Caringbah High School

Year 11 2018

Mathematics Extension 1

Preliminary Course

Assessment Task 3

General Instructions

- Reading time – 5 minutes
- Working time – 90 minutes
- Write using black pen.
- NESA calculators may be used.
- The approved board of studies reference sheet has been provided.
- In questions11 - 15, show relevant mathematical reasoning and/or calculations.
- Marks may not be awarded if sufficient reasons and/or working have not been shown.

Total marks – 65

Section I **10 marks**

Attempt Questions 1-5

Mark your answers on the Multiple Choice answer sheet provided. You may detach the sheet and write your name on it.

Section II **55 marks**

Attempt Questions 11-15

Write your answers in the answer booklets provided.

Ensure your name or student number is clearly visible.

Name: _____ **Class:** _____

Marker's Use Only							
Section I	**Section II**					**Total**	
Q 1-10	**Q11**	**Q12**	**Q13**	**Q14**	**Q15**		
/10	/11	/11	/11	/11	/11	/65	%

Section I

5 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10

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1. Given that *ABCD* is a parallelogram, consider the following statements.

I: If diagonals *AC* and *BD* are perpendicular then *ABCD* is a rhombus.
II: If diagonals *AC* and *BD* are equal then *ABCD* is a square.

Which of the statements I and II are correct?

(A) I only
(B) II only
(C) Both I and II
(D) I and II are both incorrect

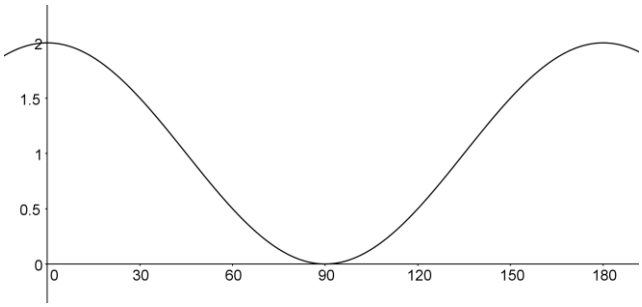
2. The centre of a circle has coordinates (–3, 4). If the circle passes through the point (1, 2), then the diameter has length

(A) $2\sqrt{2}$
(B) $4\sqrt{2}$
(C) $2\sqrt{5}$
(D) $4\sqrt{5}$

3. What are the coordinates of the focus of the parabola $x^2 = 2(y - 1)$

(A) $\left(0, \frac{1}{2}\right)$
(B) $\left(0, \frac{3}{2}\right)$
(C) $\left(\frac{1}{2}, 0\right)$
(D) $\left(\frac{3}{2}, 0\right)$

4. A possible equation for this curve is:



(A) $y = -1 + \cos 2x$ (B) $y = 1 + \cos 2x$

(C) $y = \cos(2x + 1)$ (D) $y = 1 - \cos 2x$

5. What is the acute angle between the lines $y = 7 - 4x$ and $2x - 3y - 6 = 0$, to the nearest degree?

(A) 26° (B) 48°

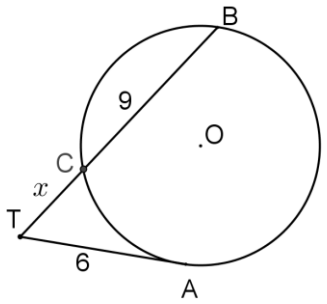
(C) 70° (D) 75°

6. An expression for the general solution to the trigonometric equation $\tan 3x = -\sqrt{3}$, where n is any integer and x is in degrees is:

(A) $x = 60n - 20$ (B) $x = 60n - 60$

(C) $x = 60n + 60$ (D) $x = 60n + 20$

7. The line *TA* is a tangent to the circle at *A*. *TB* is a secant cutting the circle at *B* and *C*.



$TA = 6\ cm$

$BC = 9\ cm$

$TC = x\ cm$

What is the value of *x*?

(A) 2 (B) 3

(C) 4 (D) 12

8. The solution to the inequality $6 - x - x^2 \leq 0$ is:

(A) $x \leq -3$ or $x \geq 2$ (B) $-3 \leq x \leq 2$

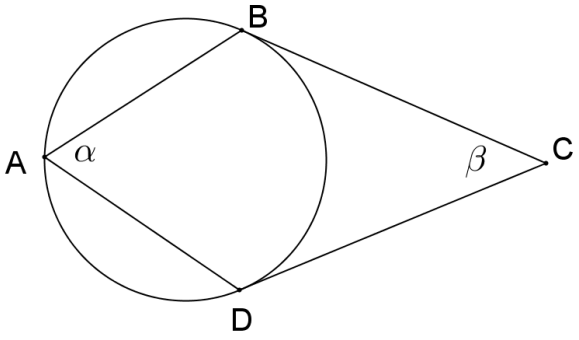
(C) $x \leq -2$ or $x \geq 3$ (D) $-2 \leq x \leq 3$

9. What is the correct condition for $y = mx + b$ to be a tangent to $x^2 = 4ay$?

(A) $am - b = 0$ (B) $am^2 - b = 0$

(C) $am + b = 0$ (D) $am^2 + b = 0$

10. In the diagram below, *BC* and *DC* are tangents.



Which statement below is correct?

(A) $\alpha + \beta = 180°$ (B) $2\alpha + \beta = 180°$

(C) $\alpha + 2\beta = 180°$ (D) $2\alpha - \beta = 180°$

End of Section I

Section II

60 marks
Attempt Questions 11–15
Allow about 1 hour 15 minutes for this section

Answer each question in a SEPARATE writing booklet.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (11 marks)

(a) If α and β are roots of the equation $3x^2 - 6x + 12 = 0$, find the value of

i) $\alpha^2\beta + \alpha\beta^2$ **1**

ii) $\alpha^2 + \beta^2$ **1**

(b) For the parabola $2y = x^2 - 4x$,

i) Find the coordinates of the vertex. **1**

ii) Find the coordinates of the focus. **1**

iii) Sketch the curve, clearly showing the vertex, focus and the directrix. **1**

(c) Differentiate $f(x) = x^2 + 3x - 1$ from first principles. **2**

(d) Find $\lim\limits_{n\to\infty} \dfrac{3n^2 + 2n - 1}{5n^2 + 4}$ **1**

(e) Find the equation of the normal to the curve $y = \dfrac{8}{x+3}$ at the point $(1, 2)$. **3**

End of Question 11

Question 12 (11 marks) Use a SEPARATE writing booklet.

(a) Solve the inequality $\frac{2x-3}{x-4} \geq -3$ **2**

(b) Shade the region on the number plane where the following inequalities hold simultaneously. (Draw a $\frac{1}{3}$ page diagram.) **2**

$y \leq x^2 - 4$ and $y > 4 - x^2$

(c) Find values for b and c such that $x^2 - x \equiv (x+3)^2 + bx + c$. **2**

(d) Express $\sqrt{3}\cos\theta - \sin\theta$, in the form $R\cos(\theta + \varphi)$, where $R > 0$ and $0° < \varphi < 90°$. **2**

(e) $f(x) = \begin{cases} |x+1|, & \text{for } x < 0 \\ a^x + b, & \text{for } x \geq 0 \end{cases}$

i) Find the values of a and b if $f(0) = 4$ and $f(1) = 7$. **2**

ii) Evaluate $f(-3) + f(2)$ **1**

End of Question 12

Question 13 (11 marks) Use a SEPARATE writing booklet.

(a) Let A be the point (–3, 8) and let B be the point (5, –6). Find the coordinates of the point P that divides the interval AB externally in the ratio 3:2. **1**

(b) Find the equation of the locus of a point, $P(x, y)$, that moves such that its distance from the point $C(2, -3)$ and its distance from the point, $D(-3, 7)$ is in the ratio 2:3. **2**

(c) Evaluate $\lim_{x \to 2} \dfrac{x^2 - 4}{x - 2}$ **1**

(d) Find the natural domain of the function $f(x) = \sqrt{2x^2 - 9x + 4}$ **2**

(e) Consider the function $f(x) = \dfrac{9 - x^2}{x^2 - 4}$

i) Show that the function is even. **1**

ii) State the equations of the asymptotes. **2**

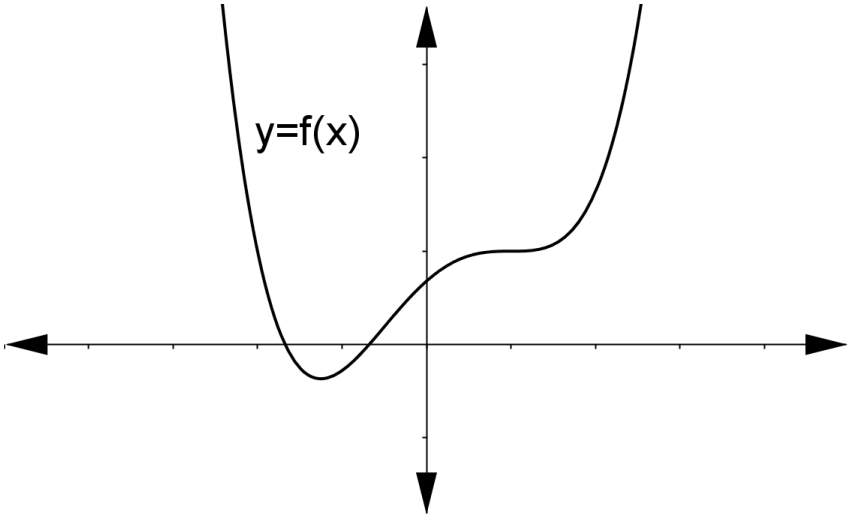
iii) Sketch the graph of $y = f(x)$. (Draw a $\frac{1}{3}$ page diagram) **2**

End of Question 13

Question 14 (11 marks) Use a SEPARATE writing booklet.

(a) The curves $y = 2x^3 + 3$ and $y = x^2 – 2x + 6$ meet at the point (1, 5). Find the acute angle between these curves at this point of intersection. **3**

(b) The graph of $y = f(x)$ is given below **2**



Copy this diagram of $y = f(x)$ onto your answer sheet.
On the same set of axes or directly below it draw a neat sketch of $y = f'(x)$.

(c) Form a Cartesian equation by eliminating the parameter, θ, from these parametric equations. **2**

$$x = 3\tan\theta \qquad y = 2\sec\theta$$

(d) Consider the expression $(m – 1)x^2 + (m + 1)x + (m – 1)$

i) Show that the discriminant in terms of m is **1**

$$\Delta = –3m^2 + 10m – 3$$

ii) Hence find the values of m for which the expression is positive for all values of x. **3**

End of Question 14

Question 15 (11 marks) Use a SEPARATE writing booklet.

(a) Show that $\sin A\cos A\cos 2A = \frac{1}{4}\sin 4A$ **1**

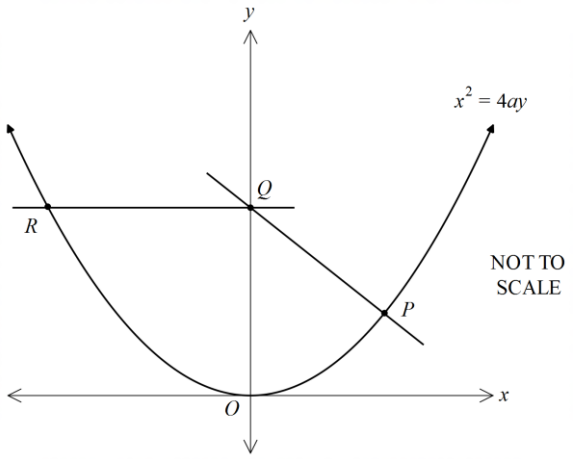
(b) i) Solve the equation $\cos 2\theta = 0.5$, for $0° \le \theta \le 360°$ **2**

ii) Solve the equation $\sin 2\theta = \sqrt{2}\cos\theta$, for $0° \le \theta \le 360°$ **2**

(c) Find the equation of the tangent to the curve $y = 2x^3 - 5x + 1$ at the point $A(2, 7)$. **2**

x

(d) The diagram below shows the graph of the parabola $x^2 = 4ay$.
The normal to the parabola at the variable point $P(2ap, ap^2)$, $p > 0$, cuts the y-axis at Q.
Point R lies on the parabola.



i) Find the coordinates of R given that QR is parallel to the x-axis and $\angle PQR > 90º$. **1**

ii) Let M be the midpoint of RQ. Find the Cartesian equation of the locus of M. **3**

End of Question 15

End of Examination

Multiple Choice Answer Sheet

Name _____

Colour the oval A, B, C or D that best answers the question.

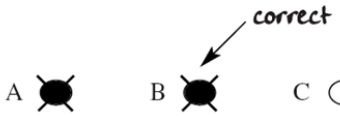
Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.



1. A ○ B ○ C ○ D ○
2. A ○ B ○ C ○ D ○
3. A ○ B ○ C ○ D ○
4. A ○ B ○ C ○ D ○
5. A ○ B ○ C ○ D ○
6. A ○ B ○ C ○ D ○
7. A ○ B ○ C ○ D ○
8. A ○ B ○ C ○ D ○
9. A ○ B ○ C ○ D ○
10. A ○ B ○ C ○ D ○

2018 Mathematics Ext 1 AT3

1) A 2) D 3) B 4) B 5) C
6) A 7) B 8) A 9) D 10) B

Question 11

a) i) $\alpha+\beta=2 \quad \alpha\beta(\alpha+\beta)=8$

$\alpha\beta=4$

ii) $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=-4$

b) $2y=(x^2-4x+4)-4$

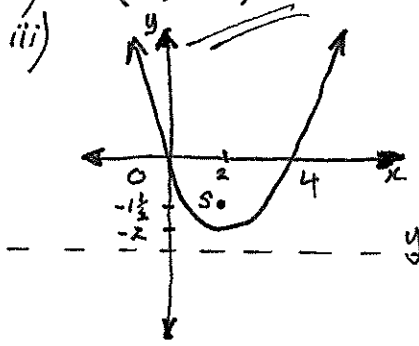
$2(y+2)=(x-2)^2 \quad 4a=2$

$a=\frac{1}{2}$.

i) $V(2,-2)$

ii) $S(2,-1\frac{1}{2})$

iii)



c) $f(x)=x^2+3x-1$

$f(x+h)=(x+h)^2+3(x+h)-1$

$=x^2+2xh+h^2+3x+3h-1$

$\frac{dy}{dx}=\lim_{h\to 0}\frac{[(x^2+2xh+h^2+3x+3h-1)-(x^2+3x-1)]}{h}$

$=\lim_{h\to 0}\frac{2xh+h^2+3h}{h}$

$=\lim_{h\to 0}\ 2x+h+3$

$=2x+3$

d) $\lim_{n\to\infty}\frac{\frac{3n^2}{n^2}+\frac{2n}{n^2}-\frac{1}{n^2}}{\frac{5n^2}{n^2}+\frac{4}{n^2}}$

$=\frac{3}{5}$

e) $y=8(x+3)^{-1}$

$y'=-8(x+3)^{-2}$

$x=1 \quad m_T=\frac{-8}{16}$

$=-\frac{1}{2} \quad \therefore m_N=+2$

Eq'n of normal

$(y-y_1)=m(x-x_1)$

$(y-1)=2(x-1)$

$y=2x$

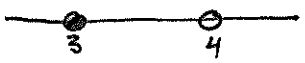
Question 12

a) $\frac{2x-3}{x-4}\geq -3 \qquad x\neq 4$

critical pts $2x-3=-3x+12$

$5x=15$

$x=3$



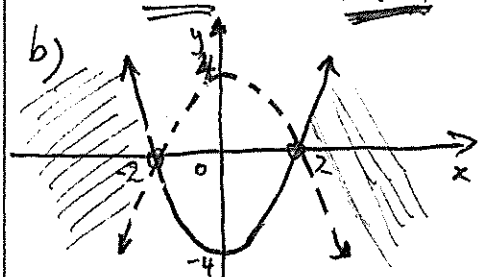
If $x=0$ LHS $=\frac{3}{4}$ ✓

$x=3\frac{1}{2}$ LHS $=-8$

$x=5$ LHS $=7$ ✓

$x\leq 3$ or $x>4$

b)



c) $x^2-x\equiv (x+3)^2+bx+c$

If $x=0$

$0\equiv 9+c \quad \therefore c=-9$

If $x=-3$

$12\equiv -3b-9 \quad \therefore b=-7$

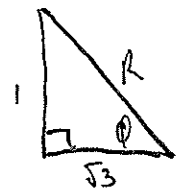
d) $\sqrt{3}\cos\theta-\sin\theta$

$R\cos\theta\cos\phi-R\sin\theta\sin\phi$

$R\cos\phi=\sqrt{3} \quad R\sin\phi=1$

$1^2+(\sqrt{3})^2=R^2 \quad \therefore R=2$

$\tan\phi=\frac{1}{\sqrt{3}} \quad \therefore \phi=30°$



$2\cos(\theta+30°)$

e) i) $x=0 \quad a^0 + b = 4 \quad \therefore b = 3$

$x=1 \quad a^1 + b = 7 \quad \therefore a = 4$

ii) $f(-3) + f(2) = 2 + (4^2 + 3)$

$= 21$

Question B

a) $A(-3, 8) \quad B(5, -6) \quad m:n = 3:-2$

$P\left(\frac{3\times 5 + 2\times -3}{3-2}, \frac{3\times -6 - 2\times 8}{3-2}\right)$

$P(21, -34)$

b) $\frac{d_1}{d_2} = \frac{2}{3}$ so $9d_1^2 = 4d_2^2$

$9\left[(x-2)^2 + (y+3)^2\right] = 4\left[(x+3)^2 + (y-7)^2\right]$

$9\left[x^2 - 4x + 4 + y^2 + 6y + 9\right] = 4\left[x^2 + 6x + 9 + y^2 - 14y + 49\right]$

$9x^2 - 36x + 36 + 9y^2 + 54y + 81 = 4x^2 + 24x + 36 + 4y^2 - 56y + 196$

$5x^2 - 60x + 5y^2 + 110y = 115$

$x^2 - 12x + y^2 + 22y = 23$

$(x^2 - 12x + 36) + (y^2 + 22y + 121) = 23 + 36 + 121$

$(x-6)^2 + (y+11)^2 = 180$

c) $\lim_{x\to 2} \frac{(x-2)(x+2)}{(x-2)} = 4$

d) $2x^2 - 9x + 4 \geq 0$

$(2x-1)(x-4) \geq 0$

$x \leq \frac{1}{2}$ or $x \geq 4$

e) i) $f(a) = \frac{9 - a^2}{a^2 - 4} \quad f(-a) = \frac{9 - (-a)^2}{(-a)^2 - 4}$

$= \frac{9 - a^2}{a^2 - 4}$

$= f(a)$

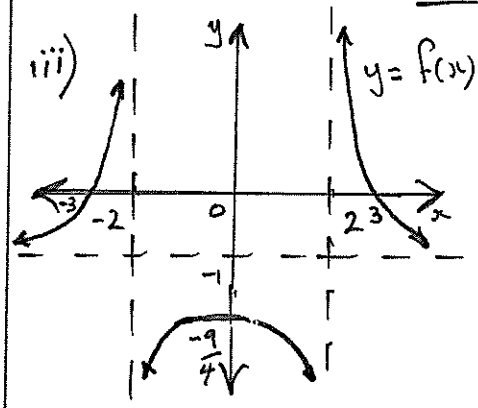
$\therefore f(x)$ is even.

ii) $x^2 - 4 \neq 0 \quad \therefore x \neq 2, x \neq -2.$

as $x \to \infty \quad \frac{\frac{9}{x^2} - 1}{1 - \frac{4}{x^2}} \to -1$

$\therefore$ asymptotes $x = \pm 2, \quad y = -1$

iii)



Question 14

a) $y = 2x^3 + 3 \qquad y = x^2 - 2x + 6$

$y' = 6x^2 \qquad y' = 2x - 2$

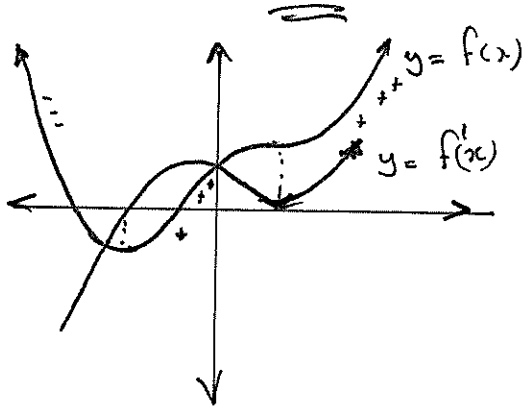
$x=1 \quad m_1 = 6 \qquad m_2 = 0$

$\tan\theta = \left|\frac{6 - 0}{1 + 6\times 0}\right|$

$= 6$

$\theta = 80^\circ 32'$

b)



c) $\tan\theta = \frac{x}{3} \qquad 1 + \tan^2\theta = \sec^2\theta$

$\sec\theta = \frac{y}{2} \qquad 1 + \frac{x^2}{9} = \frac{y^2}{4}$

$\frac{x^2}{9} - \frac{y^2}{4} = -1$

d) i) $(m-1)x^2 + (m+1)x + (m-1)$

$\Delta = b^2 - 4ac$

$= (m+1)^2 - 4(m-1)(m-1)$

$= m^2 + 2m + 1 - 4m^2 + 8m - 4$

$= -3m^2 + 10m - 3$

ii) need $a > 0$ $\quad \Delta < 0$

$m - 1 > 0$ $\quad -3m^2 + 10m - 3 < 0$

$m > 1$ $\quad 3m^2 - 10m + 3 > 0$

$(3m-1)(m-3) > 0$

$m < \frac{1}{3}$ or $m > 3$

but as $m > 1$

solution is $m > 3$

Question 15

a) $\sin 2A = 2\sin A\cos A$

so $\sin A\cos A\cos 2A = \frac{1}{2}\sin 2A\cos 2A$

$\sin 4A = 2\sin 2A\cos 2A$

so $\frac{1}{2}\sin 2A\cos 2A = \frac{1}{2}\times\frac{1}{2}\sin 4A$

$= \frac{1}{4}\sin 4A$

b) ii) $\sin 2\theta = \sqrt{2}\cos\theta \quad 0° \le \theta \le 360°$

$2\sin\theta\cos\theta - \sqrt{2}\cos\theta = 0$

$\sqrt{2}\cos\theta(\sqrt{2}\sin\theta - 1) = 0$

$\cos\theta = 0$ or $\sin\theta = \frac{1}{\sqrt{2}}$

$\therefore \theta = 90°, 270°, 45°, 135°$

c) $y = 2x^3 - 5x + 1$

$y' = 6x^2 - 5$ at $x = 2$ $m = 19$

Eqn $(y - 7) = 19(x - 2)$

$y = 19x - 31$

d) From ref. sheet

i) Eqn of normal: $x + py = ap^3 + 2ap$.

$x = 0 \quad y = ap^2 + 2a$

$\therefore x^2 = 4a^2p^2 + 8a^2$

$= 4a^2(p^2 + 2)$

$x = \pm 2a\sqrt{p^2+2}$

$\therefore R\left(-2a\sqrt{p^2+2},\ ap^2 + 2a\right)$.

ii) $M\left(-a\sqrt{p^2+2},\ ap^2 + 2a\right)$

$x = -a\sqrt{p^2+2} \quad y = a(p^2+2)$

$p^2 + 2 = \frac{y}{a}$

$x = -a\sqrt{\frac{y}{a}}$

$x^2 = \frac{a^2y}{a}$

$x^2 = ay \quad$ (for $x < 0$)

b) i) $\cos 2\theta = 0.5 \quad 0° \le \theta \le 360°$

$2\theta = 60, 300, 420, 660 \quad 0° \le 2\theta \le 720°$

$\theta = 30°, 150°, 210°, 330°$