$

$3 \times {}^{10}C_k \, x^k \rightarrow k = 8$

$3 \times {}^{10}C_8 \, x^8$

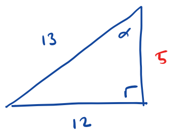
$-4x \times {}^{10}C_k \, x^k \rightarrow 1 + k = 8$

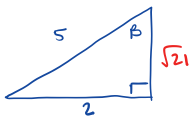
$k = 7$

$-4 \times {}^{10}C_7 \, x^8$

$\therefore$ coefficient is $(3 \times {}^{10}C_8) + (-4 \times {}^{10}C_7) = -345$

d) If $\alpha = \sin^{-1}\left(\frac{12}{13}\right)$ and $\beta = \sin^{-1}\left(\frac{2}{5}\right)$, find the exact value of $\cos(\alpha - \beta)$ (2 marks)





$\cos\alpha = \frac{5}{13}$ $\cos\beta = \frac{\sqrt{21}}{5}$

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$= \frac{5}{13} \times \frac{\sqrt{21}}{5} + \frac{12}{13} \times \frac{2}{5}$$

$$= \frac{\sqrt{21}}{13} + \frac{24}{65}$$

$$= \frac{5\sqrt{21} + 24}{65}$$

e) i) Prove that $\frac{\sin 2A}{1 - \cos 2A} = \cot A$ (1 mark)

ii) Hence find the values of a and b if $\cot\frac{3\pi}{8} = a + \sqrt{b}$ for integers a and b. (2 marks)

i) $$\text{LHS} = \frac{2\sin A\cos A}{1 - (1 - 2\sin^2 A)}$$

$$= \frac{2\sin A\cos A}{2\sin^2 A}$$

$$= \frac{\cos A}{\sin A}$$

$$= \cot A$$

ii) $$\cot\frac{3\pi}{8} = \frac{\sin\frac{3\pi}{4}}{1 - \cos\frac{3\pi}{4}}$$

$$= \frac{1/\sqrt{2}}{1 + 1/\sqrt{2}}$$

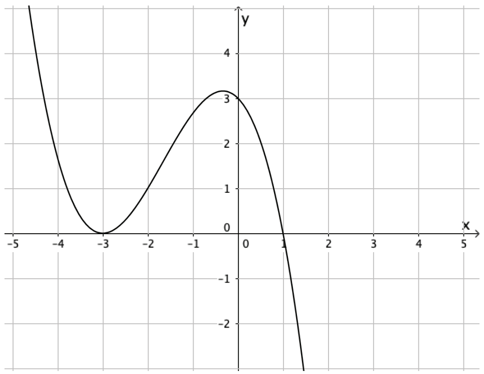
$$= \frac{1}{\sqrt{2} + 1}$$

$$= \sqrt{2} - 1$$

$\therefore$ $a = -1$ and $b = 2$

13 a) The diagram shows the graph of the function $f(x)$.

Sketch each of the following functions on separate set of axes, showing all key features.

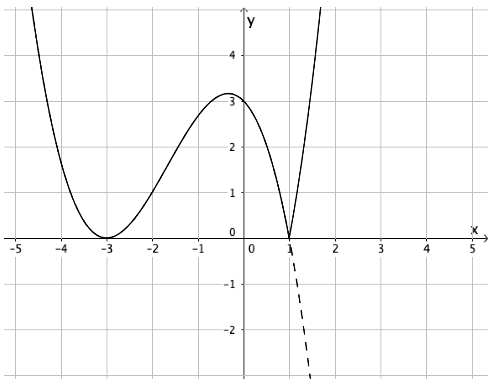


i) $y = |f(x)|$ (1 mark)

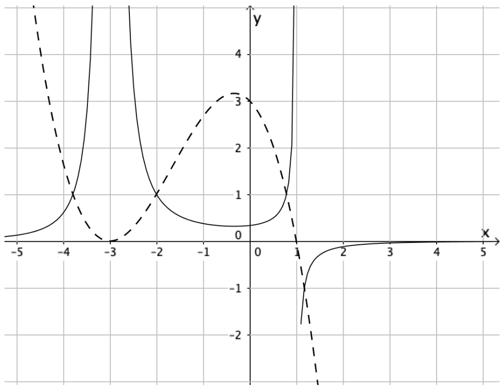
ii) $y = \frac{1}{f(x)}$ (2 marks)

iii) $y^2 = f(x)$ (2 marks)

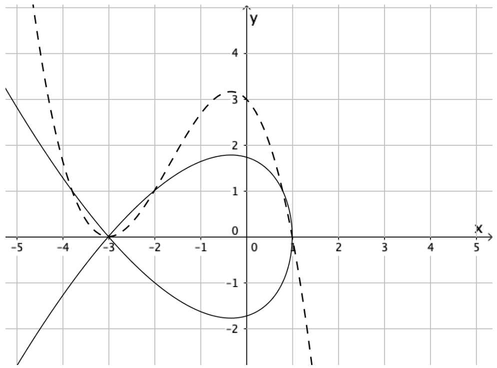
i)



ii)



iii)



b) a sculptor is rolling modelling clay on a board. The clay is in the shape of a cylinder and has a constant volume 30π cm^3. As he rolls it into a thinner cylinder, the radius decreases at a constant rate of 2cm per minute. Find the rate of increase of the height h of the cylinder with respect to time, at the moment when the radius is 5 cm. (3 marks)

$V = \pi r^2 h$ $\qquad \frac{dr}{dt} = -2$ cm / min

$30\pi = \pi r^2 h$

$h = \frac{30}{r^2}$

$h = 30r^{-2}$

$\frac{dh}{dr} = -60r^{-3}$

$\frac{dh}{dt} = \frac{dh}{dr} \times \frac{dr}{dt}$

$= -60r^{-3} \times -2$

$= 120/r^3$

when $r = 5$ cm $\qquad dh/dt = 120/5^3$

$= 0.96$ cm / min

c) Water is being heated in a kettle. At time t seconds, the temperature of the water is T°C.

The rate of increase of the temperature of the water at any time after the kettle is switched on, is modelled by the equation $\frac{dT}{dt} = k(122 - T)$, where k is a positive constant.

The temperature of the water is at 22°C when the kettle is switched on.

i) Show that $T = 122 - 100e^{-kt}$ is both a solution to the differential equation and satisfies the initial conditions. (2)

ii) When the temperature of the water reaches 100°C, the kettle switches off. If it takes 8 seconds for the temperature to increase 10°C once the kettle is switched on, find how long it takes for the kettle to switch off, to the nearest second. (2 marks)

i) $T = 122 - 100e^{-kt}$ $\qquad \longrightarrow \qquad 100e^{-kt} = 122 - T$

$\frac{dT}{dt} = (-k)(-100)e^{-kt}$

$= k(100e^{-kt})$

$= k(122 - T)$

when $t = 0$, $T = 122 - 100e^0$

$= 122 - 100$

$= 22$

ii) when $t = 8$, $T = 32$

$32 = 122 - 100e^{-8k}$

$90 = 100e^{-8k}$

$\frac{9}{10} = e^{-8k}$

$\ln\left(\frac{9}{10}\right) = -8k$

$k = -\frac{1}{8}\ln\left(\frac{9}{10}\right)$

$= 0.013170\ldots$

when $T = 100$, $t = ?$

$100 = 122 - 100e^{-kt}$

$-22 = -100e^{-kt}$

$\frac{11}{50} = e^{-kt}$

$\ln\left(\frac{11}{50}\right) = -kt$

$t = -\frac{\ln(11/50)}{k}$

$t = 114.96737\ldots$

$\therefore$ The kettle switches off after 115 seconds.

d) i) Show that $\frac{1+\tan x}{1-\tan x} - \frac{1-\tan x}{1+\tan x} = 2\tan 2x$ (2 marks)

ii) Hence, find the exact value of $\frac{1+\tan\frac{7\pi}{12}}{1-\tan\frac{7\pi}{12}} - \frac{1-\tan\frac{7\pi}{12}}{1+\tan\frac{7\pi}{12}}$. (1 mark)

i) $\text{LHS} = \frac{(1+\tan x)^2 - (1-\tan x)^2}{(1-\tan x)(1+\tan x)}$

$= \frac{1 + 2\tan x + \tan^2 x - (1 - 2\tan x + \tan^2 x)}{1 - \tan^2 x}$

$= \frac{4\tan x}{1-\tan^2 x}$

$= 2 \times \frac{2\tan x}{1-\tan^2 x}$

$= 2 \times \tan 2x$

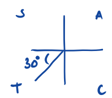
$= \text{RHS}$

ii) $x = \frac{7\pi}{12} = 105°$

$= 2\tan 2x$

$= 2\tan 2(105°)$

$= 2\tan 210°$



$= 2 \times \frac{1}{\sqrt{3}}$

$= \frac{2}{\sqrt{3}}$