Caringbah High School

Year 11 2017

Mathematics Extension 1

Preliminary Course

Semester 2 Exam

General Instructions

- Reading time – 3 minutes.
- Working time – 90 minutes.
- Write using black or blue pen.
- Board-approved calculators may be used.
- Relevant mathematical reasoning and/or calculations **MUST** be shown.
- Marks may not be awarded for answers which do not show relevant working and/or reasoning.
- A reference sheet is provided.

Total marks – 65

Write your answers in the answer booklets provided.

Start each question in a new booklet.

Ensure your name is clear.

Name: _____ **Class:** _____

Question 1: (13 marks)

Marks

a) Solve $x - 8 \leq \frac{20}{x}$. **3**

b) Sketch the graph of the function $y = 3^{x+1}$ **2**

c) Find the exact value of cos 165°. **1**

d) Find the derivative of the following expressions, expressing your answer in simplest form.

i) $5x^3 - 3x^2 + 2$ **1**

ii) $\sqrt[3]{x}$ **1**

iii) $(5x - 2)^3$ **1**

iv) $(3x + 1)(2x - 3)^4$ **2**

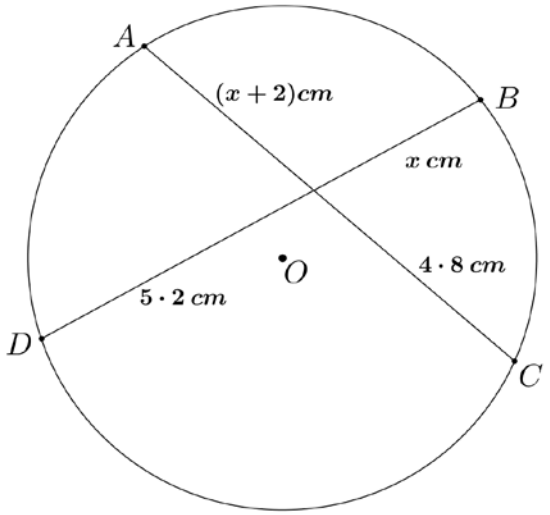
v) $\frac{2x + 1}{\sqrt{x - 3}}$ **2**

Question 2: (13 marks)

Marks

a) Factorise $p^6 + 8$ **1**

b) Let $f(x) = 2x^2 - 3x$. Find $f'(x)$ by differentiating from first principles. **2**

c) 

Find the value for x, giving reasons. **2**

d) The point B (5, -2) divides the interval joining the points A (3, 7) and $C(x, y)$ internally in the ratio 2:3. **2**

Find the coordinates of point C.

e) Sketch $y = \dfrac{x+1}{x-2}$, clearly showing any asymptotes and intercepts on axes. **3**

f) Solve $\sec^2\theta = 7\tan\theta - 1$ for $0° \leq \theta \leq 180°$, to nearest degree. **3**

Question 3: (13 marks)

Marks

a) State the domain and range of the graph of $y = \sqrt{x-9}$. **2**

b) Find the equation of the straight line that passes through the intersection of the lines **3**

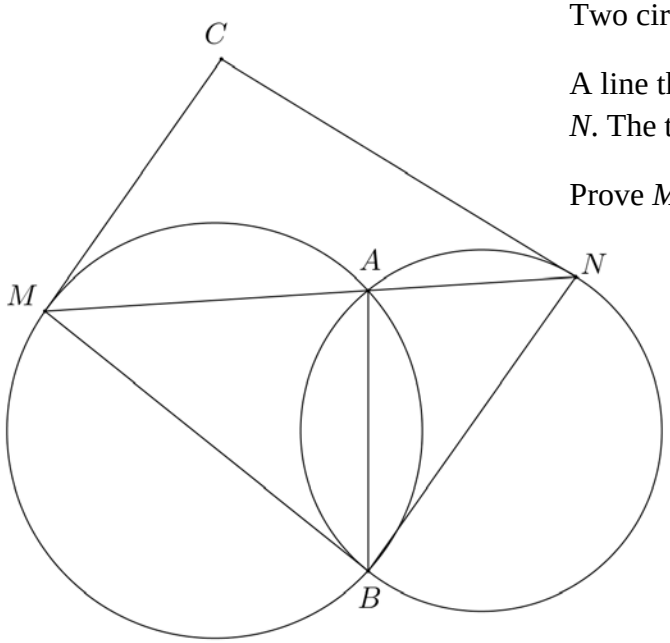
$$x + 3y - 2 = 0 \text{ and } 2x - y - 5 = 0$$

and also passes through the point (5, -2). Express your answer in general form.

c) Find the values for k such that the line $12x + 5y + k = 0$ is a tangent to the circle **3**

$(x-1)^2 + (y+5)^2 = 9$.

d) **3**



Two circles intersect at A and B.

A line through A cuts the circles at M and N. The tangents at M and N meet at C.

Prove M, N, C and B are concyclic.

e) i) Show that $x + 3 - \dfrac{4}{x+1} \equiv \dfrac{x^2 + 4x - 1}{x+1}$ **1**

ii) Hence state the oblique asymptote of the graph $y = \dfrac{x^2 + 4x - 1}{x+1}$ **1**

Question 4: (13 marks)

Marks

a) α and $\beta$are roots of the equation $2x^2 - 3x + 5 = 0$. Find the values of:

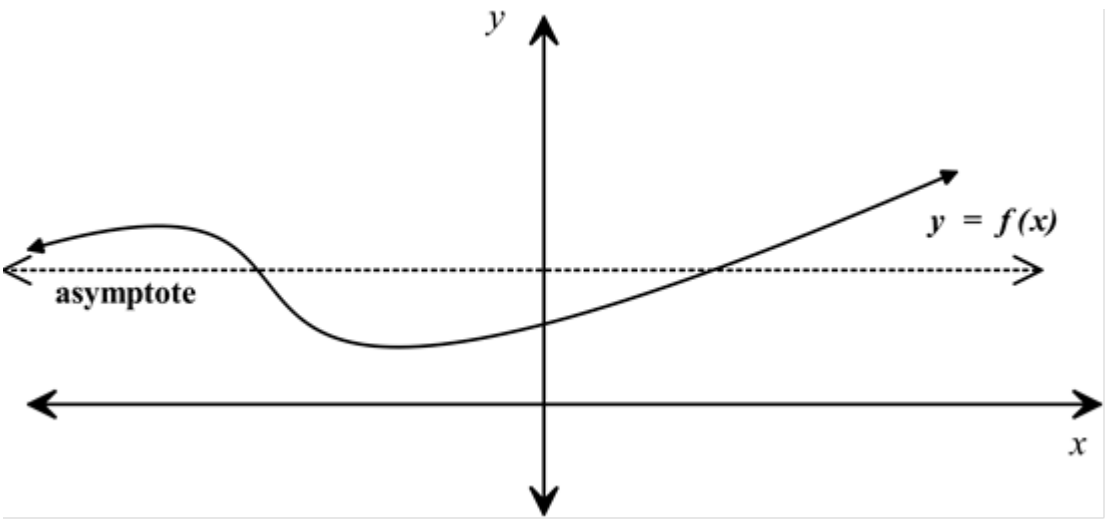
i) $\alpha\beta$ **1**

ii) $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$ **2**

iii) $(\alpha - \beta)^2$ **2**

iv) $\alpha^3 + \beta^3$ **2**

b) Copy or trace the function $y = f(x)$ into your answer booklet. Draw a neat sketch of the derivative function $y = f'(x)$, showing its relationship to $y = f(x)$. **3**



c) Shade the region on the Cartesian number plane where the following inequalities hold simultaneously. **3**

$x^2 + y^2 < 4$ and $-1 < x \leq 2$.

Question 5: (13 marks)

a) Find the point that divides the interval between the points $A(-3, -1)$ and $B(5, 9)$ externally in the ratio 5:3. **2**

b) Find the equation of the normal to the curve $y = x^3 - 2x^2 + 4x + 1$ at the point $(2, 9)$. **3**

c) i) By solving simultaneously, show that the two parabolas **2**

$y = 4x^2 + 6x - 1$ and $y = x^2 + 2x + 3$ intersect at the point (-2, 3).

ii) Find the acute angle between these two parabolas at this point of intersection. **2**

Express your answer correct to the nearest minute.

d) Express $x^4 - 2x^3 + 9x^2 - 8x - 20$ in the form $(x^2 - x)^2 + b(x^2 - x) + c$. **4**

Hence solve $x^4 - 2x^3 + 9x^2 - 8x - 20 = 0$.

End of Exam

2017 Maths Ext 1
Year 11 Sem 2

Question 1

a) $x \neq 0 \quad x^2 - 8x - 20 \leq 0$

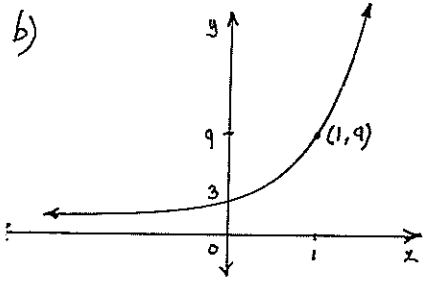
$(x-10)(x+2) \leq 0$

check $x=-3$ T $\quad x=1$ T

$x=-1$ F $\quad x=11$ F

$x \leq -2$ or $0 < x \leq 10$

b)



c) $\cos 165° = \cos 120° \cos 45 - \sin 120 \sin 45$

$= -\frac{1}{2} \times \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}$

$= -\frac{1+\sqrt{3}}{2\sqrt{2}}$

d) i) $f'(x) = 15x^2 - 6x$

ii) $f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$ or $\frac{1}{3\sqrt[3]{x^2}}$

iii) $f'(x) = 15(5x-2)^2$

iv) $f'(x) = 8(3x+1)(2x-3)^3 + 3(2x-3)^4$

$= (2x-3)^3(24x+8+6x-9)$

$= \frac{(2x-3)^3(30x-1)}{}$

v) $f'(x) = \frac{(x-3)^{\frac{1}{2}} \times 2 - (2x+1) \times \frac{1}{2}(x-3)^{-\frac{1}{2}}}{x-3}$

$= \frac{\frac{1}{2}(x-3)^{-\frac{1}{2}}\left[4(x-3) - (2x+1)\right]}{x-3}$

$= \frac{2x-13}{2(x-3)^{\frac{3}{2}}}$

Question 2

a) $p^6 + 8 = (p^2+2)(p^4 - 2p^2 + 4)$

b) $f(x) = 2x^2 - 3x$

$f(x+h) = 2(x+h)^2 - 3(x+h)$

$= 2x^2 + 4xh + 2h^2 - 3x - 3h$

$f'(x) = \lim_{h \to 0} \frac{(2x^2+4xh+2h^2-3x-3h) - (2x^2-3x)}{h}$

$= \lim_{h \to 0} \frac{4xh + 2h^2 - 3h}{h}$

$= \lim_{h \to 0} 4x + 2h - 3$

$= 4x - 3$

c) $(x+2) \times 4.8 = 5 \cdot 2x$ (product of intercepts of chords)

$0.4x = 9.6$

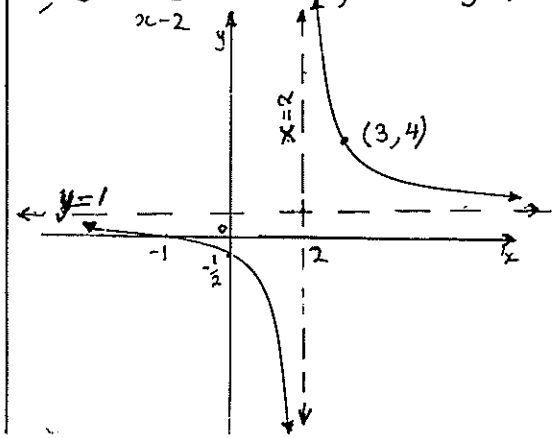
$x = 24$

d) $\frac{2x+3\times3}{5} = 5 \qquad \frac{2y+3\times7}{5} = -2$

$2x = 16 \qquad 2y = -31$

$x = 8 \qquad y = -15\frac{1}{2}$

e) $y = \frac{x+1}{x-2} \qquad x \neq 2$, as $x \to \infty \; y \to 1$



f) $\sec^2\theta = 7\tan\theta - 1$

$1 + \tan^2\theta = 7\tan\theta - 1$

$\tan^2\theta - 7\tan\theta + 2 = 0$

let $x = \tan\theta$

$x^2 - 7x + 2 = 0$

$x = \frac{7 \pm \sqrt{49-8}}{2}$

$= \frac{7 \pm \sqrt{41}}{2}$

$\theta = \tan^{-1}\left(\frac{7 \pm \sqrt{41}}{2}\right)$

$\theta = 82°$ or $17°$

Question 3

a) D: $x \geq 9$

R: $y \geq 0$

b) $L_1 + kL_2 = 0 \qquad (5,-2)$

$(x_1+3y_1-2) + k(2x_1-y_1-5) = 0$

$(5-6-2) + k(10+2-5) = 0$

$-3 + 7k = 0$

$k = \frac{3}{7}$

$(x+3y-2) + \frac{3}{7}(2x-y-5) = 0$

$7x + 21y - 14 + 6x - 3y - 15 = 0$

$13x + 18y - 29 = 0$

c) centre $(1,-5)$ radius $= 3$.

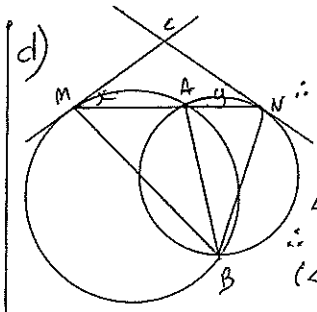
$d = \frac{|12\times1 + 5\times(-5) + k|}{\sqrt{12^2+5^2}}$

$3 = \frac{|k-13|}{13}$

$k - 13 = 39 \quad$ or $\quad k - 13 = -39$

$k = 52 \quad$ or $\quad k = -26$

d)



Let $\angle CMN = x$

$\therefore \angle MBA = x$ ($\angle$ in alt. segment)

Let $\angle CNA = y$

$\therefore \angle NBA = y$ ($\angle$ in alt. segment)

So $\angle MBN = x + y$ (adjacent angles)

$\angle MCN = 180 - (x+y)$ ($\angle$ sum of $\triangle CMN$)

$\therefore \angle MBN + \angle MCN = 180$

$\therefore$ MNCB are concyclic.

(opp $\angle$s of cyclic quad are supplementary)

e) i) $x + 3 - \frac{4}{x+1} = \frac{(x+3)(x+1)}{x+1} - \frac{4}{x+1}$

$= \frac{x^2+4x+3-4}{x+1}$

$= \frac{x^2+4x-1}{x+1}$

ii) As $x \to \infty \quad \frac{4}{x+1} \to 0$

$\therefore$ asymptote is $y = x+3$

Question 4

a) $\alpha + \beta = \frac{3}{2} \qquad \alpha\beta = \frac{5}{2}$

i) $\alpha\beta = \frac{5}{2}$

ii) $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\beta^2 + \alpha^2}{\alpha^2\beta^2}$

$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$

$= \frac{(\frac{3}{2})^2 - 2\times\frac{5}{2}}{(\frac{5}{2})^2}$

$= -\frac{11}{25}$

iii) $(\alpha-\beta)^2 = \alpha^2+\beta^2-2\alpha\beta$

$= (\alpha+\beta)^2 - 4\alpha\beta$

$= \left(\frac{3}{2}\right)^2 - 4\times\frac{5}{2}$

$= -\frac{31}{4}$

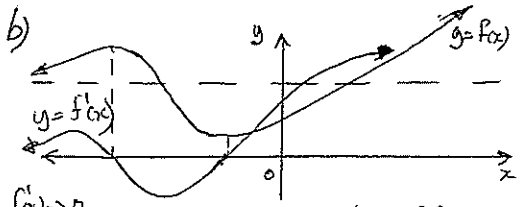
iv) $\alpha^3+\beta^3 = (\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$

$= (\alpha+\beta)((\alpha^2+\beta^2)-\alpha\beta)$

$= \frac{3}{2}\times\left[\left(\frac{3}{2}\right)^2 - 2\times\frac{5}{2} - \frac{5}{2}\right]$

$= -\frac{63}{8}$

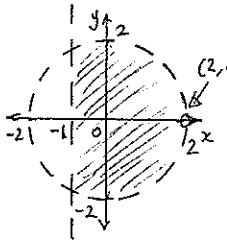
b)



$f'(x)\to 0$ as $x\to-\infty$

as $x\to\infty$ $f(x)$ is straight

c)



(2,0) is NOT included.

Question 5

$x = \frac{5\times5-3\times3}{5-3}$ $y = \frac{5\times9-3\times1}{5-3}$

$= \frac{25+9}{2}$ $= \frac{48}{2}$

P(17, 24)

b) $y = x^3-2x^2+4x+1$

$y' = 3x^2-4x+4$

$x=2$

$m_T = 12-8+4$

$= 8$

$m_N = -\frac{1}{8}$

Eqn of normal

$(y-9) = -\frac{1}{8}(x-2)$

$8y-72 = -x+2$

$x+8y-74=0$

c) i) $4x^2+6x-1 = x^2+2x+3$

$3x^2+4x-4=0$

$\frac{(3x+6)(3x-2)}{3} = 0$

$x=-2$ or $x=\frac{2}{3}$

at $x=-2$ $y = (-2)^2+2\times(-2)+3$

$= 4-4+3$

$= 3$

ii) $\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$

$y_1' = 8x+6$

$m_1 = -10$

$y_2' = 2x+2$

$m_2 = -2$

$\tan\theta = \left|\frac{-10+2}{1+20}\right|$

$\therefore \theta = 20^\circ 51'$

d) $x^4-2x^3+9x^2-8x-20 \equiv (x^2-x)^2+b(x^2-x)+c$

Let $x=0$ LHS $=-20$ RHS $=c$ $\therefore c=-20$

Let $x=2$ LHS $=0$ RHS $=4+2b+c=0$

$\therefore b=8$

$(x^2-x)^2+8(x^2-x)-20=0$

$u^2+8u-20=0$

$(u+10)(u-2)=0$ $u=2$ $u=-10$

$x^2-x=-10$ or $x^2-x=2$

$x^2-x+10=0$ $x-x-2=0$

No solution $x=-1$, $x=2$