Caringbah High School

Student Name

2015 Year 11
Semester 2 Exam

Mathematics Extension 1

General Instructions

- All questions must be attempted
- Start each question on a new page.
- Answers without necessary working may not attract full marks.
- Marks may not be awarded for careless or badly arranged work.
- Diagrams and working which lead to a correct solution may attract some marks.
- Approved calculators may be used.

Time Allowed

1 hour 30 min. + 3 min. reading time

Student Name: …………………………..

Question 1 (13 marks)

a) The point P divides the interval joining $A(-1,-2)$ to $B(9,3)$ internally in the ratio $4:1$. Find the coordinates of P. (2)

b) Find the domain of the function (2)

$$f(x)=\sqrt{2x+3}+\sqrt{x-\frac{1}{2}}$$

c) Sketch the piece meal function ($\frac{1}{3}$ page size) (3)

$$f(x)=\begin{cases} x^2+1 & x\geq 0 \\ x-3 & x<0 \end{cases}$$

d) The gradient of the tangent to $y=3x^2-mx+1$ has a value of 2 at $x=-1$, find the value of m. (2)

e)

i) Prove that (2)

$$\frac{1}{\sqrt{x+h}+\sqrt{x}}=\frac{\sqrt{x+h}-\sqrt{x}}{h}$$

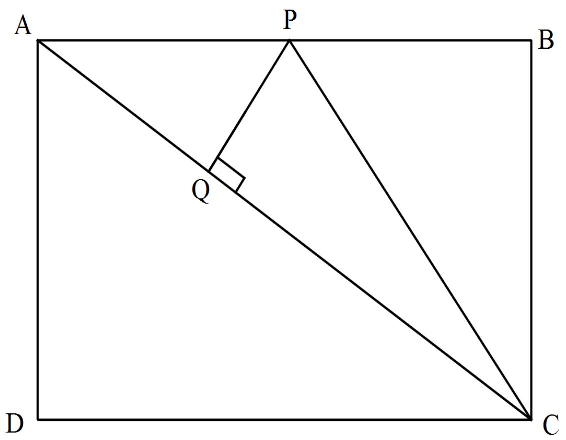
ii) Use the result above to find $\frac{d}{dx}\left(\sqrt{x}\right)$ from first principles. (2)

Student Name: …………………………..

Question 2 (13 marks)

a) In the diagram $ABCD$ is a square, PC bisects $\angle ACB$ and Q is the foot of the perpendicular from P to AC

(Diagram not to scale)

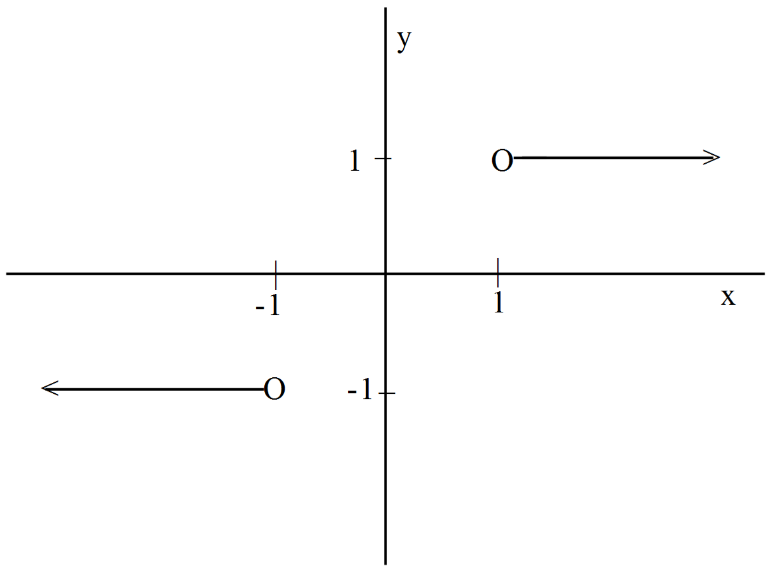


Copy the diagram into your booklet.

i) Prove $\Delta PBC \equiv \Delta PQC$ (3)

ii) Prove that the area of ΔAPQ is $\frac{x^2}{6}\text{cm}^2$ given that $BC = x\,cm$ and $PB = \frac{1}{3}AB$ (3)

b) Sketch a possible function that could have the gradient function as graphed below. ($\frac{1}{3}$ page size) (2)



c) Find the derivative of the following

i) $y = \frac{1}{3}x^3 - 2x^2 + 6$ (1)

ii) $y = \frac{(4x+3)^3}{x+1}$ (2)

iii) $y = (4x-1)\sqrt{2x+4}$ (2)

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Question 3 (13 marks)

a) On the same diagram ($\frac{1}{3}$ page size) shade in the region for which (3)

$$\begin{cases} y < x^2 \\ y \geq 0 \\ y \leq \sqrt{9-x^2} \end{cases}$$

are simultaneously true.

b) In the diagram, the vertices of ΔABC lie on the circle with centre O .
The point D lies on BC such that ΔABD is isosceles $AD = BD$ and $\angle ABC = x$

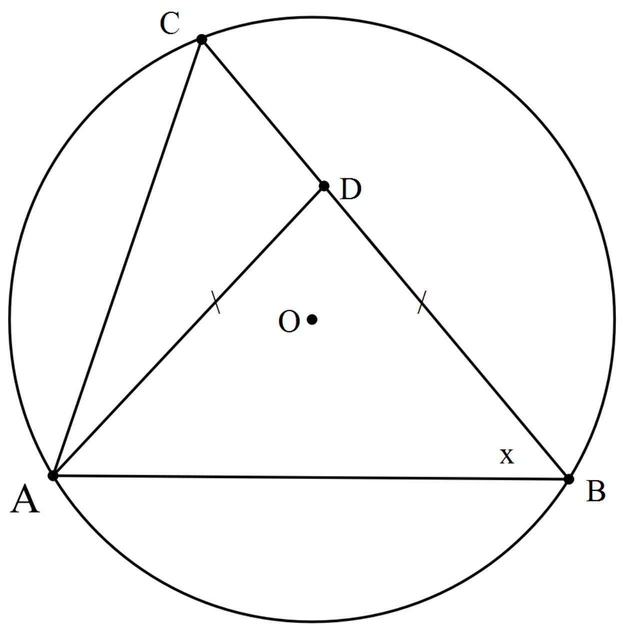

Diagram not to scale

Copy or trace this diagram into your writing booklet.

i) Explain why $\angle AOC = 2x$ (2)

ii) Prove $ACDO$ is a cyclic quadrilateral. (2)

c) Solve for x: $4^x - 6(2^x) + 8 = 0$ (2)

d) The roots of the equation $x + \frac{1}{x} = 5$ are α and β.

Find the value of

i) $\alpha + \frac{1}{\alpha}$ (1)

ii) $\alpha + \beta$ (1)

iii) $\alpha^2 + \beta^2$ (2)

Question 4 (13 marks)

a) For the function $y = x^2 - 4x$, find :

i) the co ordinates of points of intersection with the x axis. (1)

ii) the gradient of the tangents at these points of intersection. (2)

iii) Find the acute angle formed by the intersection of these two tangents. (2)

b) Find the values of K for which $x^2 + Kx + (K+1)^2 = 0$ has two roots. (2)

c) The general form of the line, l, through the intersection of the two lines

$$5x-2y+3=0 \quad \text{and} \quad 2x+6y-7=0$$

is given by $l:(5x-2y+3)+k(2x+6y-7)=0$

(You do not need to show this)

i) Show the gradient of l is $\dfrac{5+2k}{2-6k}$ (1)

ii) Hence find the equation of the line, l, if it is parallel to $2y-x+4=0$ (2)
Write your equation in general form.

d) Prove that the line $5x-12y=52$ is tangent to the circle with equation (3)

$x^2+y^2=16$.

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Question 5 (13 marks)

a) Consider the function defined by

$$f(x)=\frac{2x-2}{x^2+x-2}$$

i) For what values of x is $f(x)$ discontinuous? (2)

ii) At each point of discontinuity found in part i) determine whether $f(x)$ has a limit, and if so, give the value of the limit. (2)

iii) Write an equation for each vertical and horizontal asymptote to the graph of $y=f(x)$. Justify your answer. (2)

iv) Draw a sketch of $f(x)=\dfrac{2x-2}{x^2+x-2}$ showing above features ($\frac{1}{3}$ page size) (3)

b)

i) Find the value of k such that $y=x+k$ is a tangent to $y=x^2-x-2$ (2)

ii) What are the co ordinates of the point of contact ? (2)

END OF EXAM

2015

YEAR 11 EXT 1 SEM 2 EXAM MARK BREAKDOWN | **NAME**

ALGEBRA		**TRIG**		**GEOMETRY**		**FUNCTIONS**		**CIRCLE GEOM**	
1e(i)	/2	4a(iii)	/2	2a	/6	1b	/2	3b	/4
4a(i)	/1					1c	/3		
4c	/3					2b	/2		
total	/6	**total**	/2	**total**	/6	3a	/3		
						5a(i)	/2		
						5a(iii)	/2		
						5a(iv)	/3		
						total	/17	**total**	/4

CALCULUS		**COORD GEOM**		**QUAD FUNCTION**		**SUMMARY**	
1d	/2	1a	/2	3c	/2	ALGEBRA	/6
1e(ii)	/2	4d	/3	3d	/4	TRIG	/2
2c(i)	/1			4b	/2	GEOMETRY	/6
2c(ii)	/2					FUNCTIONS	/17
2c(iii)	/2					CIRCLE GEOM	/4
4a(ii)	/2					CALCULUS	/17
5a(ii)	/2					COORD GEOM	/5
5b	/4					QUAD FUNCTION	/8
total	/17	**total**	/5	**total**	/8		
						TOTAL	**/65**

YEAR 11 Ext 1 Semester 2 EXAM 2015

Solutions

Question 1

(a) $\begin{matrix} x_1 & y_1 \\ (-1, & -2) \end{matrix}$ $\begin{matrix} x_2 & y_2 \\ (9, & 3) \end{matrix}$

$\begin{matrix} 4 & : & 1 \\ m & & n. \end{matrix}$

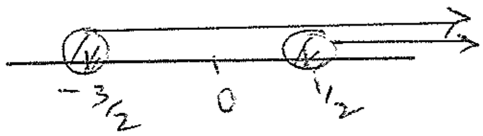
$$x = \frac{1(-1)+4(9)}{5}$$
$$= \frac{35}{5}$$
$$= 7$$

$$y = \frac{1(-2)+4(3)}{5}$$
$$= \frac{10}{5}$$
$$= 2$$

(b) $f(x) = \sqrt{2x+3} + \sqrt{x-1/2}$

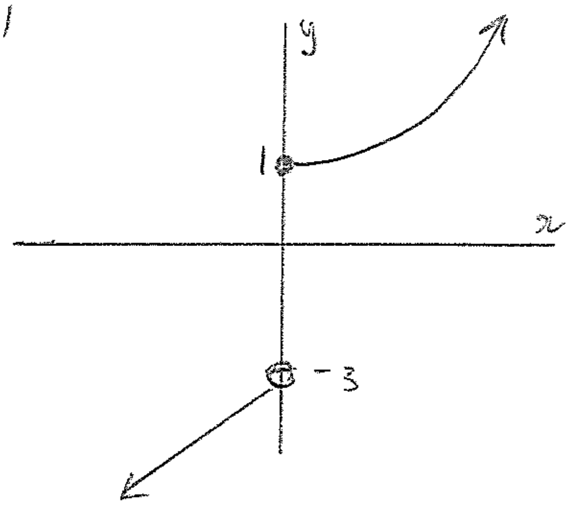
$2x+3 \geq 0 \qquad x - 1/2 \geq 0$

$x \geq -3/2 \qquad x \geq 1/2$



D: $x \geq 1/2$.

(c)



(d) $y = 3x^2 - mx + 1$

$y' = 6x - m$

$2 = 6(-1) - m$

$m = 8$

(e) (i) $\dfrac{1}{\sqrt{x+h}+\sqrt{x}} \times \dfrac{\sqrt{x+h}-\sqrt{x}}{\sqrt{x+h}-\sqrt{x}}$

$$= \frac{\sqrt{x+h}-\sqrt{x}}{x+h-x}$$
$$= \frac{\sqrt{x+h}-\sqrt{x}}{h}.$$

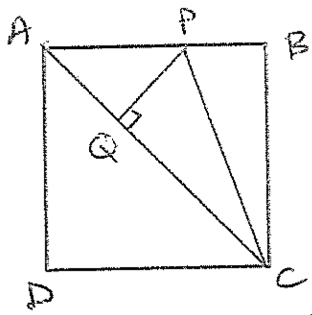
(ii) $\dfrac{\sqrt{x+h}-\sqrt{x}}{h}.$

$$= \frac{1}{\sqrt{x+h}+\sqrt{x}}$$

$$\lim_{h \to 0} = \frac{1}{\sqrt{x}+\sqrt{x}}$$
$$= \frac{1}{2\sqrt{x}}$$

Question 2

a) (i)



In $\triangle PBC$ and $\triangle PQC$

$\angle PQC = 90^\circ$ (given $PQ \perp AC$)

(ii) $\angle PBC = 90^\circ$ (prop of square)

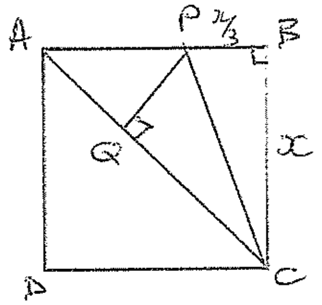
$\therefore \angle PQC = \angle PBC$

$\angle QCP = \angle PCB$ (given)

PC is common side

$\triangle PBC \equiv \triangle PQC$ (A.A.S)

(iii)



Area $\triangle ABC = \frac{1}{2}x^2$

Area $\triangle PBC = \frac{1}{2} \times x \times \frac{x}{3}$

$= \frac{x^2}{6}$

Area $\triangle PBC$ = Area $\triangle PQC$ (cong $\triangle$s)

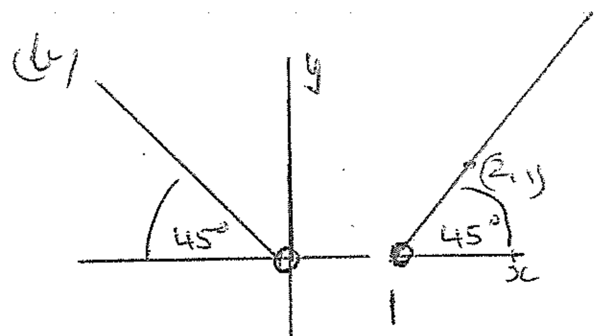
Area $\triangle ABC$ − 2Area $\triangle PBC$

$\frac{1}{2}x^2 - 2\left[\frac{x^2}{6}\right]$

$= \frac{1}{2}x^2 - \frac{x^2}{3}$

$= \frac{x^2}{6}$ cm^2

(b)



Note not necessarily on x axis

(c) $y = \frac{1}{3}x^3 - 2x^2 + 6$

(i) $y' = x^2 - 4x$

(ii) $y = \dfrac{(4x+3)^3}{x+1}$

$$y' = \frac{(x+1).3(4x+3)^2.4 - (4x+3)^3.1}{(x+1)^2}$$

$$= \frac{(4x+3)^2[12(x+1) - (4x+3)]}{(x+1)^2}$$

$$= \frac{(4x+3)^2[8x+9]}{(x+1)^2}$$

(iii) $y = (4x-1)(2x+4)^{1/2}$

$$y' = (2x+4)^{1/2}.4 + (4x-1).\tfrac{1}{2}(2x+4)^{-1/2}$$

$$= 4(2x+4)^{1/2} + \frac{4x-1}{(2x+4)^{1/2}}$$

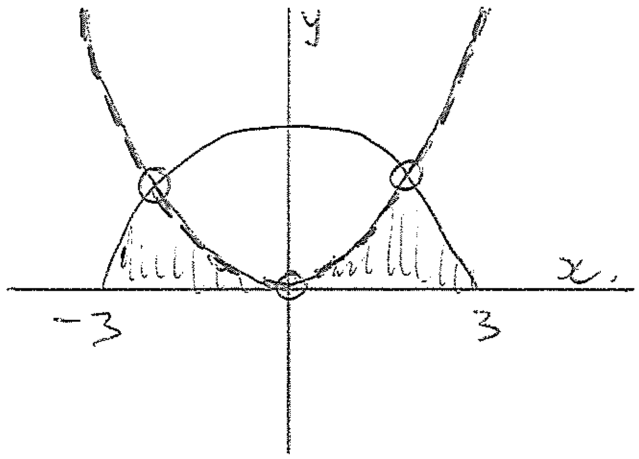
$$= \frac{4(2x+4) + 4x - 1}{\sqrt{2x+4}}$$

$= (12x+15)/\sqrt{2x+4}$

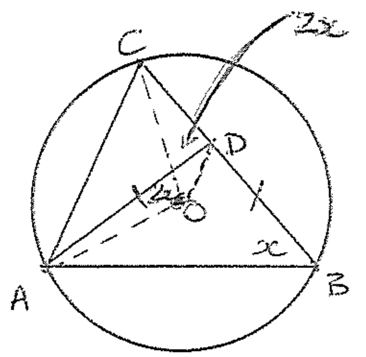
$\dfrac{3(4x+5)}{\sqrt{2x+4}}$

Question 3.

(a)



(b)



(i) $\angle AOC = 2x$

Angle at the centre twice the angle at the circ standing on same chord AC or arc AC

(ii) Join OD

$\angle DAB = x$ base angles of Isos $\triangle ADB$ equal

$\angle CDA = 2x$ ext $\angle$ of $\triangle ABD$ = to 2 opp int $\angle$'s

$\angle CDA = \angle COA$
$= 2x$

CAOD cyclic quad angles at circ standing on same chord are equal.

(d) $x + \frac{1}{x} = 5$

$x + \frac{1}{x} - 5 = 0$

$x^2 - 5x + 1 = 0$

(i) 5

(ii) 5

(iii) $\alpha^2 + \beta^2$

$= (\alpha + \beta)^2 - 2\alpha\beta$

$= 5^2 - 2(1)$

$= 23$

(c) $4^x - 6(2^x) + 8 = 0$

$(2^x)^2 - 6(2^x) + 8 = 0$

Let $m = 2^x$

$m^2 - 6m + 8 = 0$

$(m-4)(m-2) = 0$

$m = 4, \quad m = 2$

$2^x = 4 \qquad 2^x = 2$

$x = 2, \quad x = 1$

Question 4

(a) $y = x^2 - 4x$

(i) when $y = 0$

$x(x-4) = 0$

$x = 0 \quad , \quad x = 4$

(ii) $y' = 2x - 4$

at $x = 0$

$M_T = -4$

at $x = 4$

$M_T = 4$

(iii) $\tan\phi = \left| \dfrac{m_1 - m_2}{1 + m_1 m_2} \right|$

$= \left| \dfrac{4 - -4}{1 + 4(-4)} \right|$

$\tan\phi = \dfrac{8}{15}$

$\phi = 28^0$

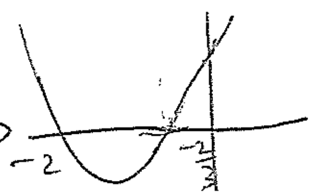
(b) $K^2 - 4(1)(K+1)^2 > 0$

$K^2 - 4(K^2 + 2K + 1) > 0$

$K^2 - 4K^2 - 8K - 4 > 0$

$\therefore \; 3K^2 + 8K + 4 < 0$

$(3K+2)(K+2) < 0$



$-2 < K < -2/3$.

Question 4

(c) $(5x-2y+3)+k(2x+6y-7)=0$

$5x-2y+3+2kx+6ky-7k=0$

i) $-y(2-6k)+x(5+2k)+3-7k=0$

$y(2-6k) = (5+2k)x+3-7k$

$y = \left(\frac{5+2k}{2-6k}\right)x + \frac{3-7k}{2-6k}$

$\therefore M = \frac{5+2k}{2-6k}.$

(ii) $2y-x+4=0$

$2y = x-4$

$y = x/2 - 2$

$M = 1/2$

$\frac{5+2k}{2-6k} = \frac{1}{2}$

$10+4k = 2-6k$

$8 = -10k$

$k = -4/5.$

Required eqn

$5x-2y+3-\frac{4}{5}(2x+6y-7)=0$

$25x-10y+15-8x-24y+28=0$

$17x-34y+43=0$

(d) $5x-12y-52=0$

$a=5,\ b=-12,\ c=-52$

$(0,0)$

P.D. $\frac{|5(0)-12(0)-52|}{\sqrt{5^2+(-12)^2}}$

$= \frac{52}{13}$

$= 4.$

now radius of circle is 4 and ⊥ dist from centre to point of contac is 4

$\therefore 5x-12y=52$ tangent to circle.

Question 5

$$f(x) = \frac{2x-2}{x^2+x-2}$$

(a)

(i) $f(x) = \dfrac{2x-2}{(x+2)(x-1)}$

Discontinuous at $x = -2$, $x = 1$

(ii) $\lim_{x \to -2} \dfrac{2(x-1)}{(x+2)(x-1)}$

$= 2/0$

$\to \infty$

$\lim_{x \to 1} \dfrac{2(x-1)}{(x+2)(x-1)}$

$= 2/3$

(iii) Vert asymptote

at $x = -2$

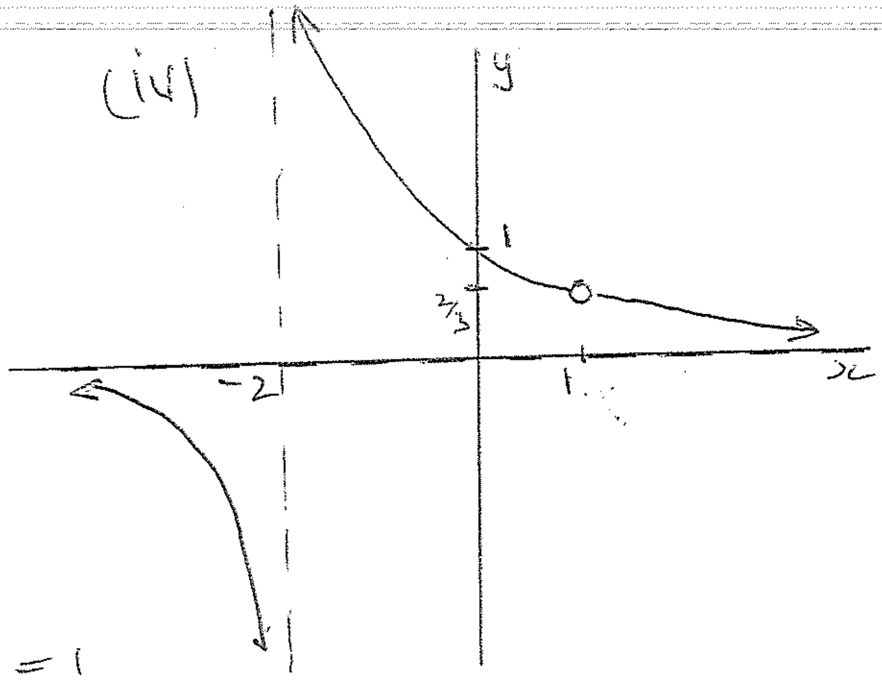
from (ii)

Horizontal

$\lim_{x \to \infty} \dfrac{\frac{2x}{x^2} - \frac{2}{x^2}}{\frac{x^2}{x^2} + \frac{x}{x^2} - \frac{2}{x^2}}$

$= 0$. $\therefore$ x axis.

(iv)



(b) $y = x^2 - x - 2$

$y = x + K$

(i) $x^2 - x - 2 = x + K$

$x^2 - x - 2 - x - K = 0$

$x^2 - 2x - 2 - K = 0$.

$\Delta = 0$ for one soln.

$\Delta = (-2)^2 - 4(1)(-2-K)$

$0 = 4 + 8 + 4K$.

$0 = 12 + 4K$

$K = -3$.

(ii) $x^2 - 2x - 2 + 3 = 0$

$x^2 - 2x + 1 = 0$

$(x-1)^2 = 0$

$x = 1$ $y = -2$