Caringbah High School

Year 11 Mathematics Extension 1
Semester 2 Examination
2014

Time Allowed: $1\frac{1}{2}$ hours + 5 minutes reading time

Instructions

- All questions must be attempted.
- Start each question on a new page.
- Answers without necessary working may not attract full marks.
- Marks may not be awarded for careless or badly arranged work.
- Diagrams and working which lead to a correct solution may attract some marks.
- Approved calculators may be use

Question 1 (10 marks) Use the Question 1 Writing Booklet.

(a) Find $\lim_{x \to 1} \dfrac{x^2 - 1}{x - 1}$ **1**

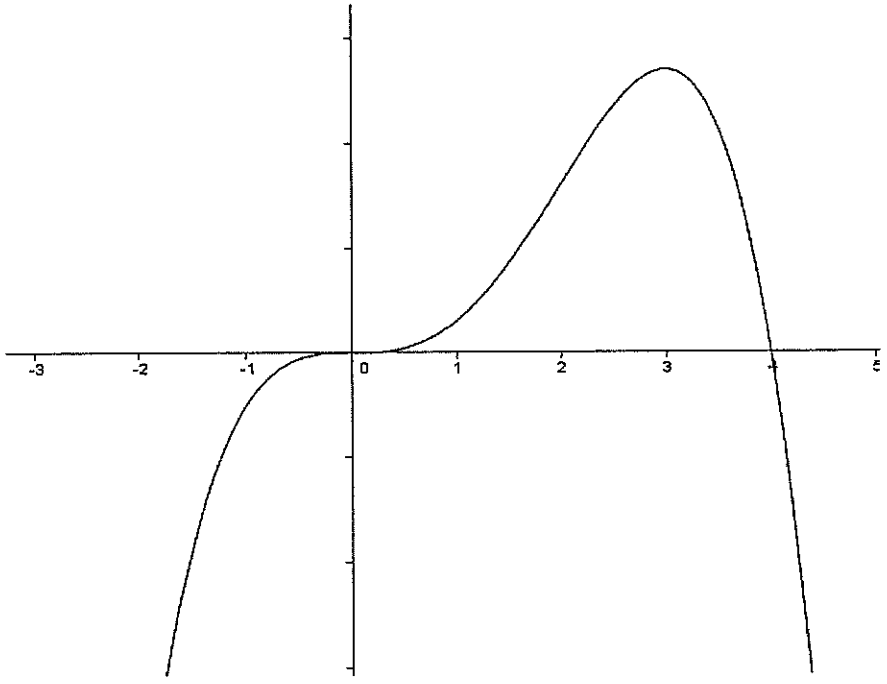
(b)

(i) Differentiate $y = \dfrac{x^2 - 3}{2x - 3}$ **2**

(ii) Find the value of $f'(2)$ **1**

(iii) Hence find the equation of the tangent to the function at this point. **2**

(c) Draw the graph of the gradient function to the following curve. **2**



(d) Determine whether the following quadratic equation has equal rational roots, unequal rational roots, unequal rational roots or no real roots. **2**

$4x^2 + 4x + 1 = 0$

Question 2 (10 marks) Use the Question 2 Writing Booklet.

(a) Find the values of a so that the line $x + a = 0$ is a tangent to the circle centre $(1, -2)$ and radius 3 units. **2**

(b) Find the point that divides the interval $(-2, 3)$ and $(4, -2)$ externally in the ratio 1:2. **2**

(c) Given three points $A(-1,1)$ $B(1,2)$ and $C(2, -3)$

(i) Find the exact distances AC and AB **2**

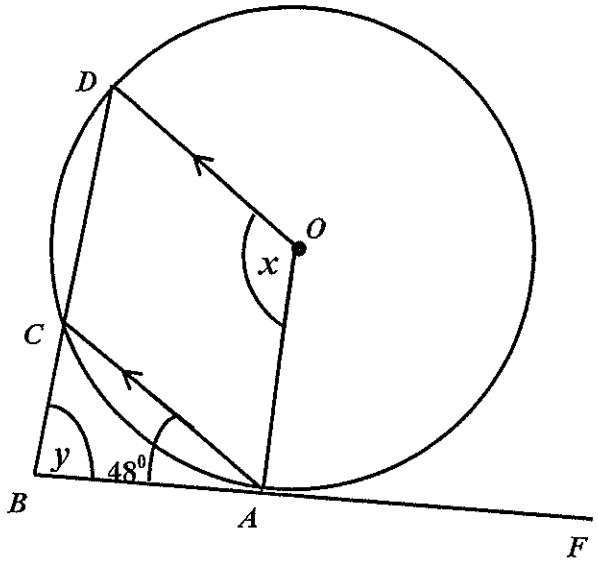
(ii) Find the angle between the lines AB and AC. **2**

(iii) Find the area of the triangle formed by joining ABC. **2**

Question 3 (10 marks) Use the Question 3 Writing Booklet.

(a) *BF* is a tangent to the circle centre *O* at *A*. 3
$\angle CAB = 48°$, $\angle DOA = x°$, $\angle ABC = y°$
Find the values of *x and y* giving reasons.

Write your answer on page 8



(b) Solve $\frac{3x}{3x-4} < 3$ 2

(c) The general form of the line through the point of intersection M of two lines

$2x-2y+5=0$ and $x+3y+2=0$ is

$l: 2x-2y+5+k(x+3y+2)=0$

(i) Show that the gradient of l is $\frac{2+k}{2-3k}$ 1

(ii) Hence find the equation of the line through *M* parallel to the line $3y-x-5=0$. Write your equation in general form. 2

(d) Shade in the region for which both the following inequalities hold simultaneously. 2

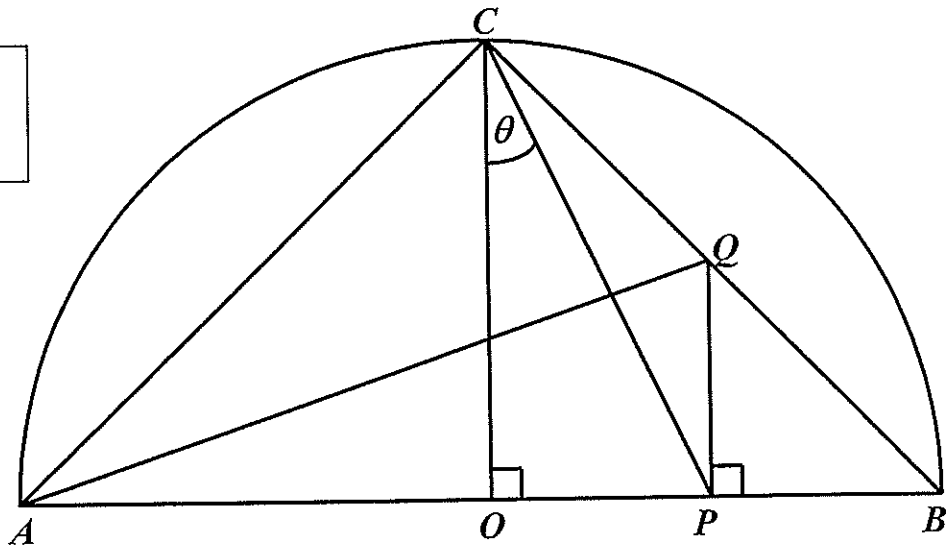
$y \geq x^2$, $y < \frac{8}{x}$

Question 4 (10 marks) Use the Question 4 Writing Booklet.

(a) Differentiate by first principles $y = \sqrt{x}$ 3

(b) The point O is the centre of the semi-circle. OC and QP are perpendicular to AB and $\angle OCP = \theta$. 3
Prove that $\angle PCA = \angle PQA = 45^\circ + \theta$

Write your answer on page 9



(c) Find the value of m if the equation $x^2 - 3mx + (m+3) = 0$

(i) has the sum of the roots equal to their product. 2

(ii) has one root the reciprocal of the other. 2

Question 5 (10 marks) Use the Question 5 Writing Booklet.

(a) If $f(x)=\begin{cases} x & x \geq 0 \\ x^3 & x<0 \end{cases}$

(i) Sketch the curve for $-2 \leq x \leq 2$ 2

(ii) Find $f(2)-f(-2)$ 1

(iii) Is $f(x)$ able to be differentiated at $x=0$? Give a reason as to why it is or is not. 1

(b) $f(x)=\dfrac{x}{x^2-1}$

(i) Find the domain and range for $f(x)$. 2

(ii) Explain why $f(x)$ is odd, even or neither. 1

(iii) Sketch the function showing any asymptotes and intercepts. 2

(c) Show that $\lim\limits_{x\to\infty}\dfrac{x^3+2x^2-3}{2x^3+x}$ is $\dfrac{1}{2}$. 1

Question 6 (10 marks) Use the Question 6 Writing Booklet.

(a) Differentiate and express your answers in the most factorised form.

(i) $\left(x^2-3\right)^2\left(x^2+2\right)$ **2**

(ii) $\left(x+\frac{1}{x}\right)^3$ **2**

(b) For the equation $x^2+rx+p=0$. If one root is twice the other show that $2r^2=9p$ **2**

(c) Solve the equation $x^4+6x^3+6x^2-9x-4=0$ by expressing it in the form $(x^2+ax)^2+b(x^2+ax)+c$ and hence solve the equation $x^4+6x^3+6x^2-9x-4=0$ **4**

End of Paper

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Examination:.....

Q1 a) $\lim_{x \to 1} \frac{(x+1)(x-1)}{(x-1)} = 2.$

b) $y = \frac{x^2-3}{2x-3}$ $\quad u = x^2-3 \quad v = 2x-3$

$u' = 2x \quad v' = 2$

$y' = \frac{(2x-3)2x - 2(x^2-3)}{(2x-3)^2}$

$= \frac{4x^2-6x-2x^2+6}{(2x-3)^2}$

$= \frac{2x^2-6x+6}{(2x-3)^2}$

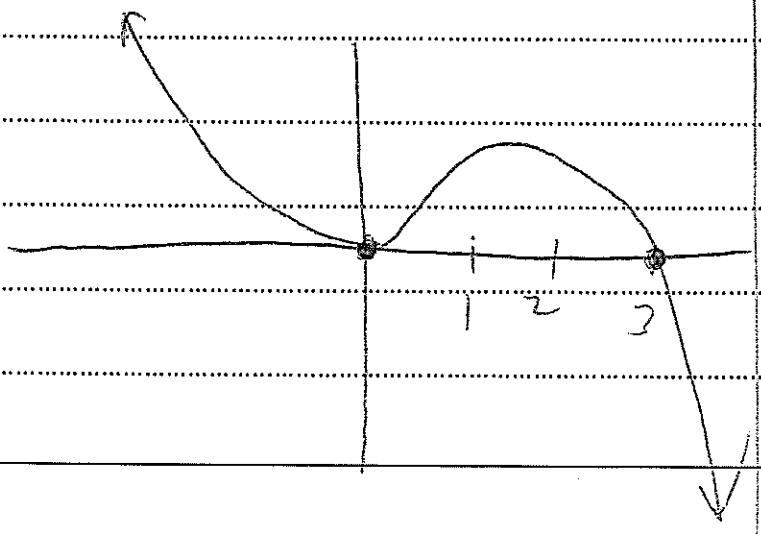
i) $f'(2) = \frac{8-12+6}{1^2} = 2.$

ii) $f(2) = \frac{4-3}{4-3} = 1$

$y - 1 = 2(x-2)$

$y = 2x - 3$

c)



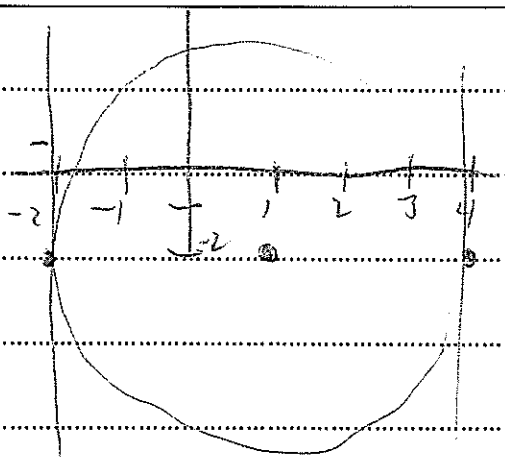
d) $4x^2 + 4x + 1 = 0$

$(2x+1)^2 = 0$

$x = -\frac{1}{2}$

So equal rational roots.

Q2

a) 

$x = 4$ or $x = -2$

y

$x - 4 = 0$ $\quad x + 2 = 0$

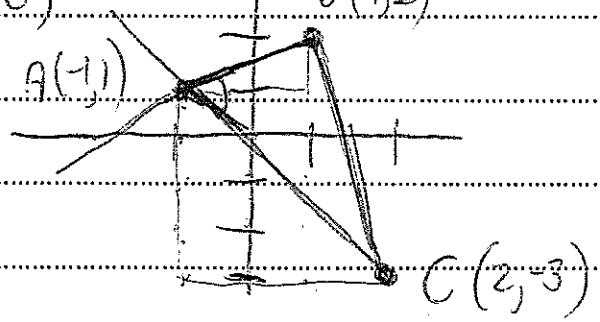
$a = -4, 2.$

b) $(-2, 3) \quad (4, -2)$

$-1 : 2$

$\left(\frac{-1 \times 4 + 2 \times -2}{1}, \frac{-1 \times -2 + 2 \times 3}{1} \right)$

$= (-8, 8)$

c) 

i) $AB = \sqrt{2^2 + 1^2}$

$= \sqrt{5}$

ii) $AC = \sqrt{3^2 + 4^2}$

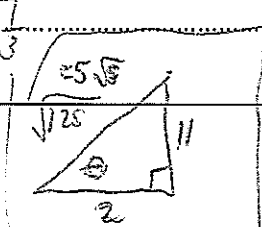
$= 5$

ii) $m_{AB} = \frac{1}{2}$

$m_{AC} = \frac{-4}{3}$

$\tan\theta = \left| \frac{\frac{1}{2} + \frac{4}{3}}{1 - \frac{2}{3}} \right| = \frac{\frac{11}{6}}{\frac{1}{3}} = 5\frac{1}{2}$

$\theta = 80^\circ (79^\circ 42')$



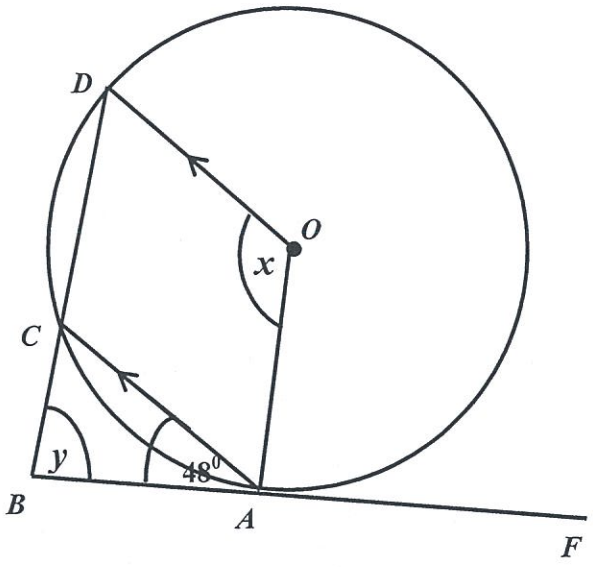
$\sin\theta = \frac{5\sqrt{5}}{11}$

iii) Area $= \frac{1}{2} \times 5 \times \sqrt{5} \times \sin 79^\circ 42'$

$= 5.5\ u^2$

Answer Sheet for Q3 (a)

BF is a tangent to the circle centre *O* at *A*. 3
$\angle CAB = 48°$, $\angle DOA = x°$, $\angle ABC = y°$
Find the values of *x and y* giving reasons.



$\angle CAO = 42°$ (radius ⊥ tangent = 90°)

$\angle DOA = x = 138°$ (cointerior AC // OD)

reflex $\angle DOA = 222°$

$\therefore \angle DCA = 111°$ (∠ at centre = 2× ∠ at circumf.)

$\therefore y = 63°$ (external angle = sum of opposite interior ∠s in Δ)

Q3

b) $\frac{3x}{3x-4} < 3 \quad x \neq \frac{4}{3}$

$3x(3x-4)^2 < 3(3x-4)^2$

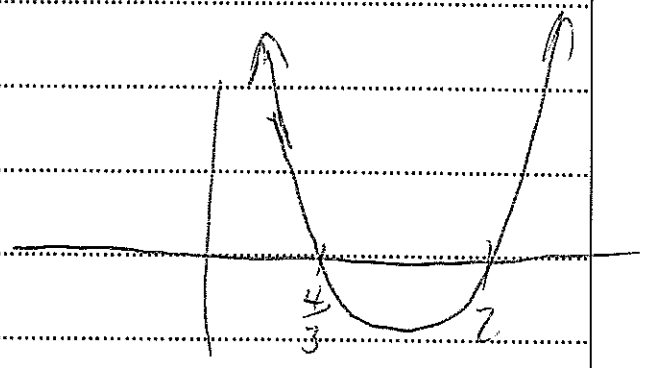
$(3x-4)$

$0 < 3(3x-4)^2 - 3x(3x-4)$

$0 < 3(3x-4)(3x-4-x)$

$0 < 3(3x-4)(2x-4)$

$0 < 6(3x-4)(x-2)$



$\therefore x < \frac{4}{3}$ and $x > 2$.

c) i) $2x - 2y + 5 + kx + 3ky + 2k = 0$

$(k+2)x - (2-3k)y + 5 + 2k = 0$

$\therefore y = \frac{k+2}{2-3k}x + \frac{5+2k}{2-3k}$

ii) $3y = x + 5 \qquad y = \frac{1}{3}x + \frac{5}{3}$

$m = \frac{1}{3}$

$\frac{k+2}{2-3k} = +\frac{1}{3}$

$3k + 6 = 2 - 3k$

$6k = -4$

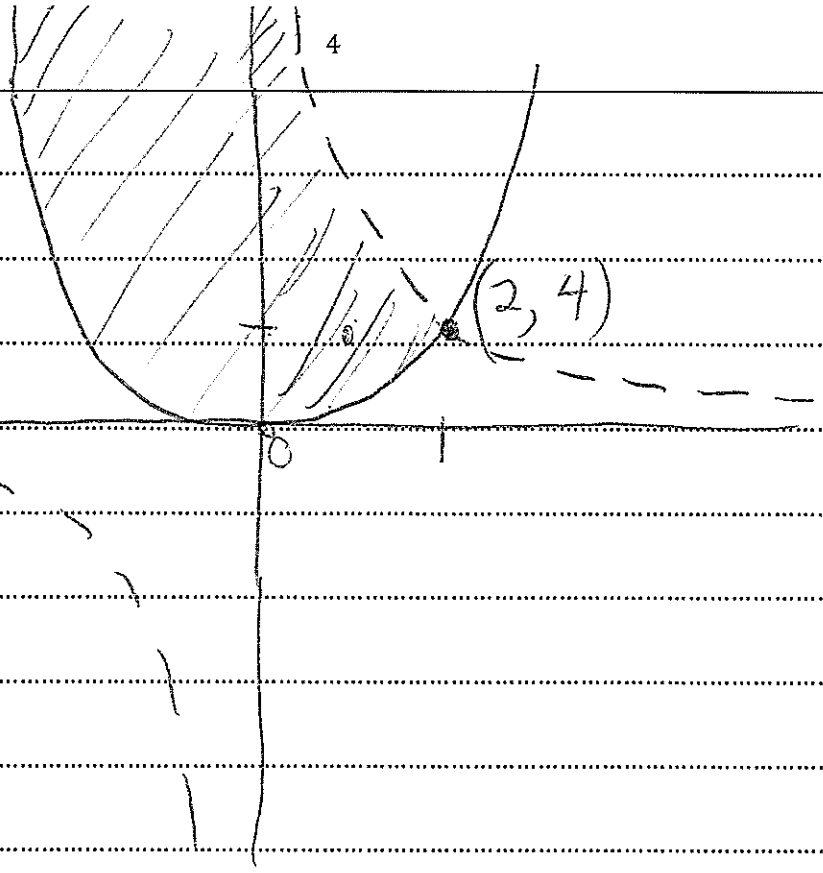
$k = -\frac{2}{3}$

$\therefore 2x - 2y + 5 - \frac{2}{3}(x + 3y + 1) = 0$

$6x - 6y + 15 - 2x - 6y - 4 = 0$

$4x - 12y + 11 = 0$

d)



You may ask for an extra Writing Booklet if you need more space to answer this question.

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Examination:.....

Q4

a) $\lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h} \times \frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$

$= \lim_{h\to 0} \frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})}$

$= \frac{1}{2\sqrt{x}}$

b) Shell Shell

c) $x^2 - 3xm + (m+3) = 0$

i) $\alpha+\beta = 3m \qquad \alpha\beta = m+3$

$3m = m+3$

$2m = 3$

$m = \frac{3}{2}$

ii) $\alpha \times \frac{1}{\alpha} = m+3$

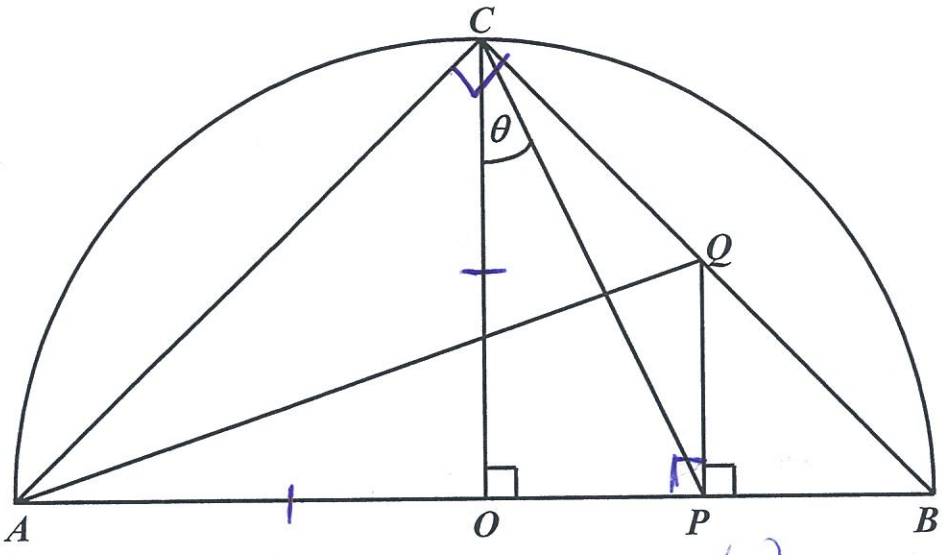
$1 = m+3$

$m = -2.$

Answer Sheet for Q4 (b)

The point O is the centre of the semi-circle. OC and QP are perpendicular to AB and $\angle OCP = \theta$. 3

Prove that $\angle PCA = \angle PQA = 45^\circ + \theta$



$\angle ACB = 90^\circ$ ($\triangle$ in a semicircle)

ACQP is a cyclic quad as opposite $\angle$'s are supplementary

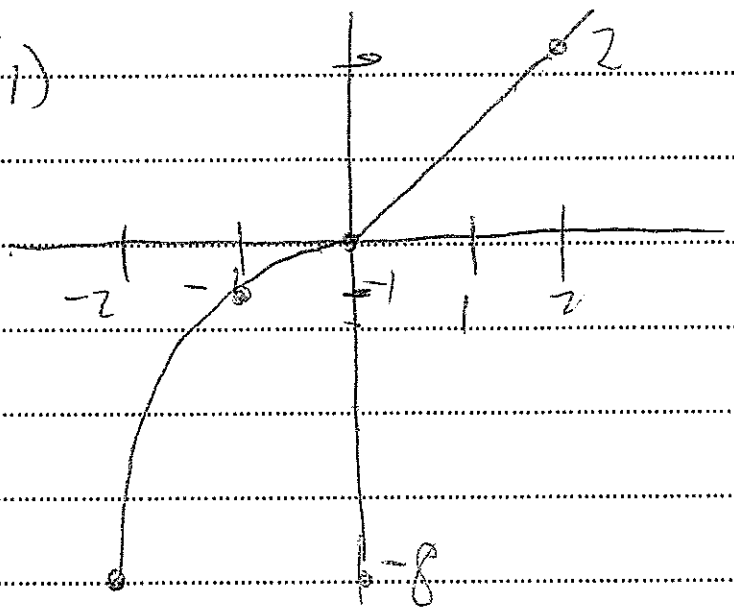
$\therefore \angle ACP = \angle AQP$ ($\angle$'s in same segment)

$\triangle AOC$ is isosceles $\therefore$ two angles (AO, OC radii)

$\therefore \angle ACO = 45^\circ$

$\therefore \angle PCA = 45^\circ + \theta$.

Q5. i)



ii) $f(2) - f(-2) = 2 - -8 = 10$

iii) NE as $f'(x) = 3x^2$ for $x < 0$ and $f'(x) = 1$ for $x > 0$

$\lim_{x \to 0^-} = 0$ $\lim_{x \to 0^+} = 1$. So different values

at $x = 0$ it is a sharp corner so no tangent can be drawn that is unique.

b) $f(x) = \frac{x}{x^2 - 1}$

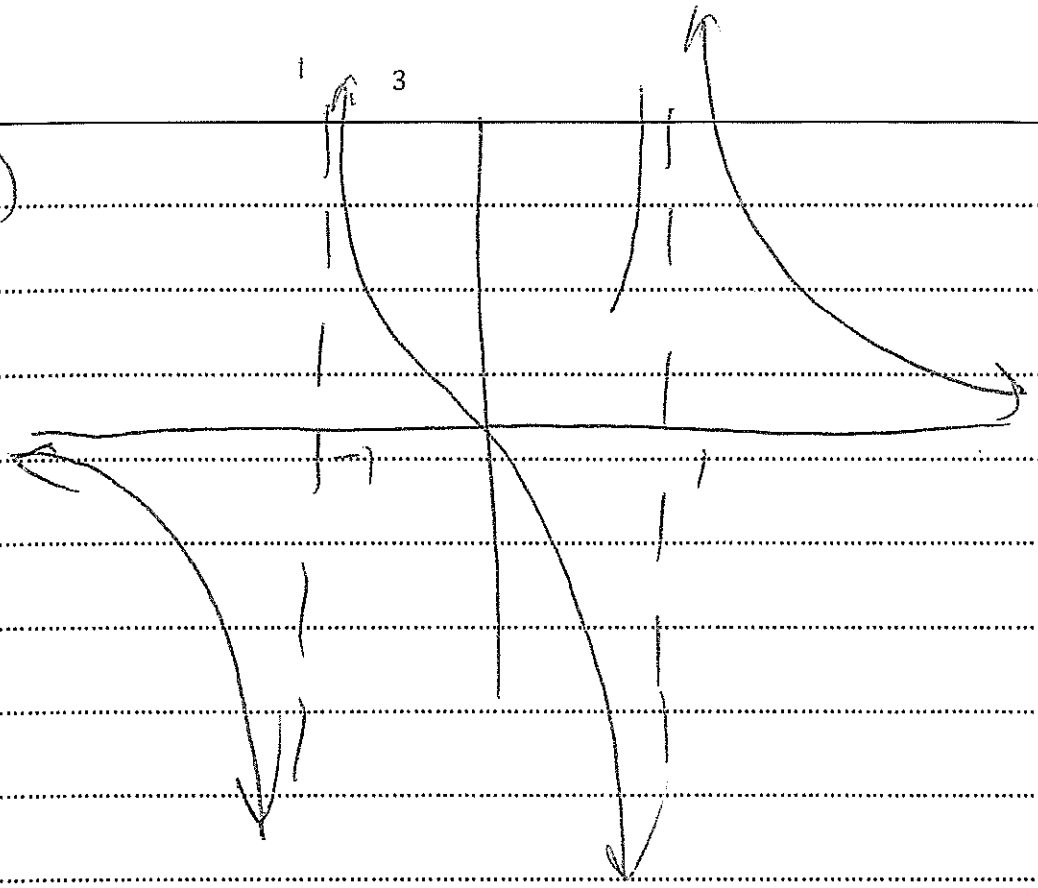
i) D: $x \neq \pm 1$ all real x except $x \neq \pm 1$

R: All real y.

ii) $f(-x) = \frac{-x}{(-x)^2 - 1} = \frac{-x}{x^2 - 1} = -f(x)$

∴ $f(x)$ is odd

iii)



c) $$\lim_{x \to \infty} \frac{\frac{x^3}{x^2} + \frac{2x^2}{x^3} - \frac{3}{x^3}}{\frac{2x^3}{x^3} + \frac{x}{x^3}}$$

$$= \lim_{x \to \infty} \frac{1 + \frac{2}{x} - \frac{3}{x^3}}{2 + \frac{1}{x^2}}$$

$$= \frac{1}{2}$$

Q6. a) i) $(x^2-3)^2(x^2+2)$

$y = 2 \times 2x(x^2-3)(x^2+2) + 2x(x^2-3)^2$

$= 2x(x^2-3)(2x^2+4+x^2-3)$

$= 2x(x^2-3)(3x^2+1)$

ii) $3\left(x+\frac{1}{x}\right)^2\left(1-\frac{1}{x^2}\right)$

b) $x^2 + rx + p = 0$

$\alpha+\beta = -r \qquad \alpha\beta = p$

But $\alpha = 2\beta$

$\therefore 3\beta = -r \qquad 2\beta^2 = p$

$\beta = \frac{-r}{3}$

$\therefore 2\left(\frac{-r}{3}\right)^2 = p$

$2r^2 = 9p$

You may ask for an extra Writing Booklet if you need more space to answer this question.

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Examination:.....

c) $x^4+6x^3+6x^2-9x-4=(x^2+ax)^2+b(x^2+ax)+c$

$x=0 \quad c=-4$

$\therefore 2ax^3=6x^3$

$\therefore 2a=6$

$a=3$

$x=1$

$0=(1+3)^2+b(1+3)-4$

$0=16+4b-4$

$4b=-12$

$b=-3$

So

$(x^2+3x)^2-3(x^2+3x)-4=0$

$u^2-3u-4=0$

$(u-4)(u+1)=0$

$x^2+3x-4=0 \qquad x^2+3x+1=0$

$(x+4)(x-1)=0 \qquad x=\frac{-3\pm\sqrt{5}}{2}$

$x=1,-4$

are the solutions.

Year 11 S 2

Mathematics Ext 1 Outcome Sheet

Name:_____

Question	Geometry		Functions and Graphs		Quadratic Function		Calculus		Coordinate		Total
1			1a	/1	1d	/2	1b i	/2	1b iii	/2	
							1b ii	/1			
							1c	/2			/10
2									2a	/2	
									2b	/2	
									2c i	/2	
									2c ii	/2	
									2c iii	/2	/10
3	3a	/3	3c i	/1	3b	/2					
			3c ii	/2							
			3d	/2							/10
4	4b	/3			4c i	/2	4a	/3			
					4c ii	/2					/10
5			5a i	/2							
			5a ii	/1							
			5a iii	/1							
			5b i	/2							
			5b ii	/1							
			5b iii	/2							
			5c	/1							/10
6					6b	/2	6a i	/2			
					6c	/4	6a ii	/2			/10
Total	Geometry	/6	Functions & Graphs	/16	Quad Functions	/12	Calculus	/12	Coordinate	/12	/60
Percent		%		%		%		%		%	%