

NAME: _____



Blacktown Boys' High School

2024 Year 11

Yearly Examination

Mathematics Advanced

**General
Instructions**

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided for this paper
- All diagrams are not drawn to scale
- In Questions in Section II, show all relevant mathematical reasoning and/or calculations

**Total marks:
70**

Section I – 10 marks (page 3-5)

- Attempt Questions 1 - 10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 8-20)

- Attempt Questions 11 - 26
- Allow about 1 hour 45 minutes for this section

Assessor: P Govender

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Section I

10 marks

Attempt Questions 1–10

Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1–10.

- 1 Which of the following lines is perpendicular to $3x - 4y + 8 = 0$ and has the same y -intercept?

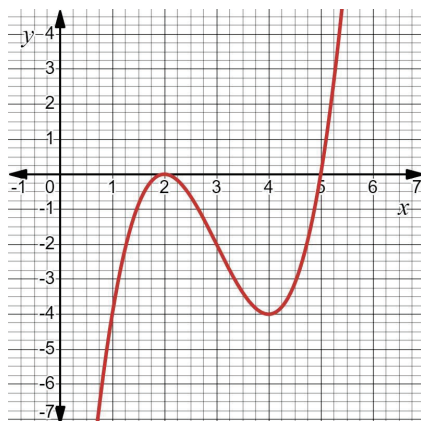
A. $y = -\frac{4}{3}x - 2$

B. $y = \frac{4}{3}x - 2$

C. $y = -\frac{4}{3}x + 2$

D. $y = \frac{4}{3}x + 2$

- 2 The graph of $y=f(x)$ is shown.



Which of the following could be the equation of this graph?

A. $y = (x + 2)^2(x + 5)$

B. $y = (x + 5)^2(x + 2)$

C. $y = (x - 2)^2(x - 5)$

D. $y = (x - 5)^2(x - 2)$

- 3 What is the solution to the equation $\log_5(3x - 5) = 2$?

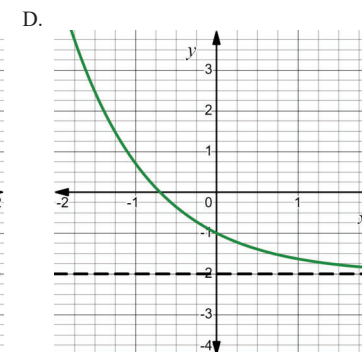
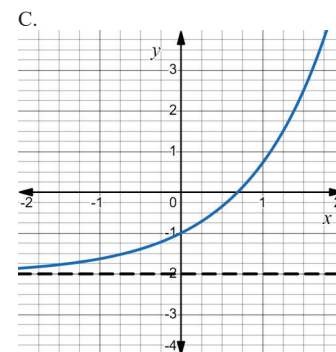
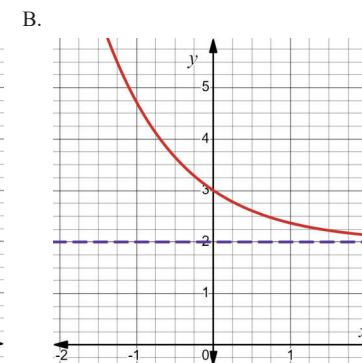
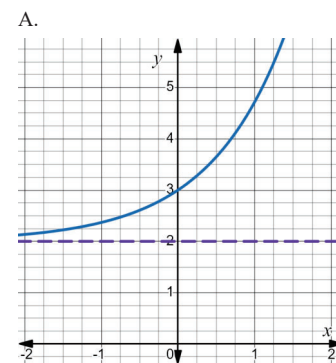
A. $x = \frac{37}{3}$

B. $x = 10$

C. $x = 25$

D. $x = \frac{20}{3}$

- 4 Which of the following is the graph of $y = e^x + 2$?



5 Which of the following is equivalent to $\cos^2 x + \sin^2 x + \cot^2 x$?

- A. $\sec^2 x$
- B. $\operatorname{cosec}^2 x$
- C. 1
- D. $\tan^2 x$

6 What is the domain of $f(x) = \frac{1}{\sqrt{1-2x}}$?

- A. $[0.5, \infty)$
- B. $(0.5, \infty)$
- C. $(-\infty, 0.5]$
- D. $(-\infty, 0.5)$

7 Which of the following is a probability distribution?

A.

x	0	1	2	3	4
$P(X=x)$	0.55	0.20	0.15	0.05	0.01

B.

x	0	1	2	3	4
$P(X=x)$	0.45	0.45	0.45	0.45	0.45

C.

x	0	1	2	3	4
$P(X=x)$	0.50	0.25	0.10	0.05	0.10

D.

x	0	1	2	3	4
$P(X=x)$	0.60	-0.25	0.10	0.45	0.10

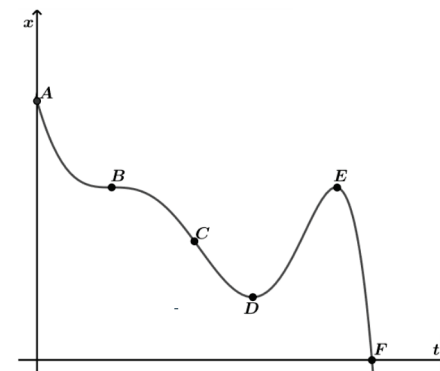
8 The vertical and horizontal asymptotes for the graph $y = \frac{1}{4x+2} + 3$ are:

- A. $x = 3, \quad y = -\frac{1}{2}$
- B. $x = -3, \quad y = \frac{1}{2}$
- C. $x = \frac{1}{2}, \quad y = -3$
- D. $x = -\frac{1}{2}, \quad y = 3$

9 The derivative of $y = (x+2)^5(x-2)^4$ is given by which of the following:

- A. $(x+2)^3(x-2)^4(9x-2)$
- B. $(x+2)^4(x-2)^3(9x-2)$
- C. $(x+2)^3(x-2)^4(9x+2)$
- D. $(x+2)^4(x-2)^3(9x+2)$

10 For the displacement graph below, at which of the points will the velocity be equal to zero?



- A. A and F only
- B. B, D and E only
- C. A, D, E and F only
- D. D and E only

End of Section I
Exam continues on next page

Mathematics Advanced

Section II Answer Booklet

60 marks
Attempt Questions 11 – 25
Allow about 1 hour 45 minutes for this section

- Instructions**
- Answer the Questions in the spaces provided. These spaces provide guidance for the expected length of response.
 - Your response should include relevant mathematical reasoning and/or calculations
 - Extra writing space is provided at the back of this booklet.
If you use this space, clearly indicate which question you are answering.

Question 11 (2 marks)

Solve $|7x - 2| = 19$. 2

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Question 12 (3 marks)

Solve the following for x .

(a) $\log_x 243 = 5$ 1

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(b) $3 \log_3(5) - 3 \log_3(2) + \log_3(x) = 2$ 2

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Question 13 (7 marks)

The probability distribution of a random variable X is shown in the table below.

x	1	2	3	4	5
$P(X = x)$	$3p$	0.35	0.20	0.25	p

- (a) Calculate the expected value $E(x)$.

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- (b) Show that the variance $Var(X) = 1.31$

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- (c) Find the standard deviation to 3 decimal places.

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- (d) Find $P(X \leq 4)$.

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- (e) Find $P(2 < X \leq 5 | X \leq 4)$

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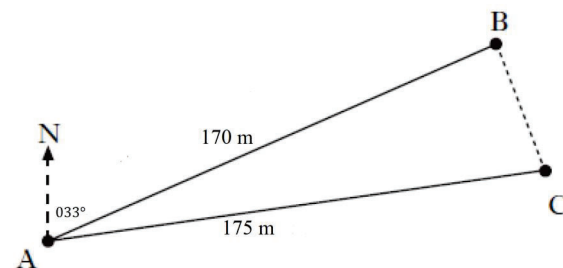
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Question 14 (4 marks)

Sarah is participating in a race, as shown in the diagram below. She needs to run 170 metres from A to B on a bearing of 033° . Instead, she ran 175 metres from A to C on a bearing of 053° .



- (a) What is the size of $\angle BAC$?

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- (b) Calculate the length of BC to two decimal places.

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- (c) What is the bearing from C to B ? Round the answer to the nearest degree.

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Question 15 (4 marks)

At the Year 8Y class at Blacktown Boys High School, 25 students play cricket and 17 play tennis. Altogether there are 30 students in the Year 8Y class and each student plays at least one sport.

- (a) Draw a Venn diagram to represent this scenario. 1

- (b) One student is chosen at random. Given that the student plays cricket, what is the probability that the student does NOT play tennis? 1

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- (c) Two different students are chosen at random one after the other. What is the probability that both students play both tennis and cricket? 1

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- (d) One student is chosen at random. What is the probability that this student does NOT play two sports? 1

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Question 16 (4 marks)

A car manufacturer produces 20 cars, 3 of which are defective. Two cars are drawn at random.

- (a) What is the probability that the first car is defective? 1

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- (b) What is the probability that both are defective? 1

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- (c) What is the probability that at least one is defective? 2

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Question 17 (2 marks)

Differentiate the following $y = \frac{1}{2}x^2 - e^{7-9x}$ 2

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Question 18 (2 marks)

Show $\frac{d}{dx} \left(\frac{x}{\sqrt{7x-9}} \right) = \frac{7x-18}{2(7x-9)\sqrt{7x-9}}$

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Question 19 (4 marks)

Solve the following equations for x .

(a) $2^{2x} - 5(2^{x+1}) + 16 = 0$

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(b) $e^{2\ln x} = x + 12$

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Question 20 (5 marks)

A car is worth \$12000 when new. After t years, the value of the car (in dollars) is given by the formula $V = Ae^{-0.25t}$.

- (a) Find the value of A . 1

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- (b) Find the value of the car after 7 years. **1**

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- (c) Find the rate in dollars per year at which the car's value is depreciating, when $t = 7$. 2

[illegible]

- (d) Find how long it will take for the value of the car to depreciate to \$1000. Round the answer to 2 decimal places.

Question 21 (3 marks)

Find the equation of the normal to the curve $y = \frac{x}{2}(3 - 4x)^4$ at the point where $x = 1$.

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Question 22 (5 marks)

Find the exact values of x for the given domain.

(a) $\sin x = \frac{-1}{\sqrt{2}}, \quad 0 \leq x \leq 2\pi$

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(b) $2 \cos^2 2x - \cos 2x - 1 = 0, \quad 0 \leq x \leq \pi$

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Question 23 (4 marks)

A bag contains 6 red, 9 green and 3 blue marbles. Two marbles are drawn in succession without replacement.

(a) Find the probability of drawing 2 green marbles.

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(b) Find the probability of drawing a red marble followed by a blue marble.

1

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(c) Find the probability of not drawing two same coloured marbles.

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Question 24 (3 marks)

Prove that $\tan \theta (1 + \tan \theta) + (1 + \cot \theta) = \frac{\tan \theta + 1}{\sin \theta \cos \theta}$

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Question 25 (5 marks)

A particle is moving in a straight line with displacement x metres from the origin O at time t , where $x = 4t^3 - 30t^2 + 72t$.

- (a) Find the initial velocity.

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- (b) Find when the particle is at rest.

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- (c) At what time is the acceleration zero? Find the velocity and position of the particle at this time.

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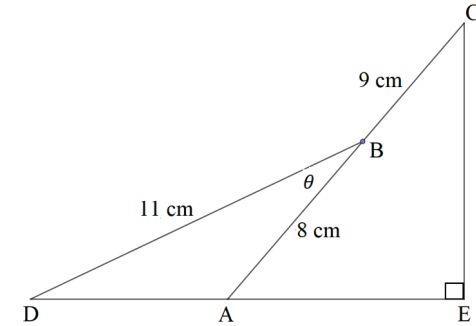
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Question 26 (3 marks)

In the diagram below, there are two triangles, $\triangle DAB$, and $\triangle AEC$. Given that $BC = 9\text{ cm}$, $AB = 8\text{ cm}$, $DB = 11\text{ cm}$, $\angle DBA = \theta$ and DAE is a straight line.



- (a) Use the cosine rule to find an expression for the length of DA in terms of θ .

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- (b) Find an expression for the length of CE in terms of θ .

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End of examination

If you use this space, clearly indicate which question you are answering.

Multiple Choice Answer Sheet

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9
A ☐ B ☒ C ☐ D ☐

A B C D

A  B  C  D 

-22-

NAME: SOLUTIONS



Blacktown Boys' High School

2024 Year 11

Yearly Examination

Mathematics Advanced

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**Total marks:
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Section II – 60 marks (pages 8-20)

- Attempt Questions 11 - 26
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- BLANK PAGE -

Assessor: P Govender

Section I

10 marks

Attempt Questions 1–10

Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1–10.

- 1 Which of the following lines is perpendicular to $3x - 4y + 8 = 0$ and has the same y-intercept?

A. $y = -\frac{4}{3}x - 2$

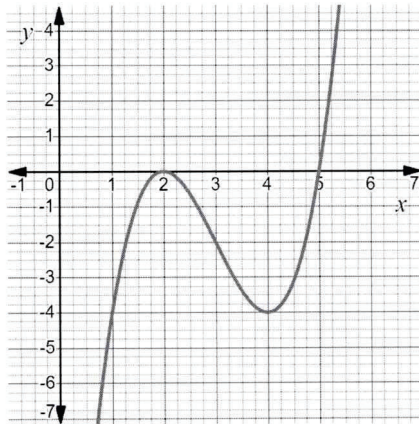
B. $y = \frac{4}{3}x - 2$

☒ C. $y = -\frac{4}{3}x + 2$

D. $y = \frac{4}{3}x + 2$

$3x - 4y + 8 = 0$
 $4y = 3x + 8$
 $y = \frac{3}{4}x + 2$
 $m_2 = -\frac{1}{m_1} = -\frac{4}{3}$
 $\therefore y = -\frac{4}{3}x + 2$

- 2 The graph of $y = f(x)$ is shown.



Which of the following could be the equation of this graph?

A. $y = (x + 2)^2(x + 5)$

B. $y = (x + 5)^2(x + 2)$

☒ C. $y = (x - 2)^2(x - 5)$

D. $y = (x - 5)^2(x - 2)$

- 3 What is the solution to the equation $\log_5(3x - 5) = 2$?

A. $x = \frac{37}{3}$

☒ B. $x = 10$

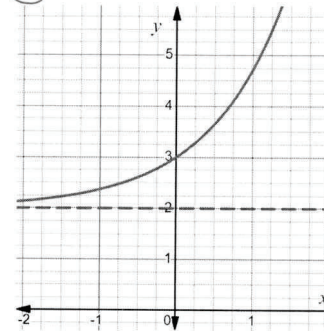
C. $x = 25$

D. $x = \frac{20}{3}$

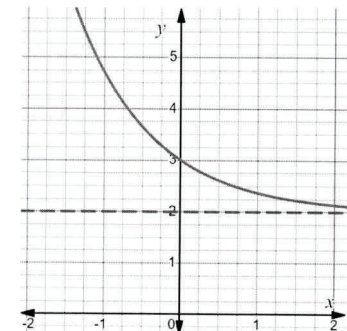
$\log_5(3x - 5) = 2$
 $3x - 5 = 5^2$
 $3x = 30$
 $x = 10$

- 4 Which of the following is the graph of $y = e^x + 2$?

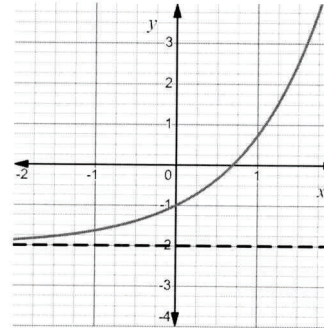
☒ A.



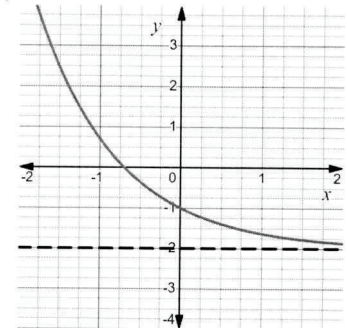
B.



C.



D.



5 Which of the following is equivalent to $\cos^2 x + \sin^2 x + \cot^2 x$?

A. $\sec^2 x$

B. $\operatorname{cosec}^2 x$

C. 1

D. $\tan^2 x$

$$\begin{aligned} \cos^2 x + \sin^2 x + \cot^2 x \\ = 1 + \cot^2 x \\ = \operatorname{cosec}^2 x \end{aligned}$$

6 What is the domain of $f(x) = \frac{1}{\sqrt{1-2x}}$?

A. $[0.5, \infty)$

B. $(0.5, \infty)$

C. $(-\infty, 0.5]$

D. $(-\infty, 0.5)$

$$\begin{aligned} 1-2x &> 0 \\ -2x &> -1 \\ x &< \frac{1}{2} \\ \therefore &(-\infty, \frac{1}{2}) \end{aligned}$$

7 Which of the following is a probability distribution?

A.

x	0	1	2	3	4
$P(X=x)$	0.55	0.20	0.15	0.05	0.01

B.

x	0	1	2	3	4
$P(X=x)$	0.45	0.45	0.45	0.45	0.45

C.

x	0	1	2	3	4
$P(X=x)$	0.50	0.25	0.10	0.05	0.10

D.

x	0	1	2	3	4
$P(X=x)$	0.60	-0.25	0.10	0.45	0.10

$$P(X=x) = 0.50 + 0.25 + 0.10 + 0.05 + 0.10 = 1$$

and $f(x)$ is a fraction and not negative.

8 The vertical and horizontal asymptotes for the graph $y = \frac{1}{4x+2} + 3$ are:

A. $x = 3, y = -\frac{1}{2}$

B. $x = -3, y = \frac{1}{2}$

C. $x = \frac{1}{2}, y = -3$

D. $x = -\frac{1}{2}, y = 3$

$$\begin{aligned} \text{Vertical asymptote: } x &= -\frac{1}{2} \\ \text{Horizontal asymptote: } y &= 3 \end{aligned}$$

9 The derivative of $y = (x+2)^5(x-2)^4$ is given by which of the following:

A. $(x+2)^3(x-2)^4(9x-2)$

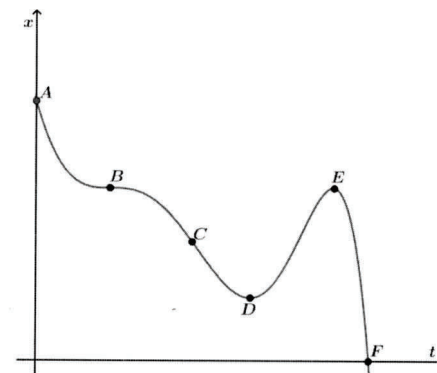
B. $(x+2)^4(x-2)^3(9x-2)$

C. $(x+2)^3(x-2)^4(9x+2)$

D. $(x+2)^4(x-2)^3(9x+2)$

$$\begin{aligned} y &= (x+2)^5(x-2)^4 \\ \frac{dy}{dx} &= 5(x+2)^4(x-2)^4 + 4(x-2)^3(x+2)^5 \\ &= (x+2)^4(x-2)^3[5(x-2) + 4(x+2)] \\ &= (x+2)^4(x-2)^3(9x-2) \end{aligned}$$

10 For the displacement graph below, at which of the points will the velocity be equal to zero?



A. A and F only

B. B, D and E only

C. A, D, E and F only

D. D and E only

End of Section I
Exam continues on next page

Mathematics Advanced

Section II Answer Booklet

60 marks

Attempt Questions 11 – 25

Allow about 1 hour 45 minutes for this section

- Instructions**
- Answer the Questions in the spaces provided. These spaces provide guidance for the expected length of response.
 - Your response should include relevant mathematical reasoning and/or calculations
 - Extra writing space is provided at the back of this booklet.
If you use this space, clearly indicate which question you are answering.

Question 11 (2 marks)

Solve $|7x - 2| = 19$.

2

$$7x - 2 = 19$$

$$\text{or } -(7x - 2) = 19$$

$$7x = 21$$

$$\text{or } -7x + 2 = 19$$

$$x = 3 \quad \checkmark \text{ 1 mark}$$

$$-7x = 17$$

$$x = -\frac{17}{7} \quad \checkmark \text{ 1 mark}$$

Question 12 (3 marks)

Solve the following for x .

(a) $\log_x 243 = 5$

1

$$x^5 = 243$$

$$x^5 = 3^5$$

$$\therefore x = 3 \quad \checkmark \text{ 1 mark}$$

(b) $3 \log_3(5) - 3 \log_3(2) + \log_3(x) = 2$

2

$$\log_3 5^3 - \log_3 2^3 + \log_3 x = 2$$

$$\log_3 125x - \log_3 8 = 2$$

$$\log_3 \left(\frac{125x}{8} \right) = 2 \quad \checkmark \text{ 1 mark}$$

$$\frac{125x}{8} = 9$$

$$x = \frac{72}{125} \quad \checkmark \text{ 1 mark}$$

Question 13 (7 marks)

The probability distribution of a random variable X is shown in the table below.

x	1	2	3	4	5
$P(X = x)$	$3p$	0.35	0.20	0.25	p

$$3 \times 0.05 = 0.15$$

$$0.05$$

- (a) Calculate the expected value $E(x)$.

2

$$3p + 0.35 + 0.20 + 0.25 + p = 1$$

$$4p + 0.8 = 1$$

$$p = 0.05 \quad \checkmark \text{ 1 mark}$$

$$E(x) = (1 \times 0.15) + (2 \times 0.35) + (3 \times 0.20) + (4 \times 0.25) + (5 \times 0.05)$$

$$E(x) = 2.7 = \mu \quad \checkmark \text{ 1 mark}$$

- (b) Show that the variance $\text{Var}(X) = 1.31$

1

$$\text{Var}(X) = \sum [x^2 p(x)] - \mu^2$$

$$= (1^2 \times 0.15) + (2^2 \times 0.35) + (3^2 \times 0.20) + (4^2 \times 0.25) + (5^2 \times 0.05) - (2.7)^2$$

$$= 8.6 - (2.7)^2$$

$$\text{Var}(X) = 1.31 \quad \dots \text{ as required } \quad \checkmark \text{ 1 mark}$$

- (c) Find the standard deviation to 3 decimal places.

1

$$\sigma = \sqrt{1.31}$$

$$= 1.1445 \dots$$

$$= 1.145 \quad (\text{to 3 d.p.}) \quad \checkmark \text{ 1 mark}$$

- (d) Find $P(X \leq 4)$.

1

$$P(X \leq 4) = p(1) + p(2) + p(3) + p(4)$$

$$= 0.15 + 0.35 + 0.20 + 0.25$$

$$= 0.95 \quad \checkmark \text{ 1 mark}$$

- (e) Find $P(2 < X \leq 5 | X \leq 4)$

2

$$P(2 < X \leq 5 | X \leq 4) = \frac{p(3) + p(4)}{p(1) + p(2) + p(3) + p(4)}$$

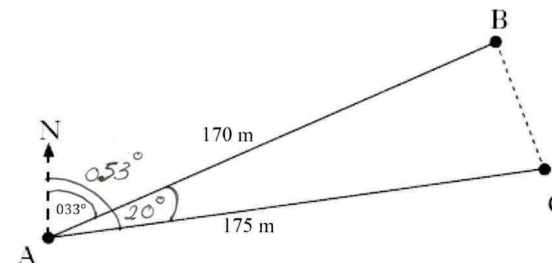
$$= \frac{0.2 + 0.25}{0.15 + 0.35 + 0.20 + 0.25} \quad \checkmark \text{ 1 mark}$$

$$= \frac{0.45}{0.95}$$

$$= 0.47 \quad \checkmark \text{ 1 mark}$$

Question 14 (4 marks)

Sarah is participating in a race, as shown in the diagram below. She needs to run 170 metres from A to B on a bearing of 033° . Instead, she ran 175 metres from A to C on a bearing of 053° .



- (a) What is the size of $\angle BAC$?

1

$$\angle BAC = 53^\circ - 33^\circ = 20^\circ \quad \checkmark \text{ 1 mark}$$

- (b) Calculate the length of BC to two decimal places.

2

$$BC^2 = 170^2 + 175^2 - 2(170)(175)\cos 20^\circ \quad \checkmark \text{ 1 mark}$$

$$BC = \sqrt{59525 - 59500 \cos 20^\circ}$$

$$BC = 60.11 \text{ m} \quad (\text{to 2 d.p.}) \quad \checkmark \text{ 1 mark}$$

- (c) What is the bearing from C to B ? Round the answer to the nearest degree.

1

$$\sin \angle BCA = \frac{\sin 20^\circ}{170}$$

$$\angle BCA = \sin^{-1} \left[\frac{170 \times \sin 20^\circ}{\sqrt{59525 - 59500 \cos 20^\circ}} \right]$$

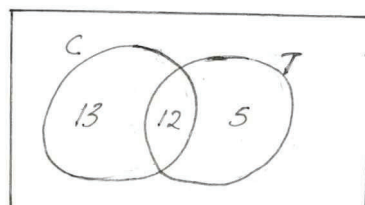
$$= 75.30^\circ$$

$$\text{Bearing from } C \text{ to } B = 180^\circ + (33^\circ + 20^\circ) + 75.30^\circ = 308^\circ \text{ T} \quad \checkmark \text{ 1 mark}$$

Question 15 (4 marks)

At the Year 8Y class at Blacktown Boys High School, 25 students play cricket and 17 play tennis. Altogether there are 30 students in the Year 8Y class and each student plays at least one sport.

- (a) Draw a Venn diagram to represent this scenario.



$$25 + 17 = 42$$

$$42 - 30 = 12$$

✓ 1 mark for all correct

- (b) One student is chosen at random. Given that the student plays cricket, what is the probability that the student does NOT play tennis?

$$P(\bar{T}/C) = \frac{13}{25}$$

✓ 1 mark

- (c) Two different students are chosen at random one after the other. What is the probability that both students play both tennis and cricket?

$$P(\text{first student plays both and second student plays both})$$

$$= \frac{12}{30} \times \frac{11}{29}$$

$$= \frac{22}{145}$$

✓ 1 mark

- (d) One student is chosen at random. What is the probability that this student does NOT play two sports?

$$P(\text{does not play both}) = P(C \text{ only or } T \text{ only})$$

$$= \frac{13}{30} + \frac{5}{30}$$

$$= \frac{18}{30}$$

$$= \frac{3}{5}$$

✓ 1 mark

Question 16 (4 marks)

A car manufacturer produces 20 cars, 3 of which are defective. Two cars are drawn at random.

- (a) What is the probability that the first car is defective?

$$P(\text{one is defective}) = \frac{3}{20}$$

✓ 1 mark

- (b) What is the probability that both are defective?

$$P(\text{both are defective}) = \frac{3}{20} \times \frac{2}{19}$$

$$= \frac{3}{190}$$

✓ 1 mark

- (c) What is the probability that at least one is defective?

$$P(\text{At least one car is defective})$$

$$= 1 - P(\text{both cars are not defective})$$

$$= 1 - \left(\frac{17}{20} \times \frac{16}{19}\right)$$

$$= 1 - \frac{68}{95}$$

$$= \frac{27}{95}$$

✓ 1 mark

$$\text{OR } P(D_1, D_2) + P(D_1, ND_2) + P(ND_1, D_2)$$

$$= \left(\frac{3}{20} \times \frac{2}{19}\right) + \left(\frac{3}{20} \times \frac{17}{19}\right) + \left(\frac{17}{20} \times \frac{3}{19}\right)$$

$$= \frac{3}{190} + \frac{51}{380} + \frac{51}{380}$$

$$= \frac{27}{95}$$

Question 17 (2 marks)

Differentiate the following $y = \frac{1}{2}x^2 - e^{7-9x}$

$$\frac{dy}{dx} = \frac{1}{2} \times 2x - (-9)e^{7-9x}$$

$$= x + 9e^{7-9x}$$

✓ 1 mark

✓ 1 mark

Question 18 (2 marks)

Show $\frac{d}{dx} \left(\frac{x}{\sqrt{7x-9}} \right) = \frac{7x-18}{2(7x-9)\sqrt{7x-9}}$

2

Let $u = x$ $v = (7x-9)^{\frac{1}{2}}$
 $u' = 1$ $v' = \frac{1}{2} \times 7 (7x-9)^{-\frac{1}{2}}$
 $v' = \frac{7}{2} (7x-9)^{-\frac{1}{2}}$

$$\frac{d}{dx} \left(\frac{x}{\sqrt{7x-9}} \right) = \frac{(7x-9)^{\frac{1}{2}} \times 1 - x \times \frac{7}{2} (7x-9)^{-\frac{1}{2}}}{[(7x-9)^{\frac{1}{2}}]^2}$$

$$= \frac{(7x-9)^{\frac{1}{2}} - \frac{7x}{2} (7x-9)^{-\frac{1}{2}}}{(7x-9)}$$

$$= \frac{\frac{(7x-9) - 7x}{2} (7x-9)^{\frac{1}{2}}}{(7x-9)}$$

$$= \frac{14x-18-7x}{2(7x-9)\sqrt{7x-9}}$$

$$= \frac{7x-18}{2(7x-9)\sqrt{7x-9}} \quad \checkmark \text{ 1 mark.}$$

$$= \text{RHS}$$

OK Using Product Rule:

Let $u = x$ $v = (7x-9)^{-\frac{1}{2}}$
 $u' = 1$ $v' = -\frac{1}{2} \times 7 (7x-9)^{-\frac{3}{2}}$

$$\frac{d}{dx} \left(\frac{x}{\sqrt{7x-9}} \right) = u'v + uv'$$

$$= 1 \times (7x-9)^{-\frac{1}{2}} + x \times \left(-\frac{7}{2} (7x-9)^{-\frac{3}{2}} \right)$$

$$= \frac{1}{\sqrt{7x-9}} - \frac{7x}{2(\sqrt{7x-9})^3}$$

$$= \frac{2(\sqrt{7x-9})^2 - 7x}{2(\sqrt{7x-9})^3}$$

$$= \frac{14x-18-7x}{2(\sqrt{7x-9})^3}$$

$$= \frac{7x-18}{2(7x-9)(\sqrt{7x-9})}$$

$$= \text{RHS}$$

Question 19 (4 marks)

Solve the following equations for x .

(a) $2^{2x} - 5(2^{x+1}) + 16 = 0$

2

$$2^{2x} - 5(2^x \times 2) + 16 = 0$$

$$2^{2x} - 10(2^x) + 16 = 0$$

Let $2^x = m$

$$m^2 - 10m + 16 = 0$$

$$(m-8)(m-2) = 0$$

$$m-8=0 \quad \text{or} \quad m-2=0$$

$$m=8 \quad \text{or} \quad m=2$$

(b) $e^{2\ln x} = x + 12$ $\therefore 2^x = 2^3$ or $2^x = 2^1$ $\therefore x=3$ or $x=1$ $\checkmark \text{ 1 mark.}$

2

$$e^{2\ln x} = x + 12$$

$$\ln e^{2\ln x} = \ln(x+12)$$

$$2\ln x = \ln(x+12)$$

$$\ln x^2 = \ln(x+12) \quad \checkmark \text{ 1 mark.}$$

$$x^2 = x + 12$$

$$x^2 - x - 12 = 0$$

$$(x-4)(x+3) = 0$$

$$x-4=0 \quad \text{or} \quad x+3=0$$

$$x=4 \quad \text{or} \quad x=-3$$

Discard $x=-3$ as $x > 0$

$$\therefore x=4 \quad \checkmark \text{ 1 mark}$$

Question 20 (5 marks)

A car is worth \$12000 when new. After t years, the value of the car (in dollars) is given by the formula $V = Ae^{-0.25t}$.

- (a) Find the value of A .

$$V = Ae^{-0.25t}$$

When $t=0$, $V = \$12000$

$$12000 = Ae^{-0.25 \times 0}$$

$$\therefore A = 12000 \quad \checkmark \text{ 1 mark}$$

- (b) Find the value of the car after 7 years.

When $t=7$, $V = 12000 \times e^{-0.25 \times 7}$

$$V = \$2085 \quad \checkmark \text{ 1 mark}$$

- (c) Find the rate in dollars per year at which the car's value is depreciating, when $t = 7$.

$$V = 12000 e^{-0.25t}$$

$$\frac{dV}{dt} = 12000 \times (-0.25) \times e^{-0.25t}$$

$$= -3000 e^{-0.25t} \quad \checkmark \text{ 1 mark}$$

When $t=7$, $\frac{dV}{dt} = -3000 \times e^{-0.25 \times 7}$

$$= -\$521 \quad \checkmark \text{ 1 mark}$$

The car is depreciating at a rate of \$521 per annum.

- (d) Find how long it will take for the value of the car to depreciate to \$1000. Round the answer to 2 decimal places.

When $V=1000$, $1000 = 12000 e^{-0.25t}$

$$e^{-0.25t} = \frac{1000}{12000}$$

$$-0.25t = \ln\left(\frac{1000}{12000}\right)$$

$$t = \frac{\ln\left(\frac{1000}{12000}\right)}{-0.25}$$

$$t = 9.94 \text{ years} \quad \checkmark \text{ 1 mark}$$

Question 21 (3 marks)

Find the equation of the normal to the curve $y = \frac{x}{2}(3-4x)^4$ at the point where $x = 1$.

$$y = \frac{x}{2}(3-4x)^4$$

Let $u = \frac{x}{2}$ $v = (3-4x)^4$

$$u' = \frac{1}{2} \quad v' = (4x-4)(3-4x)^3$$

$$v' = -16(3-4x)^3$$

$$\frac{dy}{dx} = u \times v' + v u'$$

$$= \frac{x}{2} \times [-16(3-4x)^3] + (3-4x)^4 \times \frac{1}{2}$$

$$= -8x(3-4x)^3 + \frac{1}{2}(3-4x)^4$$

$$= (3-4x)^3 \left(-\frac{1}{2}\right) [16x - (3-4x)]$$

$$= -\frac{1}{2}(3-4x)^3 (20x-3) \quad \checkmark \text{ 1 mark}$$

When $x=1$, $\frac{dy}{dx} = m_T = -\frac{1}{2}(3-4 \times 1)^3 (20 \times 1 - 3)$

$$m_T = -\frac{1}{2}(-1)(17)$$

$$m_T = +\frac{17}{2}$$

$$m_N = -\frac{2}{17} \quad \checkmark \text{ 1 mark}$$

When $x=1$, $y = \frac{1}{2}(3-4 \times 1)^4 = \frac{1}{2}$

$(1, \frac{1}{2})$: $y - \frac{1}{2} = -\frac{2}{17}(x-1)$

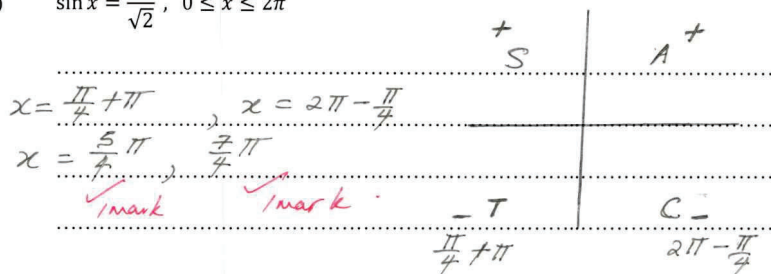
$\times 34$: $34y - 17 = -4x + 4$

$$4x + 34y - 21 = 0 \quad \checkmark \text{ 1 mark}$$

Question 22 (5 marks)

Find the exact values of x for the given domain.

(a) $\sin x = \frac{-1}{\sqrt{2}}, 0 \leq x \leq 2\pi$



(b) $2 \cos^2 2x - \cos 2x - 1 = 0, 0 \leq x \leq \pi$

$$2 \cos^2 2x - \cos 2x - 1 = 0 \quad 0 \leq 2x \leq 2\pi$$

$$(2 \cos 2x + 1)(\cos 2x - 1) = 0$$

$$2 \cos 2x + 1 = 0 \quad \text{or} \quad \cos 2x - 1 = 0$$

$$\cos 2x = -\frac{1}{2} \quad \cos 2x = 1$$

$$2x = \frac{2\pi}{3}, \frac{4\pi}{3}, 0, 2\pi$$

$$x = \frac{2\pi}{3}, \frac{4\pi}{3}, 0, \pi$$

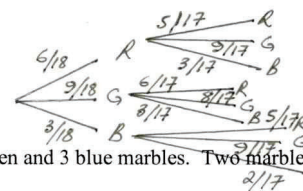
$$x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi$$

1 mark for correct answers for x for $\cos 2x = -\frac{1}{2}$

1 mark for correct answers for x for $\cos 2x = 1$

Question 23 (4 marks)

A bag contains 6 red, 9 green and 3 blue marbles. Two marbles are drawn in succession without replacement.



(a) Find the probability of drawing 2 green marbles.

$$P(GG) = \frac{9}{18} \times \frac{8}{17}$$

$$= \frac{4}{17}$$

(b) Find the probability of drawing a red marble followed by a blue marble.

$$P(R \text{ followed by } B) = \frac{6}{18} \times \frac{3}{17}$$

$$= \frac{1}{17}$$

(c) Find the probability of not drawing two same coloured marbles.

$$P(\text{Not drawing two same coloured marbles})$$

$$= 1 - [P(RR) + P(GG) + P(BB)]$$

$$= 1 - \left[\left(\frac{6}{18} \times \frac{5}{17} \right) + \left(\frac{9}{18} \times \frac{8}{17} \right) + \left(\frac{3}{18} \times \frac{2}{17} \right) \right]$$

$$= \frac{11}{17}$$

Question 24 (3 marks)

Prove that $\tan \theta (1 + \tan \theta) + (1 + \cot \theta) = \frac{\tan \theta + 1}{\sin \theta \cos \theta}$

$$\text{LHS} = \tan \theta (1 + \tan \theta) + (1 + \cot \theta)$$

$$= \frac{\sin \theta}{\cos \theta} \left(1 + \frac{\sin \theta}{\cos \theta} \right) + \left(1 + \frac{\cos \theta}{\sin \theta} \right)$$

$$= \frac{\sin \theta}{\cos \theta} + \frac{\sin^2 \theta}{\cos^2 \theta} + 1 + \frac{\cos \theta}{\sin \theta}$$

$$= \frac{\sin^2 \theta \cos \theta + \sin^2 \theta + \sin \theta \cos^2 \theta + \cos^2 \theta}{\sin \theta \cos^2 \theta}$$

$$= \frac{\sin^2 \theta (\cos \theta + \sin \theta) + \cos^2 \theta (\cos \theta + \sin \theta)}{\sin \theta \cos^2 \theta}$$

$$= \frac{(1 + \tan \theta) (\cos \theta + \sin \theta)}{\sin \theta \cos^2 \theta}$$

$$= \frac{\cos \theta + \sin \theta}{\sin \theta \cos^2 \theta}$$

$$= \frac{1 + \tan \theta}{\sin \theta \cos \theta}$$

$$= \text{RHS}$$

Question 25 (5 marks)

A particle is moving in a straight line with displacement x metres from the origin O at time t , where $x = 4t^3 - 30t^2 + 72t$.

- (a) Find the initial velocity.

$$V = \frac{dx}{dt} = 12t^2 - 60t + 72 \quad \checkmark \text{ 1 mark}$$

Initial velocity at $t=0$

$$V = 12 \times 0^2 - 60 \times 0 + 72$$

$$V = 72 \text{ m/s} \quad \checkmark \text{ 1 mark}$$

- (b) Find when the particle is at rest.

Particle is at rest when $V=0$

$$12t^2 - 60t + 72 = 0$$

$$12(t^2 - 5t + 6) = 0$$

$$(t-3)(t-2) = 0$$

$$t-3=0 \text{ or } t-2=0$$

$$t=3 \text{ or } t=2 \quad \checkmark \text{ 1 mark for both correct}$$

- (c) At what time is the acceleration zero? Find the velocity and position of the particle at this time.

$$a = \frac{dV}{dt} = 24t - 60$$

$$a=0 \Rightarrow 0 = 24t - 60$$

$$24t = 60$$

$$t = 2.5 \text{ sec} \quad \checkmark \text{ 1 mark}$$

Velocity at $t=2.5 \text{ sec}$. $V = 12(2.5)^2 - 60(2.5) + 72$

$$V = -3 \text{ m/s}$$

Position x at $t=2.5 \text{ sec}$.

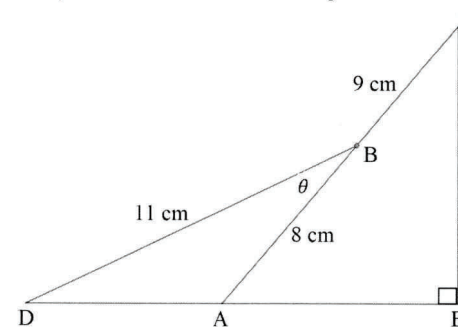
$$x = 4(2.5)^3 - 30(2.5)^2 + 72(2.5)$$

$$x = 55 \text{ m}$$

1 mark for $V = -3 \text{ m/s}$
and $x = 55 \text{ m}$

Question 26 (3 marks)

In the diagram below, there are two triangles, $\triangle DAB$, and $\triangle AEC$. Given that $BC = 9 \text{ cm}$, $AB = 8 \text{ cm}$, $DB = 11 \text{ cm}$, $\angle DBA = \theta$ and DAE is a straight line.



- (a) Use the cosine rule to find an expression for the length of DA in terms of θ .

In $\triangle DAB$: Using the Cosine Rule:

$$DA^2 = 11^2 + 8^2 - 2(11)(8) \cos \theta$$

$$DA = \sqrt{11^2 + 8^2 - 2 \times 11 \times 8 \cos \theta}$$

$$DA = \sqrt{185 - 176 \cos \theta} \quad \checkmark \text{ 1 mark}$$

- (b) Find an expression for the length of CE in terms of θ .

In $\triangle DAB$: Using Sine Rule:

$$\frac{11}{\sin(\angle DAB)} = \frac{DA}{\sin \theta}$$

$$\therefore \sin(\angle DAB) = \frac{11 \sin \theta}{DA} \quad \text{--- (2)}$$

In $\triangle CAE$: $\frac{CE}{17} = \sin(\angle CAE)$

$$CE = 17 \sin(\angle CAE) \quad \text{--- (3)}$$

$$\angle DAB = 180^\circ - \angle CAE \quad \text{--- (DAE is a straight line)}$$

$$\therefore \sin(\angle DAB) = \sin(\angle CAE) \quad \text{--- [Sine is positive in Q1 and Q2. - Related angle.]}$$

From (2) and (3): $CE = 17 \sin(\angle CAE)$

$$= 17 \sin(\angle DAB)$$

$$= \frac{17 \times 11 \sin \theta}{DA}$$

From (1): $CE = \frac{17 \times 11 \sin \theta}{\sqrt{185 - 176 \cos \theta}}$

1 mark for
correct final
answer

End of examination

* OR ALTERNATIVE SOLUTION ON PAGE 21 *

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

Q26 ALTERNATIVE SOLUTION:

$$\frac{\sin \theta}{DA} = \frac{\sin \angle DAB}{11}$$

$$\sin \angle DAB = \frac{11 \sin \theta}{DA}$$

$$\angle DAB = \sin^{-1} \left(\frac{11 \sin \theta}{DA} \right)$$

$$\sin \angle CAE = \frac{CE}{AC}$$

$$\sin \angle CAE = \frac{CE}{17}$$

$$\sin (180^\circ - \angle DAB) = \frac{CE}{17}$$

$$\sin \angle DAB = \frac{CE}{17}$$

$$CE = 17 \sin \angle DAB$$

$$CE = 17 \sin \left[\sin^{-1} \left(\frac{11 \sin \theta}{DA} \right) \right]$$

$$CE = 187 \sin \theta$$

$$\sqrt{185 - 176 \cos \theta}$$

Student Name: _____

Multiple Choice Answer Sheet

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9
 A ☐ B ☒ C ☐ D ☐

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ☒ B ☒ C ☐ D ☐

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word 'correct' and drawing an arrow as follows.

A ☒ B ☒ C ☐ D ☐
 correct

- Start Here →
1. A ☐ B ☐ C ☐ D ☐
 2. A ☐ B ☐ C ☐ D ☐
 3. A ☐ B ☐ C ☐ D ☐
 4. A ☐ B ☐ C ☐ D ☐
 5. A ☐ B ☐ C ☐ D ☐
 6. A ☐ B ☐ C ☐ D ☐
 7. A ☐ B ☐ C ☐ D ☐
 8. A ☐ B ☐ C ☐ D ☐
 9. A ☐ B ☐ C ☐ D ☐
 10. A ☐ B ☐ C ☐ D ☐