Name: ______________________

CARINGBAH HIGH SCHOOL

2019 | **Preliminary Task 3**

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 90 minutes
- Write using black pen
- NESA approved calculators and Mathomat may be used
- A reference sheet is provided
- In Questions 11–15, show relevant mathematical reasoning and/or calculations

Total marks: 65

Section I – 10 marks (pages 2–5)

- Attempt Questions 1–10
- Allow about 10 minutes for this section

Section II – 55 marks (pages 6–10)

- Attempt Questions 11–15
- Allow about 80 minutes for this section

Marker's Use Only

Question	1 – 10	11	12	13	14	15	Total	
Mark	/10	/11	/11	/11	/11	/11	/65	%

This paper must not be removed from the examination room

Section I

10 marks
Attempt Questions 1–10
Allow about 10 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1 What is the Cartesian equation of the line with parametric equations

$x = t + 2$ and $y = 2t + 1$?

(A) $2x + y + 3 = 0$

(B) $2x + y - 3 = 0$

(C) $2x - y + 3 = 0$

(D) $2x - y - 3 = 0$

2 What is the domain of the function $f(x) = \sqrt{x^2 - 2x}$?

(A) $x > 2$

(B) $x \geq 2$

(C) $x < 0$ or $x > 2$

(D) $x \leq 0$ or $x \geq 2$

3 What is the remainder when $x^3 - 6x$ is divided by $x + 3$?

(A) -9

(B) 9

(C) $x^2 - 2x$

(D) $x^2 - 3x + 3$

4 Given $x^2 - 2px + q = 0$ has two distinct real roots. Which of the following expressions is true?

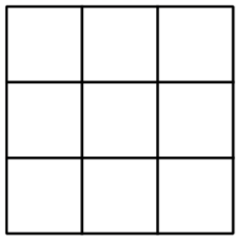
(A) $p^2 = q$

(B) $p^2 > q$

(C) $p^2 < q$

(D) $p^2 \neq q$

5 Three squares are chosen at random from the 3 × 3 grid below, and a cross is placed in each chosen square.



What is the probability that all three crosses lie in the same row, column or diagonal?

(A) $\frac{1}{28}$

(B) $\frac{2}{21}$

(C) $\frac{1}{3}$

(D) $\frac{8}{9}$

6 For what values of k does the polynomial $P(x) = (k^3 - k)x^4 + (k^2 - k)x^3 + x^2$ have degree 3?

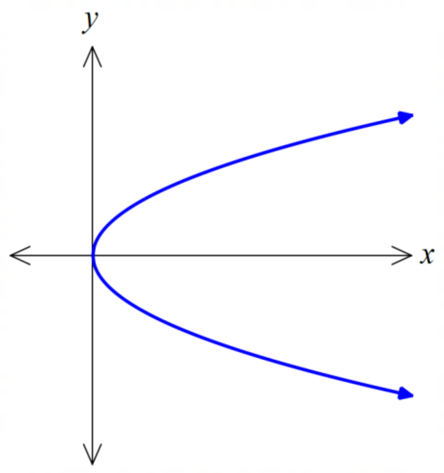
(A) $k = -1$

(B) $k = -1$ or $k = 0$

(C) $k = -1$ or $k = 1$

(D) $k = -1$, $k = 0$ or $k = 1$

7 Which equation best represents the following graph?



(A) $y = \sqrt{x}$

(B) $y = \sqrt{|x|}$

(C) $|y| = \sqrt{x}$

(D) $|y| = \sqrt{|x|}$

8 A school music band consists of 8 dancers and a singer.

The dancers are selected from 17 students in Year 9.

The singer is selected from 5 students in Year 10.

In how many ways could the band be selected?

(A) ${}^{17}C_8 + {}^{5}C_1$

(B) ${}^{17}P_8 + {}^{5}P_1$

(C) ${}^{17}C_8 \times {}^{5}C_1$

(D) ${}^{17}P_8 \times {}^{5}P_1$

9 Which group of three numbers could be the roots of the polynomial equation

$x^3 + ax^2 - 41x + 42 = 0$?

(A) 2, 3, 7

(B) 1, −6, 7

(C) −1, −2, 21

(D) −1, −3, −14

10 A store sells T-shirts in 7 different sizes. From all the shirts in the store, what is the minimum number of T-shirts that would need to be selected at random so that 2 T-shirts must be the same size?

(A) 6

(B) 7

(C) 8

(D) 9

Section II

55 marks
Attempt Questions 11–15
Allow about 80 minutes for this section

Answer each question on SEPARATE writing paper. Extra writing booklets are available.

In Questions 11 – 15, your responses should include relevant mathematical reasoning and/or calculations.

Marks

Question 11 (11 marks) Use SEPARATE writing paper.

(a) Solve the inequality $|2x-1| \leq 3$. **2**

(b) Find the number of ways in which four numerals can be chosen from the numerals 1, 2, 3, 4, 5, 6, 7, 8, 9 without repetition so that

(i) no odd numerals are chosen. **1**

(ii) at least one odd numeral is chosen. **1**

(iii) a majority of odd numerals is chosen. **2**

(c) Sketch the graph $y = |x + 1| - 2$ showing clearly the intercepts with the axes. **2**

(d) Solve $\dfrac{3x}{x-2} \geq 1$. **3**

End of Question 11

Marks

Question 12 (11 marks) Use SEPARATE writing paper.

(a) Solve the equation $\sin^2 \theta = \cos \theta + 1$ where $0 \leq \theta \leq 2\pi$. **3**

(b) Let $P(x) = x^4 - 5x^3 + 9x^2 - 7x + 2$.

(i) Given that $x = 1$ is a multiple root of the equation $P(x) = 0$, find the multiplicity of this root. You must justify your answer with relevant mathematical reasoning. **2**

(ii) Hence or otherwise factorise $P(x)$. **2**

(c) Consider the curve $y = x\sqrt{x+3}$.

(i) Show that $\dfrac{dy}{dx} = \dfrac{3x+6}{2\sqrt{x+3}}$. **2**

(ii) Hence or otherwise, find the coordinates on the curve where its tangent is parallel to the x-axis. **2**

End of Question 12

Marks

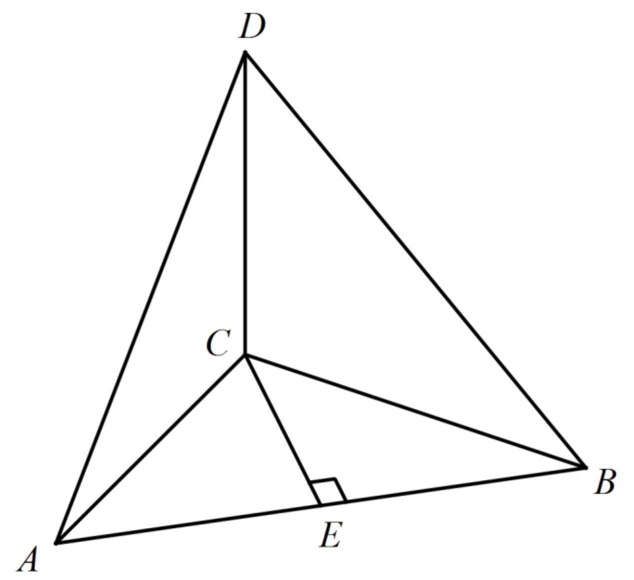
Question 13 (11 marks) Use SEPARATE writing paper.

(a) Express $\dfrac{1}{1+\sqrt{2}}+\dfrac{2}{\sqrt{2}}$ in the form $a+b\sqrt{2}$ where a and b are integers. **2**

(b) Find the equation of the inverse $f^{-1}(x)$ for $f(x)=\dfrac{1-x}{3+x}$. **2**

(c) Solve for x in $(\log_3 x)^2 = \log_3 x^2 + 8$. **3**

(d) CD is a vertical flagpole of height 10 metres. It stands with its base on horizontal ground. A and B are points on the ground due South and due East of C respectively. The angle of elevation of D from A is 45° and the angle of elevation of D from B is 30°. E is the foot of the perpendicular from C to AB.



(i) Show that $CB = 10\sqrt{3}$. **1**

(ii) Show that $\angle ABC = 30°$. **1**

(iii) Find the angle of elevation of D from E correct to the nearest minute. **2**

End of Question 13

Marks

Question 14 (11 marks) Use SEPARATE writing paper.

(a) Let $f(x) = (x-2)^2\,(x+1)^3$.

(i) Sketch the graph of $y = f(x)$ showing intercepts with the coordinate axes. **2**

(ii) Find the x value of the curve in the interval $-1 < x < 2$ where the tangent to the curve is horizontal. **3**

(b) Find the number of ways in which 2 boys and 5 girls can be arranged in a circle if:

(i) there is no restriction. **1**

(ii) the boys and girls are in separate groups. **1**

(iii) the boys are separated. **1**

(c) Consider the polynomial $P(x) = x^3 - px^2 + qx - pq$, where $q > 0$.

(i) Show that $(x-p)$ is a factor of $P(x)$. **1**

(ii) Show that the equation $P(x) = 0$ has only one real root. **2**

End of Question 14

Marks

Question 15 (11 marks) Use SEPARATE writing paper.

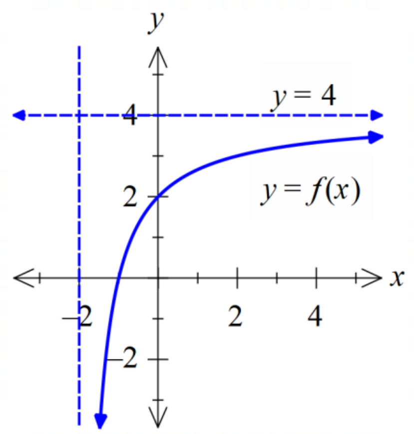
(a) Find the any points of intersection of $x^2 + y^2 = 1$ and $x + y = 1$. **2**

(b) A random variable X takes three values $a + 1$, $a + 2$ and $a - 3$. They are all equally likely.

(i) Show that $E\left(X^2\right) = \dfrac{3a^2 + 14}{3}$. **2**

(ii) Hence find Var(X). **2**

(c) The diagram shows the graph of the curve $y = f(x)$.



Using the template page provided at the back of this paper, sketch the graphs of the following curves, showing clearly the intercepts on the axes and the equations of any asymptotes.

(i) $y^2 = f\left(x\right)$ **2**

(ii) $y = \dfrac{1}{f\left(x\right)}$ **3**

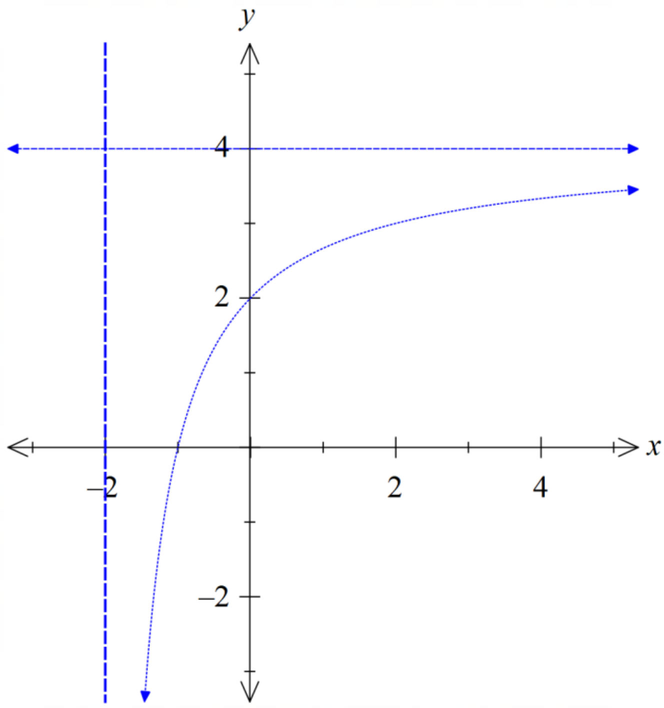
Detach the template page, put your name on it and place it inside the answer booklet for question 15.

End of Exam

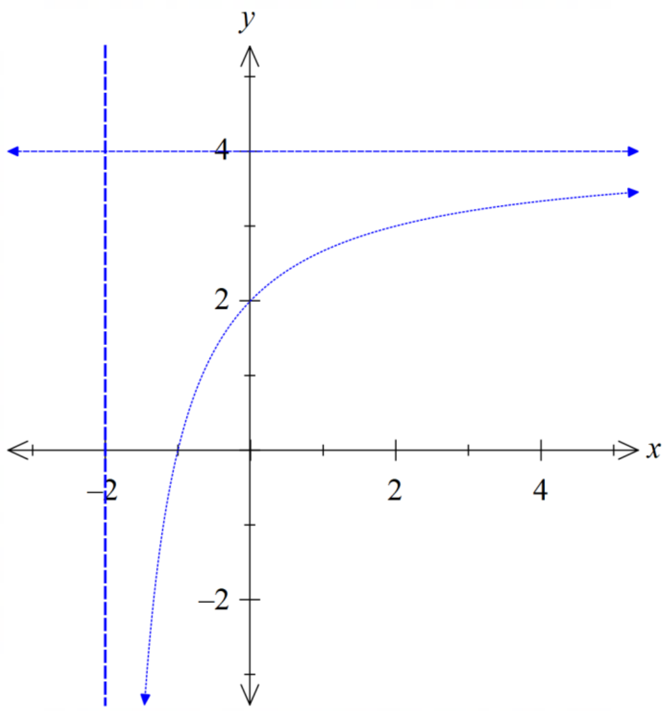
Name: ____________________

Section II – Question 15 (c) graph templates

(i) $y^2 = f(x)$



(ii) $y = \dfrac{1}{f(x)}$



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Name: ____________________

Mathematics Extension 1

Section I – Multiple Choice

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample **2 + 4 = ?** **(A) 2** **(B) 6** **(C) 8** **(D) 9**

A ○ **B** ● **C** ○ **D** ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● **B** ⊗ **C** ○ **D** ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows:

correct

A ⊗ **B** ⊗ **C** ○ **D** ○

Start Here

1. A ○ B ○ C ○ D ○

2. A ○ B ○ C ○ D ○

3. A ○ B ○ C ○ D ○

4. A ○ B ○ C ○ D ○

5. A ○ B ○ C ○ D ○

6. A ○ B ○ C ○ D ○

7. A ○ B ○ C ○ D ○

8. A ○ B ○ C ○ D ○

9. A ○ B ○ C ○ D ○

10. A ○ B ○ C ○ D ○

Year 11 CHS 2019 Mathematics Extension 1 (3U) Assessment Task 3 Solutions

Section I

Q1	Q2	Q3	Q4	Q5
(D)	(D)	(A)	(B)	(B)
Q6	**Q7**	**Q8**	**Q9**	**Q10**
(A)	(C)	(C)	(B)	(C)

Question 1

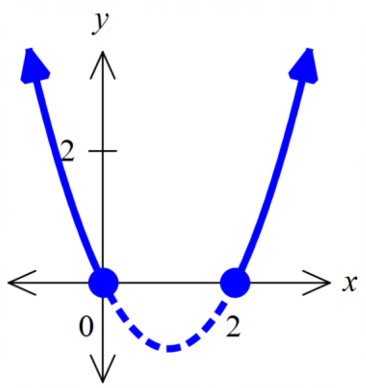
$x = t + 2 \rightarrow t = x - 2$
Substitute into $y = 2t + 1$
$y = 2(x - 2) + 1$
$y = 2x - 3$
$2x - y - 3 = 0$

$\therefore$ (D)

Question 2

$x^2 - 2x \geq 0$

$x \leq 0$ or $x \geq 2$



$\therefore$ (D)

Question 3

Using the remainder theorem.
Remainder = $(-3)^3 - 6(-3) = -9$.

$\therefore$ (A)

Question 4

two distinct real roots if $\Delta > 0$.
$(-2p)^2 - 4q > 0 \rightarrow 4p^2 - 4q > 0$
$4p^2 > 4q \rightarrow p^2 > q$.

$\therefore$ (B)

Question 5

${}^9C_3$ ways of selecting 3 squares.
There are 3 ways to select 3 crosses on the same row, 3 ways for the crosses to be on the same column, 2 ways for the diagonal = 8 total ways

Probability = $\dfrac{8}{{}^9C_3} = \dfrac{2}{21}$.

$\therefore$ (B)

Question 6

$k^2 - k = 0 \neq 0$ and $k^3 - k = 0$
$k(k-1) \neq 0$ and $k(k-1)(k+1) = 0$
$k \neq 0, 1$ and $k = 0, 1, -1$
$\therefore k = -1$

$\therefore$ (A)

Question 7

Reflect the graph $y = \sqrt{x}$ about the x-axis to get $y = -\sqrt{x}$.
This is equivalent to $y = \pm\sqrt{x}$
This is also equivalent to $|y| = \sqrt{x}$

$\therefore$ (C)

Question 8

Choose 8 dancers from 17 dancers in ${}^{17}C_8$ ways.
Choose 1 singer from 5 singers in ${}^5C_1$ ways.
For each singer I chose, there are ${}^{17}C_8$ ways to choose the dancers.
Since there are ${}^5C_1$ ways to choose the singers, therefore total ways = ${}^{17}C_8 \times {}^5C_1$

$\therefore$ (C)

Question 9

$\alpha\beta\gamma = -42 < 0$ which eliminates A and C, since
$2 \times 3 \times 7 > 0$ and $-1 \times -2 \times 21 > 0$
$\alpha\beta + \beta\gamma + \gamma\alpha = -41$
$1\times-6 + -6\times7 + 7\times1 = -41$
$-1\times-3 + -3\times-14 + -14\times-1 > 0$ eliminate D

$\therefore$ (B)

Question 10

In the worst case scenario, all 7 shirts have different size. The 8th shirt will be guaranteed to be the same size as one of the previous 7 shirts.

$\therefore$ (C)

Section II

Question 11

(a) $|2x-1|\le 3$

$-3\le 2x-1\le 3$

$-2\le 2x\le 4$

$-1\le x\le 2$

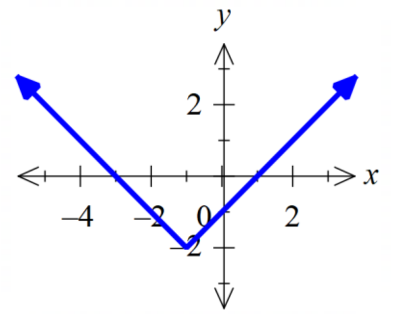
(b)

(i) No odd numbers are equivalent to all 4 numbers are even. Even numbers are 2, 4, 6, 8. So only 1 way or ${}^4C_4$ way.

(ii) at least one odd numeral
= Total ways – no odd
= ${}^9C_4 - 1 = 125$

(iii) 3 odd or 4 odd
= ${}^5C_3\times{}^4C_1+{}^5C_4$
= 45

(c)



(d) $\dfrac{3x}{x-2}\ge 1 \qquad x\ne 2$

$\dfrac{3x}{\cancel{x-2}}\times(x-2)^{\cancel{2}}\ge 1\times(x-2)^2$

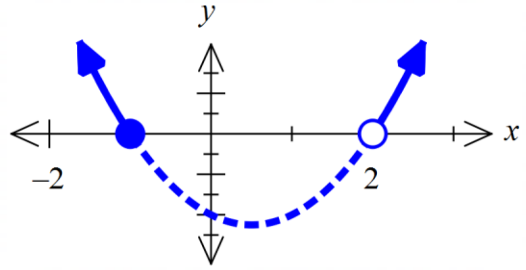
$3x(x-2)\ge(x-2)^2$

$3x(x-2)-(x-2)^2\ge 0$

$(x-2)(3x-(x-2))\ge 0$

$(x-2)(2x+2)\ge 0$

$2(x-2)(x+1)\ge 0$



$\therefore x\le -1$ or $x>2$.

Question 12

(a) $\sin^2\theta=\cos\theta+1$

$1-\cos^2\theta=\cos\theta+1$

$\cos^2\theta+\cos\theta=0$

$\cos\theta(\cos\theta+1)=0$

$\cos\theta=0$ or $\cos\theta=-1$

$\theta=\dfrac{\pi}{2},\dfrac{3\pi}{2}$ or $\theta=\pi$

(b)

(i) $P(x)=x^4-5x^3+9x^2-7x+2. \to P(1)=0$

$P'(x)=4x^3-15x^2+18x-7\to P'(1)=0$

$P''(x)=12x^2-30x+18x\to P''(1)=0$

$P'''(x)=24x-30\to P'''(1)=-6\ne 0$

Therefore $x=1$ is a root of multiplicity 3.

(ii) $\therefore (x-1)^3$ is a factor of $P(x)$

$\therefore P(x)=(x-1)^3(ax+b)$.

$P(0)=(0-1)^3(a(0)+b)=-b$,

but $P(0)=2$, $\therefore -b=2\to b=-2$

Equating the coefficient of x^4.

We get $ax^4=x^4\to a=1$

$\therefore P(x)=(x-1)^3(x-2)$.

(c)

(i) $y=x\sqrt{x+3}$

$u=x \quad v=(x+3)^{\frac{1}{2}}$

$u'=1 \quad v=\dfrac{1}{2}(x+3)^{-\frac{1}{2}}$

$\dfrac{dy}{dx}=(x+3)^{\frac{1}{2}}+\dfrac{x}{2}(x+3)^{-\frac{1}{2}}$

$=\dfrac{(x+3)^{-\frac{1}{2}}}{2}(2(x+3)+x)$

$=\dfrac{3x+6}{2\sqrt{x+3}}$

(ii) $\dfrac{dy}{dx}=0$

$\dfrac{3x+6}{2\sqrt{x+3}}=0$

$3x+6=0$

$x=-2$

$y=x\sqrt{x+3}$

$=-2\sqrt{-2+3}$

$=-2$

$\therefore (-2,-2)$

Question 13

(a) $\dfrac{1}{1+\sqrt{2}}+\dfrac{2}{\sqrt{2}}$

$\dfrac{1\times\left(1-\sqrt{2}\right)}{\left(1+\sqrt{2}\right)\times\left(1-\sqrt{2}\right)}+\dfrac{2\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}}$

$\dfrac{1-\sqrt{2}}{1-2}+\dfrac{2\sqrt{2}}{2}$

$\dfrac{1-\sqrt{2}}{-1}+\dfrac{\cancel{2}\sqrt{2}}{\cancel{2}}$

$-1+\sqrt{2}+\sqrt{2}$

$-1+2\sqrt{2}$

(b) $y=\dfrac{1-x}{3+x}\xrightarrow{\text{swap } x \text{ and } y}x=\dfrac{1-y}{3+y}$

$$x(3+y)=1-y$$
$$3x+xy=1-y$$
$$3x+xy+y=1$$
$$xy+y=1-3x$$
$$y(x+1)=1-3x$$
$$y=\frac{1-3x}{1+x}$$
$$f^{-1}(x)=\frac{1-3x}{1+x}$$

(c) $(\log_3 x)^2=\log_3 x^2+8$

$$(\log_3 x)^2-2\log_3 x-8=0$$
$$(\log_3 x-4)(\log_3 x+2)=0$$
$$\log_3 x=4,\log_3 x=-2$$
$$x=3^4 \text{ or } 3^{-2}$$
$$x=81 \text{ or } \frac{1}{9}$$

(d)

(i) In ΔBCD,

$$\frac{10}{BC}=\tan 30^\circ$$
$$=\frac{1}{\sqrt{3}}$$
$$10=\frac{BC}{\sqrt{3}}$$
$$BC=10\sqrt{3}$$

(ii) ΔACD is an isosceles triangle, $\therefore AC = 10$

In ΔABC,

$$\tan B=\frac{10}{10\sqrt{3}}$$
$$=\frac{1}{\sqrt{3}}$$
$$B=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$
$$B=30^\circ$$

(iii) In ΔBCE

$$\frac{CE}{10\sqrt{3}}=\sin 30^\circ$$
$$CE=\frac{1}{2}\times 10\sqrt{3}$$
$$CE=5\sqrt{3}$$

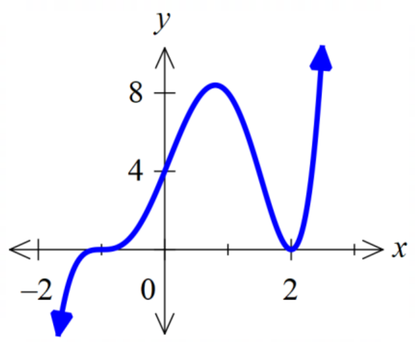
In ΔCDE

$$\tan E=\frac{10}{5\sqrt{3}}$$
$$=\frac{2}{\sqrt{3}}$$
$$E=\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$$
$$E=49^\circ 6'$$

Question 14

(a)

(i)



(ii) $y = (x-2)^2\,(x+1)^3$

$y' = 2(x-2)\,(x+1)^3 + 3(x-2)^2\,(x+1)^2$

$y' = (x-2)\,(x+1)^2\,(2(x+1) + 3(x-2))$

$y' = (x-2)\,(x+1)^2\,(2x + 2 + 3x - 6)$

$y' = (x-2)\,(x+1)^2\,(5x-4)$

$y' = 0 \rightarrow\ x=\dfrac{4}{5}$

Question 14 (continued)

(b)

(i) Fix one person, rearrange 6 people
So 6! = 720.

(ii) Fix one girl,
2! ways to rearrange the boys,
5! ways to rearrange the remaining 4 girls
& 1 group of 2 boys,
Total ways = 5! × 2! = 240

(iii) 2 boys separate = total – 2 boys separate
= 6! – 5! × 2! = 480

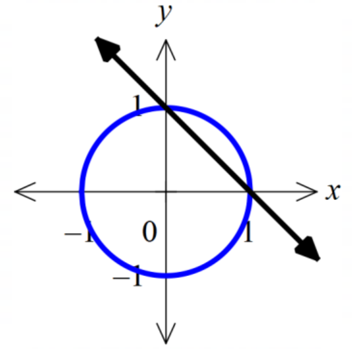
(c)

(i) $P(p) = p^3 - pp^2 + qp - pq = 0$
$\therefore (x - p)$ is a factor of $P(x)$.

(ii) $P(x) = x^2(x - p) + q(x - p)$
$= (x - p)(x^2 + q) = 0$
$x^2 = -q < 0 \rightarrow$ no solution (since $q > 0$)
$\therefore x = p$ only.

Question 15

(a) Points of intersection are (0, 1) and (1, 0) as shown from the graph below.



(b)

(i) Each outcome is equally likely, so each outcome has $\frac{1}{3}$ chance of occurring

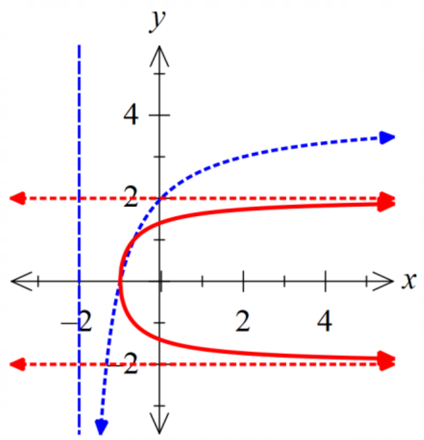
$$E(X^2) = \frac{1}{3}(a+1)^2 + \frac{1}{3}(a+2)^2 + \frac{1}{3}(a-3)^2$$
$$= \frac{a^2+2a+1+a^2+4a+4+a^2-6a+9}{3}$$
$$= \frac{3a^2+14}{3}$$

(ii)

$$E(X) = \frac{1}{3}(a+1) + \frac{1}{3}(a+2) + \frac{1}{3}(a-3)$$
$$= \frac{a+1+a+2+a-3}{3}$$
$$= \frac{3a}{3}$$
$$\mu = a$$
$$Var(X) = E(X^2) - \mu^2$$
$$= \frac{3a^2+14}{3} - a^2$$
$$= \frac{3a^2+14-3a^2}{3}$$
$$= \frac{14}{3}$$

(c)

(i) $y^2 = f(x)$



(ii) $y = \dfrac{1}{f(x)}$