Name: ______________________________

Teacher: ______________________________

Class: ______________________________

FORT STREET HIGH SCHOOL

2012

PRELIMINARY SCHOOL CERTIFICATE COURSE

ASSESSMENT TASK 3: FINAL PRELIMINARY EXAMINATION

Mathematics Extension I

TIME ALLOWED: 2 HOURS

Outcomes Assessed	Questions	Marks
Chooses and applies appropriate arithmetic, algebraic, graphical and geometric techniques to solve problems	1, 2	34
Performs routine arithmetic and algebraic manipulation involving trigonometric identities and applies appropriate trigonometry techniques	3,4	30
Determines the derivative of a function through routine application of the rules of differentiation and relates the derivative of a function to the slope of its graph	5	12
Synthesises mathematical solutions to harder problems and communicates them in appropriate form	6	14

Question	1	2	3	4	5	6	Total	%
Marks	/15	/19	/15	/15	/12	/14	/90	

Directions to candidates:

- Attempt all questions
- The marks allocated for each question are indicated
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board – approved calculators may be used
- **Each new question is to be started in a new booklet**

<u>Question 1:</u> BEGIN A NEW ANSWER BOOKLET (15 marks)

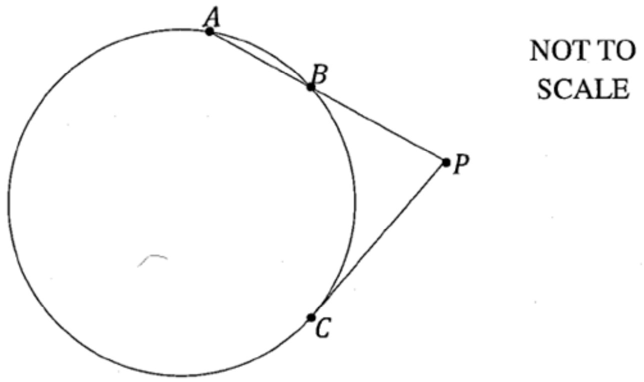
a) Simplify

i. $\dfrac{1}{\sqrt{(1.2)^2+(2.1)^2}}$ (correct to 4 significant figures) **1**

ii. $\dfrac{\sqrt{2}-1}{\sqrt{2}+1}$ (in surd form) **1**

b) Find the coordinates of the point that internally divides the interval AB in the ratio $3:2$ if A is $(-2,3)$ and B is $(3,-4)$. **2**

c) In the diagram the points A, B and C lie on the circle and AB produced meets the tangent from C at the point P.



i. Given that $PC=12, AB=7$ and $PB=x$, find x. **2**

ii. BC is the diameter of the circle passing through P, B and C. Find the length of BC. **1**

d) Find the locus of the point $P(x,y)$ that moves so that P is equidistant from the points $A(3,5)$ and $B(-1,-1)$. **2**

e) Solve $|x+1|=2x-1$ for x. **3**

f) Solve $\dfrac{2t}{1-t}\geq t$ for t. **3**

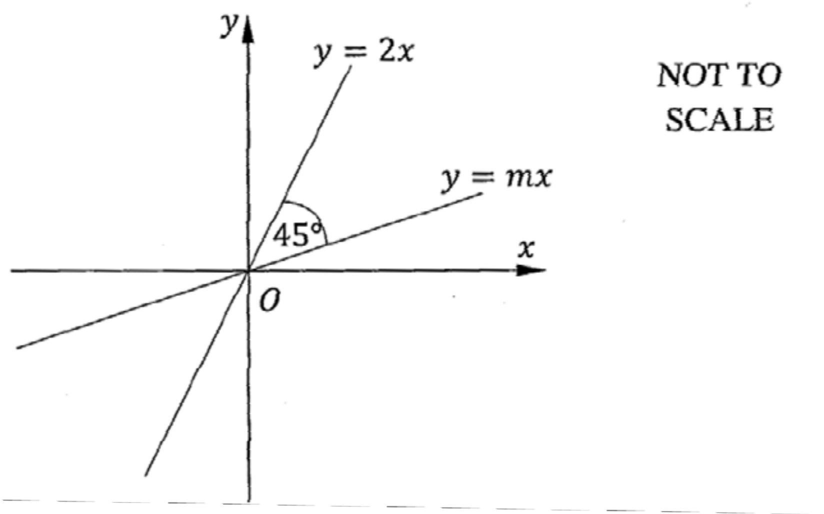
<u>Question 2:</u> BEGIN A NEW ANSWER BOOKLET (19 marks)

a) For the curve $y = 2 + \sqrt{x-1}$:

i. Draw a neat ⅓ page sketch of this curve. **2**

ii. State the domain and range for this curve. **2**

b) The angle between the lines $y = mx$ and $y = 2x$ is $45°$, where $m > 0$, as shown in the diagram below. Find the value of m. **2**



c) Find values of a, b and c such that $3x^2 - 7x - 6 \equiv a(x-2)^2 + b(x-2) + c$ for all x. **3**

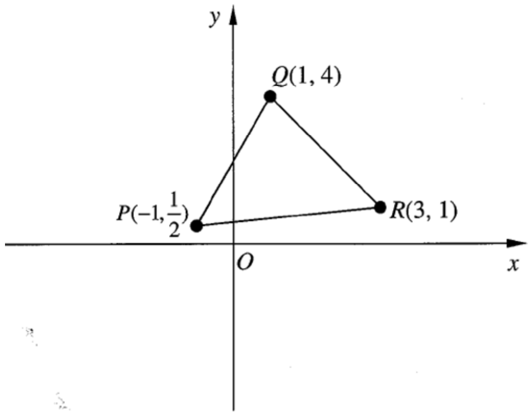
d) The polynomial $P(x) = 2x^3 - 3x^2 + x - 4$ has roots α, β and γ. Find the value of

i. $\alpha + \beta + \gamma$ **1**

ii. $\alpha^2 + \beta^2 + \gamma^2$ **2**

e) Find the focus and directrix of the parabola $y = 2x^2 - 4x + 7$. **3**

f) $P\left(-1,\frac{1}{2}\right), Q(1,4)$ and $R(3,1)$ are the vertices of a triangle, as shown below.



iii. Find the gradient of PM, where M is the midpoint of QR. **2**

iv. Show that PM is the perpendicular bisector of QR. **2**

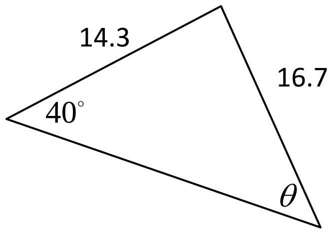
<u>Question 3:</u> BEGIN A NEW ANSWER BOOKLET (15 marks)

a) Give the exact value for

i. $\sin 60^\circ$ **1**

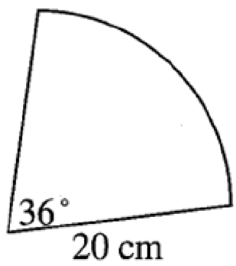
ii. $\tan \dfrac{5\pi}{6}$ **1**

b) Find the value of θ in the following triangle (to the nearest degree). **2**



c) Draw a neat ⅓ page sketch of the curve $y = 3\sin\left(2x + \dfrac{\pi}{3}\right)$ for $0 \le x \le 2\pi$ **3**

d) For the diagram given:



i. Express 36° in radian measure. **1**

ii. Find the exact area of the shape **2**

iii. Find the exact perimeter of the shape. **2**

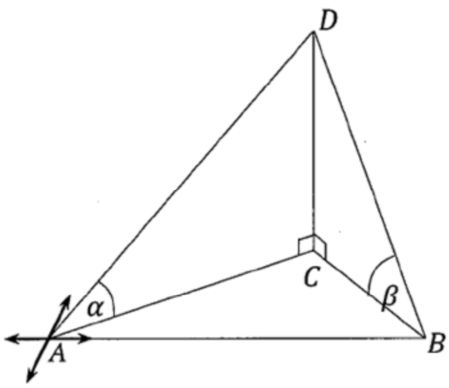
e) Two cars leave the point of no return at the same time. Car A averages $80 km/h$ along a straight road with a bearing of $025^\circ T$ and car B averages $90 km/h$ along a different straight road with a bearing of $135^\circ T$.

i. Draw a neat sketch showing their positions relative to the point of no return three hours after the cars leave the point of no return. **1**

ii. Calculate how far apart the two cars are at this time (to the nearest km). **2**

<u>Question 4:</u> BEGIN A NEW ANSWER BOOKLET (15 marks)

a) A vertical pole, CD, is positioned so that the angles of elevation of the top of the pole from points A and B on the ground are α and β respectively. The ground is a level horizontal surface and the triangle ABC is right-angled at C. This information is shown in the diagram below.



If point B is due east of point A and point C is on a bearing of $060^\circ T$ from point A, show that $\dfrac{\tan\alpha}{\tan\beta} = \dfrac{1}{\sqrt{3}}$. **3**

b) Prove $\dfrac{\cot\theta\cos\theta}{\cot\theta+\cos\theta} = \dfrac{\cos\theta}{1+\sin\theta}$. **3**

c)

i. Write down the angle expansion for $\tan(\theta-\alpha)$. **1**

ii. Hence find the exact value of $\tan 165^\circ$. **2**

d) Use the transformation for $r\cos(2x+\alpha)$ to express $2\sqrt{3}\cos(2x)-\sin(2x)$ as a single term. State the amplitude, period and phase shift for this expression. **3**

e) Find the general solutions for $\cos 2\theta = \sin\theta$. **3**

<u>Question 5:</u> BEGIN A NEW ANSWER BOOKLET (12 marks)

a) Use the rules of differentiation to differentiate with respect to x:

i. $x^2 - 7x + 1$ **1**

ii. $\left(2x^5 - 3\right)^4$ **2**

iii. $\left(x^4 + 5\right)\left(x - 7\right)^3$ **3**

b) Differentiate $f\left(x\right) = \frac{1}{x}$ from first principles. **3**

c) The function $f\left(x\right) = x^3 - 3x + 1$ has a root near $x = -2$. Use one iteration of Newton's Method to find a better solution. **3**

<u>Question 6:</u> BEGIN A NEW ANSWER BOOKLET (14 marks)

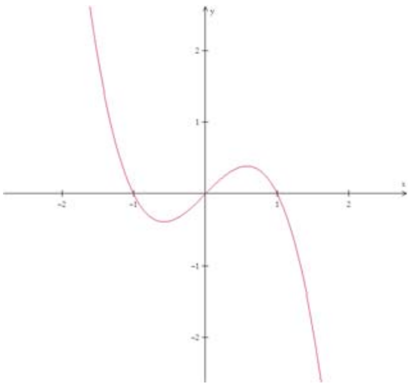
a) Express the polynomial $P\left(x\right) = x^3 + bx^2 + cx + d$ in its factored form given it has a root at $x = 3$, a y-intercept of 3 and when $P\left(x\right)$ is divided by $\left(x - 2\right)$ it has a remainder of -3. 4

b) Using the substitution $t = \tan\frac{\theta}{2}$, solve the equation $2\cos^2\theta + 3\cos\theta + 1 = 0$ for $0 \leq \theta \leq 2\pi$ **4**

c) Find the equation of the normal to the parabola $y = x^2 + 4x - 5$ at the point $\left(-1, -8\right)$ and hence find the co-ordinates of the point where it again meets the parabola. **3**

d) Solve $\cos\left(5x - \frac{\pi}{3}\right) = \sin 3x$ for $0 \leq x \leq 2\pi$. **3**

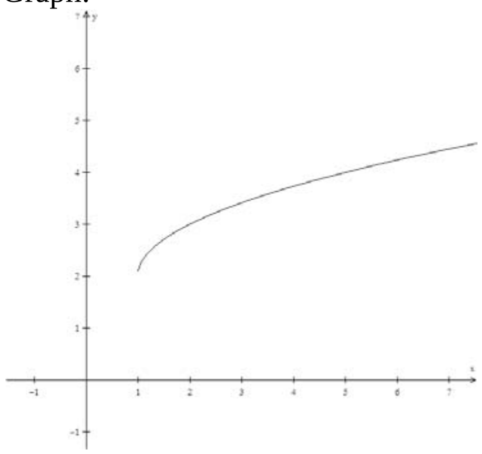
	MARKING	COMMENTS
Question 1:		
a) i) 0.4134	❶answer with sig figs correct	• Many wrote 0.413 - the 0 before the dp is not significant.
ii) $\frac{\sqrt{2}-1}{\sqrt{2}+1}\times\frac{\sqrt{2}-1}{\sqrt{2}-1}$ $=\frac{\left(\sqrt{2}-1\right)^2}{2-1}$ $=2-2\sqrt{2}+1$ $=3-2\sqrt{2}$	❶answer	• Many failed to simplify correctly
b) $x=\frac{mx_2+nx_1}{m+n}$ $=\frac{3\times3+2\times-2}{3+2}$ $=\frac{5}{5}$ $=1$; $y=\frac{my_2+ny_1}{m+n}$ $=\frac{3\times-4+2\times3}{3+2}$ $=\frac{-6}{5}$ $=-1\frac{1}{5}$	❶correct subst in formula	
Hence point is $\left(1,-1\frac{1}{5}\right)$	❶ correct point	
c) i) $AP.PB=CP^2$ $x\left(x+7\right)=12^2$ $x^2+7x=144$ $x^2+7x-144=0$ $\left(x+16\right)\left(x-9\right)=0$	❶ correct circle property and equation	• This formula was poorly known.
$x=-16,9$ Reject -16 as x is a length, so x=9.	❶answer	
ii) BC a diameter $\Rightarrow\angle BPC=90^\circ$ ($\angle$ in semicirc) Hence: $BC^2=x^2+12^2$ $=225$ $BC=15$	❶answer	
d) For $\left\|PA\right\|=\left\|PB\right\|$ where $\left\|PA\right\|^2=\left(x-3\right)^2+\left(y-5\right)^2$ and $\left\|PB\right\|^2=\left(x+1\right)^2+\left(y+1\right)^2$	❶ correct set-up	

	MARKING	COMMENTS
$(x-3)^2+(y-5)^2=(x+1)^2+(y+1)^2$ $x^2-6x+9+y^2-10y+25=x^2+2x+1+y^2+2y+1$ $8x+12y-32=0$ $2x+3y-8=0$	❶correct equation	• Full marks given for $8x+12y-32=0$ but students should realise that any answer is expected to be in simplest form.
e) Two cases: $x+1=2x-1$ $\quad$ $-(x+1)=2x-1 =$ $2=x$ $\quad$ $0=3x$	❶ cases correct	
check: $LHS=\|2+1\| =3$, $RHS=4-1 =3 =LHS$ $(accept)$ *check*: $LHS=\|0+1\| =1$, $RHS=0-1 =-1 \neq LHS$ $(reject)$	❶checking done	• Many failed to test cases.
Hence $x=2$ is the only solution.	❶answer	
f) $\frac{2t}{1-t}\geq t$ $(1-t)^2\frac{2t}{(1-t)}\geq t(1-t)^2$ $2t(1-t)\geq t(1-t)^2$	❶ method set-up correctly	
$2t(1-t)-t(1-t)^2\geq 0$ $(1-t)\left[2t-t(1-t)\right]\geq 0$ $(1-t)\left(2t-t+t^2\right)\geq 0$ $(1-t)\left(t+t^2\right)\geq 0$ $t(1-t)(1+t)\geq 0$		• Many algebraic errors even when students know the correct method.
	❶ justification and/or evidence to support answer	
Noting $t\neq 1$ from original expression, hence $t\leq -1$ *and* $0\leq t<1$.	❶answer	• Mark deducted of equality sign for $t<1$.

Question 2:

a) For $y = 2 + \sqrt{x-1}$

i) Graph:



ii) Domain: $x \geq 1$

Range: $y \geq 2$

b) $\tan\theta = \dfrac{|m_2 - m_1|}{1 + m_1 m_2}$

$\tan 45 = \dfrac{|m-2|}{1+2m}$

$1 + 2m = |m-2|$

Two cases:

$1 + 2m = m - 2 \qquad 1 + 2m = -(m-2)$

$m = -3 \qquad 3m = 1$

$reject\ as\ m > 0 \qquad m = \dfrac{1}{3}$

$\therefore m = \dfrac{1}{3}$

c) $RHS = a(x^2 - 4x + 4) + bx - 2b + c$

$= ax^2 - 4ax + 4a + bx - 2b + c$

$= ax^2 + (b - 4a)x + 4a + 2b + c$

Equating co-efficients:

$x^2: \quad a = 3$

$x: \quad -7 = b - 4a$

$b = 5$

$const: \quad -6 = 4a - 2b + c$

$c = -8$

i.e. $a = 3, b = 5, c = -8$

MARKING

❶ basic shape & direction correct

❶ vertex correct

❶ domain correct

❶ range correct

❶ correct use of angle between two lines formla

❶ correct answer with justification

❶ correct expansion leading to correct value for *a*

❶ *b* correct

❶ *c* correct

COMMENTS

- Many could not identify and sketch the basic $\sqrt{\ }$ equation & graph
- This critical point for the curve was often poorly done.
- Well done for those who graphed correctly.
- Many did not use the 2 cases form the absolute value.
- Some did not indicate the condition $m > 0$ and thus get the correct *m* value.
- Many perfect square expansion errors!
- Many sign errors when calculation the *b* value
- The constant term often was missing one term.

	MARKING	COMMENTS
d) For $P(x)=2x^3-3x^2+x-4$		
i) $\alpha+\beta+\gamma=\frac{-b}{a}$		• Generally well done.
$=\frac{3}{2}$	❶answer	
ii) $\alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\alpha\gamma)$	❶ correct three term expansion for a square	• Mostly well done, but some had $-2\alpha\beta\gamma$ instead of the paied roots.
$=\left(\frac{-b}{a}\right)^2-2\left(\frac{c}{a}\right)$		
$=\left(\frac{3}{2}\right)^2-2\left(\frac{1}{2}\right)$		• Many simple calculation errors.
$=1\frac{1}{4}$	❶answer	
e) $y=2x^2-4x+7$		
$y=2\left(x^2-2x+1-1\right)+7$		
$=2(x-1)^2+5$	❶ correct completion of the squares	• The factor of 2 caused many issues with incorrect algebra.
$\frac{1}{2}(y-5)=(x-1)^2$		
So Vertex is at $(1,5)$ and $4a=\frac{1}{2}$		• Many used the *a* value to adjust the *x* value rather than the *y* value!
$a=\frac{1}{8}$		
Hence focus is $\left(1,5\frac{1}{8}\right)$ and directrix is	❶ focus correct	• This is a point!
$y=4\frac{7}{8}$	❶ directrix correct	• This is an equation (line)!
f)	❶ midpoint correct	
i) Midpint is $\left(\frac{1+3}{2},\frac{4+1}{2}\right)$ or $\left(2,\frac{5}{2}\right)$		• Generally well done.
Hence gradient of PM : $m=\frac{\frac{5}{2}-\frac{1}{2}}{2-^-1}$	❶ gradient correct	• Generally well done.
$=\frac{2}{3}$		
ii) Gradient QR: $m=\frac{4-1}{1-3}$		
$=\frac{3}{-2}$		
Hence $m_{PM}\times m_{QR}=\frac{2}{3}\times\frac{3}{-2}$	❶ shows perpendicular	• Generally well done.
$=-1$		

So as M is the midpoint of QR (from i) and $PM \perp QR$, then PM is the perpendicular bisector of QR (as reqd).

MARKING: ❶ conclusion linked correctly

COMMENTS: • Many either missed or went to great lengths showing the bisection component.

Question 3:

a) Exact values are

i) $\sin 60^\circ$

$$= \frac{\sqrt{3}}{2}$$

❶answer

• Some students used composite angle expansion rather than basic trig identities.

ii) $\tan \frac{5\pi}{6}$

$$= -\tan\frac{\pi}{6}$$

$$= -\frac{1}{\sqrt{3}}$$

❶answer

b) $\frac{\sin\theta}{14.3} = \frac{\sin 40^\circ}{16.7}$

$$\sin\theta = \frac{14.3\sin 40^\circ}{16.7}$$

❶ sine rule correct with substitutions

• Generally well done.

$$\theta = \sin^{-1}\left(\frac{14.3\sin 40^\circ}{16.7}\right)$$

$$= 33.39521023$$

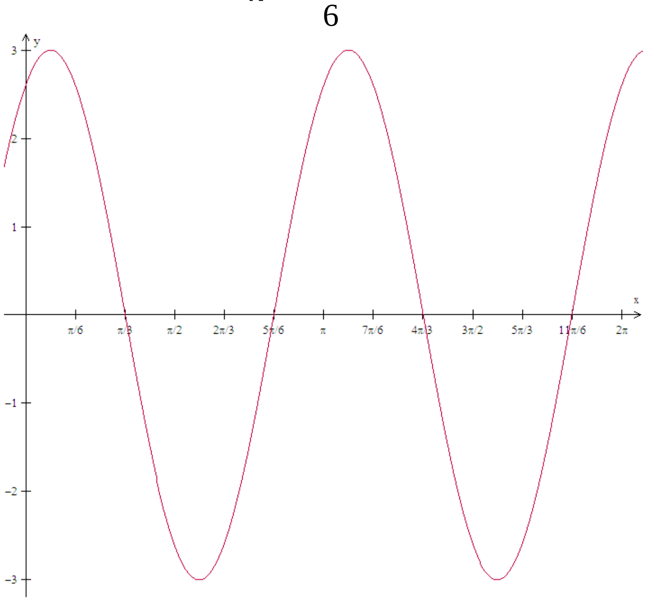
$$\approx 33^\circ \left(nearest\,^\circ\right)$$

❶answer

c) $y = 3\sin\left(2x + \frac{\pi}{3}\right)$: Amplitude is 2, period is π and shifted $0 = 2x + \frac{\pi}{3}$ (i.e. to the left).

$$x = -\frac{\pi}{6}$$

• Poorly attempted – many did not factor the 2 out to find the correct shift.



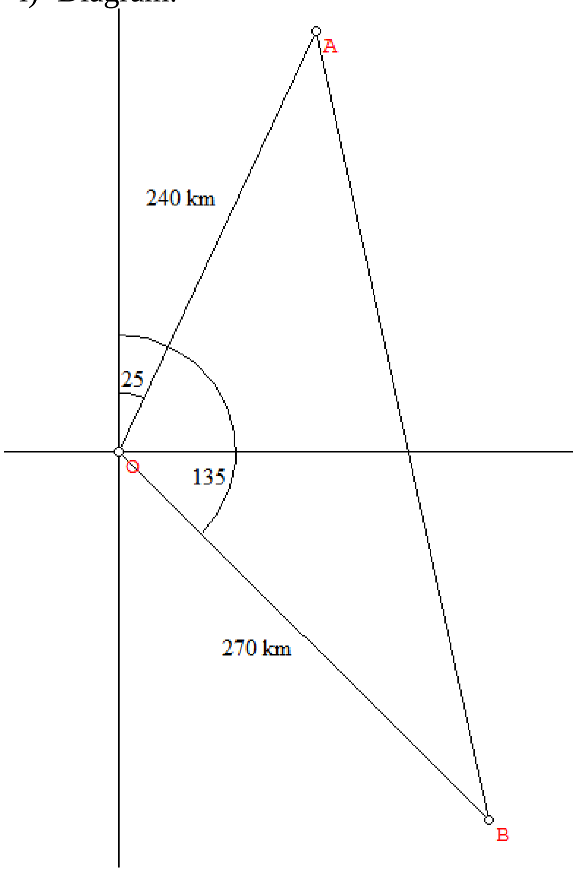
❶ amplitude correct

❶ period correct

❶ phase shift correct

• Some stated the correct period, but then did not reflect this correctly in their graph.

• Many need to take greater care graphing.

	MARKING	COMMENTS
d)		
i) $36\times\frac{\pi}{180}$		
$=\frac{\pi}{5}$	❶answer	
ii) $A=\frac{1}{2}r^2\theta$	❶correct formula & substitution	• Many students did not know the radian formulae or could not simplify their answer.
$=\frac{1}{2}\times 20^2\times\frac{\pi}{5}$		
$=40\pi\,u^2$	❶answer	
iii) $P=2r+r\theta$		
$=2\times 20+20\times\frac{\pi}{5}$	❶answer	
$=40+4\pi$	❶ exact value correct	
e)		
i) Diagram: 	❶ diagram correct	
ii) $\angle AOB=135-25$		
$=110$	❶ cosine rule & substitutions correct	• Many careless errors including calculator in radians, and incorrect distances OA, OB
Hence		
$d^2=240^2+270^2-2\times 240\times 270\times\cos 110^\circ$		
$=174825.8106$		
$d=418.1217652$		
$\approx 418\,km$	❶answer	

Question 4:	MARKING	COMMENTS
a) In ΔABC (linking Δ): with $\angle C = 90^\circ$ and $\angle CAB = 90 - 60$ (from bearing), then $\tan 30^\circ = \frac{BC}{AC}$. $\therefore \frac{BC}{AC} = \frac{1}{\sqrt{3}}$	❶ set-up for linking triangle	Students who allocated exact lengths in ΔABC were awarded 1 mark.
In ΔADC: $\tan\alpha = \frac{CD}{AC}$ In ΔADB: $\tan\beta = \frac{CD}{BC}$	❶ tan relationships in side triangles	
Hence: $\frac{\tan\alpha}{\tan\beta} = \frac{\frac{CD}{AC}}{\frac{CD}{BC}} = \frac{BC}{AC} = \frac{1}{\sqrt{3}}$ as reqd.	❶ algebra to reqd solution correct.	
b) $LHS = \frac{\frac{\cos\theta}{\sin\theta}.\cos\theta}{\frac{\cos\theta}{\sin\theta} + \cos\theta}$	❶ sin/cos substitutions correct	Mostly well done.
$= \frac{\frac{\cos^2\theta}{\sin\theta}}{\frac{\cos\theta + \cos\theta\sin\theta}{\sin\theta}}$	❶ fraction manipulations correct	
$= \frac{\cos^2\theta}{\sin\theta}.\frac{\sin\theta}{\cos\theta(1+\sin\theta)}$ $= \frac{\cos\theta}{(1+\sin\theta)}$ $= RHS$	❶ LHS...RHS format correct	
c) i) $\tan(\theta - \alpha) = \frac{\tan\theta - \tan\alpha}{1 + \tan\theta\tan\alpha}$	❶answer	

	MARKING	COMMENTS
ii) $\tan 165^\circ = -\tan 15^\circ$ $= -\tan(60-45)$ $= -\dfrac{\tan 60 - \tan 45}{1+\tan 60 \tan 45}$	❶ use of previous part (can use 45 and 30, but resulting fractions are more complex)	Essential to use tan of difference (the "Hence" in this part); marks were not awarded for using the sum
$= -\dfrac{\sqrt{3}-1}{1+\sqrt{3}}\left(\times\dfrac{\sqrt{3}-1}{\sqrt{3}-1}\right)$ $= -\dfrac{\left(\sqrt{3}-1\right)^2}{3-1}$ $= -\dfrac{1}{2}\left(3-2\sqrt{3}+1\right)$ $= \sqrt{3}-2$	❶answer	You must rationalize the denominator
d) $r\cos(2x+\alpha) = r\cos 2x\cos\alpha - r\sin 2x\sin\alpha$ With $r\cos\alpha = 2\sqrt{3}$ and $r\sin\alpha = 1$, then $\dfrac{r\sin\alpha}{r\cos\alpha} = \dfrac{1}{2\sqrt{3}}$ $\tan\alpha = \dfrac{1}{2\sqrt{3}}$ $\alpha = \tan^{-1}\dfrac{1}{2\sqrt{3}}$ $\approx 16.1^\circ$	❶ finds α	
$r^2\left(\sin^2\alpha+\cos^2\alpha\right) = 1^2 + \left(2\sqrt{3}\right)^2$ $r^2 = 13$ $r = \sqrt{13}$	❶ finds r	
Hence amplitude $=\sqrt{13}$, period is $\dfrac{2\pi}{2}$ or π and the phase shift is $\dfrac{1}{2}\tan^{-1}\left(\dfrac{1}{2\sqrt{3}}\right)$ which is $\approx 8.05^\circ\left(or\ 0.14^c\right)$.	❶ interprets & gives values correctly	Phase shift with $2x$ not interpreted well
e) $\cos 2\theta = \sin\theta$ $1-2\sin^2\theta = \sin\theta$ $0 = 2\sin^2\theta + \sin\theta - 1$ $= 2\sin^2\theta + 2\sin\theta - \sin\theta - 1$ $= 2\sin\theta(\sin\theta+1) - (\sin\theta+1)$ $= (\sin\theta+1)(2\sin\theta-1)$	❶ getting quadratic equation	

	MARKING	COMMENTS
Hence: $(\sin\theta+1)=0$ $\sin\theta=-1$ $\theta=n\pi+(-1)^n\sin^{-1}(-1)$ Two cases: odd and even: Even case: $n=2m$ $\theta=2m\pi+\frac{\pi}{2}$ $=(4m+1)\frac{\pi}{2}$ Odd case: $n=2m+1$ $\theta=(2m+1)\pi-\frac{\pi}{2}$ $=(4m+2-1)\frac{\pi}{2}$ $=(4m+1)\frac{\pi}{2}$	❶answer	
$(2\sin\theta-1)=0$ $2\sin\theta=1$ $\sin\theta=\frac{1}{2}$ $\theta=n\pi+(-1)^n\sin^{-1}\left(\frac{1}{2}\right)$ $=n\pi+(-1)^n\left(\frac{\pi}{6}\right)$ Two cases: odd and even: Even case: $n=2m$: $\theta=2m\pi+\frac{\pi}{6}$ $=(12m+1)\frac{\pi}{6}$ Odd case: $n=2m+1$: $\theta=(2m+1)\pi-\frac{\pi}{6}$ $=(12m+5)\frac{\pi}{6}$	❶ generator correct	Marks were awarded for appropriate general solutions that generated correct angles.
	Alternative Method Marks: ❶ set-up correct	

	MARKING	COMMENTS
Alternative (simpler) method: $\cos 2\theta = \cos\left(\frac{\pi}{2}-\theta\right)$ $\cos 2\theta = c = \cos\left(\frac{\pi}{2}-\theta\right)$ Gives 2 equations: $2\theta = 2n\pi \pm \cos^{-1} c \quad \frac{\pi}{2}-\theta = \cos^{-1} c$ Combining: $2\theta = 2n\pi \pm \left(\frac{\pi}{2}-\theta\right)$ Negative case:		
$2\theta = 2n\pi - \left(\frac{\pi}{2}-\theta\right)$ $\theta = (4n-1)\frac{\pi}{2}$ $\left(\frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}, ...\right)$ Positive case:	❶ negative case generator correct	
$2\theta = 2n\pi + \frac{\pi}{2} - \theta$ $3\theta = (4n+1)\frac{\pi}{2}$ $\theta = (4n+1)\frac{\pi}{6}$ Note: this positive case generates the solutions: $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{9\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \frac{21\pi}{6}...$ which include the negative case solutions $\left(\frac{9\pi}{6} \equiv \frac{3\pi}{2}; \frac{21\pi}{6} \equiv \frac{7\pi}{2}, etc...\right)$ thus the general solution is $\theta = (4n+1)\frac{\pi}{6}, n = 0, \pm 1, \pm 2, ...$	❶positive case generator correct	

	MARKING	COMMENTS
Also Note: The sin generators from the first method give: $\left(12m+1\right)\frac{\pi}{6}$ gives $\frac{\pi}{6},\frac{13\pi}{6},\frac{25\pi}{6},...,$ $\left(12m+5\right)\frac{\pi}{6}$ gives $\frac{5\pi}{6},\frac{17\pi}{6},\frac{29\pi}{6},...$ and with the $\theta=\left(4n+1\right)\frac{\pi}{2}$ generator common to both methods, these three generate every third term of the solution $\theta=\left(4n+1\right)\frac{\pi}{6}, n=0,\pm1,\pm2,...$		

Question 5:	MARKING	COMMENTS
a)		
i) $2x-7$	❶answer	• This question generally well done.
ii) $4\left(2x^5-3\right)^3.10x^4$	❶chain rule correct	
$=40x^4\left(2x^5-3\right)^3$	❶answer	
iii) $u=x^4+5 \quad v=\left(x-7\right)^3$	❶set-up correct	
$du=4x^3 \quad dv=3\left(x-7\right)^2$		
$\dfrac{dy}{dx}=u.dv+v.du$		
$=\left(x^4+5\right).3\left(x-7\right)^2+\left(x-7\right)^3.4x^3$	❶ product rule correctly substituted	
$=\left(x-7\right)^2\left[3\left(x^4+5\right)+4x^3\left(x-7\right)\right]$		
$=\left(x-7\right)^2\left(7x^4-28x^3+15\right)$	❶answer in simplest form	
b) $\dfrac{dy}{dx}=\lim\limits_{\delta x\to 0}\dfrac{f\left(x+\delta x\right)-f\left(x\right)}{\delta x}$		
$=\lim\limits_{\delta x\to 0}\dfrac{\frac{1}{x+\delta x}-\frac{1}{x}}{\delta x}$	❶ first principles formula with correct substitution	
$=\lim\limits_{\delta x\to 0}\dfrac{\frac{x-\left(x+\delta x\right)}{x\left(x+\delta x\right)}}{\delta x}$		
$=\lim\limits_{\delta x\to 0}\dfrac{x-x-\delta x}{x\delta x\left(x+\delta x\right)}$		
$=\lim\limits_{\delta x\to 0}\dfrac{-\delta x}{x\delta x\left(x+\delta x\right)}$	❶ algebra correct to be able to cancel δx	
$=\lim\limits_{\delta x\to 0}\dfrac{-1}{x\left(x+\delta x\right)}$	❶ limit resolved correctly	
$=\dfrac{-1}{x^2}$		
c) $f\left(-2\right)=\left(-2\right)^3-3\left(-2\right)+1$		
$=-1$		
$f'\left(x\right)=3x^2-3$	❶ values for $x=-2$ correct	
$f'\left(-2\right)=3\left(-2\right)^2-3$		
$=9$		

	MARKING	COMMENTS
Hence $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$	❶ formula correct	• The main error was getting the function and derivative values in the wrong order.
$= -2 - \frac{-1}{9}$		
$= -1\frac{8}{9}$	❶answer	

Question 6:

	MARKING	COMMENTS
a) For $P(x) = x^3 + bx^2 + cx + d$		
y-intercept of 3 $\Rightarrow d = 3$		
root at $x = 3 \Rightarrow P(3) = 0$ $0 = 27 + 9b + 3c + d$ $-30 = 9b + 3c$ ① Remainder of -3 when division by $(x-2)$ $\Rightarrow P(2) = -3$	❶ interprets given data correctly to give initial equations	
$-3 = 8 + 4b + 2c + d$ $-14 = 4b + 2c$ ② Eliminating c: ③=①×2 : $18b + 6c = -60$ ④=②×3: $12b + 6c = -42$	❶ forms equations to solve for b, c correctly	
③-④: $6b = -18$ $b = -3$ In ①: $c = -1$ $\therefore P(x) = x^3 - 3x^2 - x + 3$	❶ values for b, c correct	
$$\begin{array}{r} x^2 - 1 \\ (x-3)\overline{\big)\, x^3 - 3x^2 - x + 3} \\ \underline{x^3 - 3x^2 } \\ -x + 3 \\ \underline{-x + 3} \\ \cdot \quad \cdot \end{array}$$	❶ poly division or equivalent to get answer in factored form	
Hence $P(x) = (x-3)(x-1)(x+1)$		Many failed to give $P(x)$ in factored form as required.

	MARKING	COMMENTS
b) Substituting: $0=2\left(\frac{1-t^2}{1+t^2}\right)^2+3\left(\frac{1-t^2}{1+t^2}\right)+1$ $0=2\left(1-t^2\right)^2+3\left(1-t^2\right)\left(1+t^2\right)+\left(1+t^2\right)^2$ $=2\left(1-2t^2+t^4\right)+3\left(1-t^4\right)+1+2t^2+t^4$ $=2-4t^2+2t^4+3-3t^4+1+2t^2+t^4$ $=6-2t^2$	❶ correct t substitutions	Many students need to learn the 't' results; these had to be used as specified in the question.
$3=t^2$ $t=\pm\sqrt{3}$ $\therefore \tan\left(\frac{\theta}{2}\right)=\pm\sqrt{3}$ $\frac{\theta}{2}=\tan^{-1}\left(\pm\sqrt{3}\right)$	❶ correct t values	The $\pm$ was frequently missed.
$=\frac{\pi}{3},\frac{2\pi}{3},\frac{4\pi}{3},\frac{5\pi}{3}$ $\theta=\frac{2\pi}{3},\frac{4\pi}{3}\left(0\le\theta\le2\pi\right)$	❶correct θ values	
Checking $\theta=\pi$ in original equation: $LHS=2\cos^2\pi+3\cos\pi+1$ $=2\left(-1\right)^2+3\left(-1\right)+1$ $=0$ $=RHS$ $\therefore \theta=\pi$ is also a solution $\therefore \theta=\frac{2\pi}{3},\pi,\frac{4\pi}{3}\left(0\le\theta\le2\pi\right)$	❶ Checks and includes $\theta=\pi$ as a solution	Some students did not check $\theta=\pi$.
c) $y'=2x+4$ $@\,x=-1:y'=2$ So normal has gradient $\frac{-1}{2}$ $y-{}^-8=\frac{-1}{2}\left(x-{}^-1\right)$ $2y+16=-x-1$ $x+2y+17=0$ is the equation of the normal.	❶ equation of normal	Too many careless errors in this part.
Then, re-arranging: $2y=-x-17$ And the parabola becomes $\left(\times2\right)$: $2y=2x^2+8x-10$, so adding: $0=2x^2+9x+7$ $=2x^2+2x+7x+7$ $=2x\left(x+1\right)+7\left(x+1\right)$		

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$$0=(x+1)(2x+7)$$

So $x=-1,\dfrac{-7}{2}$

❶ solves equations for x values

Many failed to realize that $x=1$ had to be one of the solutions – given in the question!

As $(-1,-8)$ is the original point, the other point has a y value

$$y=\left(\frac{-7}{2}\right)^2+4\left(\frac{-7}{2}\right)-5$$

$$=\frac{-27}{4}$$

or $\left(-3\dfrac{1}{2},-6\dfrac{3}{4}\right)$

❶ point correct

d) $\cos\left(5x-\dfrac{\pi}{3}\right)=\sin(3x)$

Poorly done – too many students did not use general solutions to generate all the solutions in the given domain.

$$\cos\left(5x-\frac{\pi}{3}\right)=c=\cos\left(\frac{\pi}{2}-3x\right)$$

$$\therefore 5x-\frac{\pi}{3}=2n\pi\pm\left(\frac{\pi}{2}-3x\right)$$

❶ resolves to cases (or equivalent)

Positive case:

$$5x-\frac{\pi}{3}=2n\pi+\frac{\pi}{2}-3x$$

$$8x=2n\pi+\frac{5\pi}{6}$$

$$=(12n+5)\frac{\pi}{6}$$

$$x=(12n+5)\frac{\pi}{48}$$

❶ solutions from one case (or equivalent) correct

$$=\frac{5\pi}{48},\frac{17\pi}{48},\frac{29\pi}{48},\frac{41\pi}{28},$$

$$\frac{53\pi}{48},\frac{65\pi}{48},\frac{77\pi}{48},\frac{89\pi}{48}$$

Negative case:

$$5x-\frac{\pi}{3}=2n\pi-\frac{\pi}{2}+3x$$

$$2x=2n\pi-\frac{\pi}{6}$$

$$=(12n-1)\frac{\pi}{6}$$

❶ solutions from other case (or equivalent) correct

$$x=(12n-1)\frac{\pi}{12}$$

$$=\frac{11\pi}{12},\frac{23\pi}{12}$$