Name: ________________

Teacher: ________________

Class: ________________

FORT STREET HIGH SCHOOL

2013

PRELIMINARY HIGH SCHOOL CERTIFICATE COURSE
ASSESSMENT TASK 3:
FINAL PRELIMINARY EXAM
Mathematics Extension 1

TIME ALLOWED: 2 HOURS

Outcomes Assessed	Questions	Marks
Demonstrates appropriate mathematical techniques basic algebra, equations, geometry and functions. Chooses and applies appropriate techniques to solve quadratic functions. Manipulates algebraic expressions to solve problems with Locus and the Parabola.	1, 2 and 3	45
Chooses and applies appropriate techniques to problems involving Differentiation.	4	20
Chooses and applies appropriate techniques to problems involving trigonometry.	5	20

Question	**1**	**2**	**3**	**4**	**5**	**Total**	**%**
Marks	**/15**	**/15**	**/15**	**/20**	**/20**	**/85**	

Directions to candidates:

- Attempt all questions
- The marks allocated for each question are indicated
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board – approved calculators may be used
- Each new question is to be started in a new booklet.

Question 1 15 marks

Marks

a.

Rationalise $\dfrac{1+\dfrac{\sqrt{3}}{3}}{1-\dfrac{\sqrt{3}}{3}}$ 2

b.

Determine whether the following functions are odd, even or neither 3

i. $f(x)=\sqrt{4-x^2}$ where $|x|\leq 2$

ii. $f(x)=\dfrac{x}{1+x^2}$

iii. $f(x)=x^2-4x+5$

c.

Prove that $\dfrac{\cos\theta}{\sqrt{(1+\tan^2\theta)}}+\dfrac{\sin\theta}{\sqrt{(1+\cot^2\theta)}}=1$ 2

d.

As a battleship sails due north, a column of smoke is observed in a direction $N19°40'E$. After the ship sailed $480m$ further, the same smoke is observed in a direction $S73°20'E$ .Draw a diagram to illustrate the information. Find the distance from the battleship to the smoke at the first point of observation. 3
(to the nearest meter)

e.

Consider the circle $x^2+y^2-2x-14y+25=0$

i. Show that if the line $y=mx$ intersects the circle at two distinct points then $(1+7m)^2-25(1+m^2)>0$ 2

ii. For what values of m is the line $y=mx$ a tangent to the circle? 2

f.

The point $P(-3,8)$ divides the interval AB externally in the ration of $k:1$. If A is the point $(6,-4)$ and B is the point $(0,4)$, find the value of k. 1

Question 2 15 marks

Marks

a.

Shade the region of the plane specified by $|x+2y|<4$ 2

b.

i. Show that $\tan 75° = 2+\sqrt{3}$ 2

ii. The lines $y = mx$ and $x = y\sqrt{3}$ meets at an angle of $75°$. Find the value of m. 3

c.

A and B are points with co-ordinates $(3,0)$ and $(6,0)$ respectively. A point P moves so that the length of PB is twice that of PA.
Find the locus of P 2

d.

The line $y = 4x+6$ meets the circle $x^2+y^2 = 8y$ at A and B. Find the co-ordinates of M, the midpoint of AB. 3

e.

The parabolas $y = x^2$ and $y = (x+4)^2$ meet at the point P.

i. Find the co-ordinates of P 1

ii. Find the acute angle between the tangents to the curve at P. 2
(to the nearest minute)

Question 3 15 marks

Marks

a. If α and β are the roots of the quadratic equation $x^2+px+q=0$, write down the values of

i. $\alpha+\beta$ 1

ii. $\alpha\beta$ 1

iii. $\alpha^2+\beta^2$ 1

b.

Let a, b, c and d be the four roots of the equation
$x^4-2x^3-44x^2+18x+314=0$

i. Without attempting to locate the individual roots, state the value of their sum $S=a+b+c+d$ 1

ii. Similarly, state the value of $T=ab+ac+ad+bc+bd+cd$ 1

iii. From your results for S and T, or otherwise, find the value of $a^2+b^2+c^2+d^2$ 2

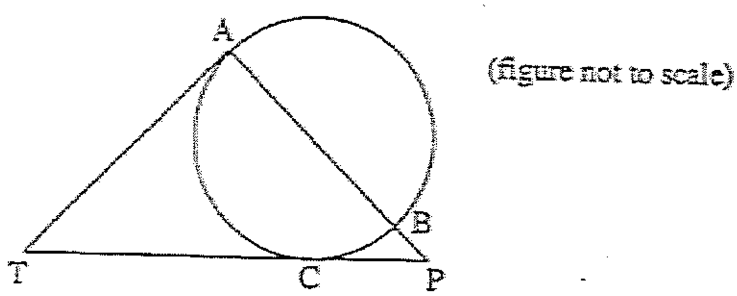
c.

State the remainder theorem for polynomials. Hence, or otherwise, factorise x^3+3x^2-9x+5 into linear factors. 3

d.

Solve the inequality $\dfrac{x^2-4}{x}>0$ 2

e.



AB is a diameter of the circle ABC. The tangents at A and C meet at T. The lines TC and AB are produced to meet at P.
Copy the diagram.

i. Prove that $\angle CAT = 90° - \angle BCP$ 2

ii. Hence, or otherwise, prove that $\angle ATC = 2\ \angle BCP$. 1

Question 4 20 marks

Marks

a.

Find the second derivative of $\frac{1}{\sqrt{x}}$ 2

b. Using

i. Evaluate the limit of 3

$$\lim_{x\to 2}\frac{x^3-8}{x-2}$$

ii. Using first principles, find the derivative of the function

$f(x)=\frac{1}{x},\ \ x\neq 0$

c.

Consider $f(x)=\frac{1-x^2}{x^4}$, $x\neq 0$

i. Find the zeros of this function, i.e. the values of x where $f(x)=0$. 1

ii. Find the turning points of this function, and determine their nature. 5

iii. Sketch this function. 2

d.

Find the equation of the tangent to the curve $y=x^4+1$. At the point where $x=1$. 2

e.

Joanne has a small business involving the delivery of fruit juice to shops. She has calculated the cost of running her van, at a speed of $v\ km/h$, is $\$\left(36+\frac{v^2}{100}\right)$ for each hour.

i. If Joanne averaged $v\,km/h$ when completing her $100km$ delivery run, show that the time taken was $\frac{100}{v}$ hours. 1

ii. If C, expressed in dollars, is the cost of Joanne's delivery run, show $C=\frac{3600}{v}+v$. 1

iii. How fast should Joanne travel in order to minimise cost? 3

Question 5 20 marks

Marks

a.

Given that $\cos x = \frac{4}{5}$, and $0 < x < \frac{\pi}{2}$, find exact values for 2

i. $\sin x$

ii. $\cos 2x$

b.

Write down the expression for $\cos(a+b)$ and hence prove 2

$\cos(2q) = 1 - 2\sin^2 q$.

c.

Prove the identity $\frac{\cos y - \cos(y+2q)}{2\sin q} = \sin(y+q)$ 2

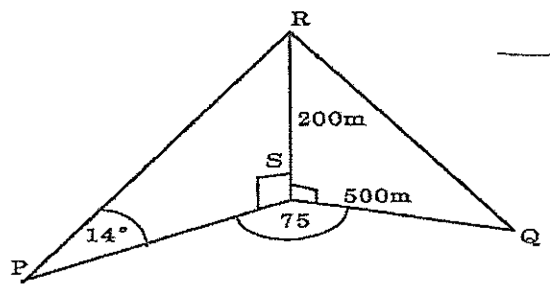
d.

Show that $\cot\theta + \tan\frac{1}{2}\theta = \cos ec\theta$ 2

e.

Find all the angles θ with $0 \le \theta \le 2\pi$ for which $\sin\theta + \cos\theta = 1$ 2

f.



3

Two house P and Q lie in the same plane as S, the foot of a hill RS. The height of the hill is known to be $200m$ and from P the angle of elevation of the top of the hill is $14°$. If P and Q subtend an angle of $75°$ at S and if Q is $500m$ from S, how far apart are the two houses? *(to the nearest metre)*

g.

Solve $\cos 2\theta = \cos\theta$ for $0 \le \theta \le 2\pi$ 3

h.

i. Expand $(a+b)(a^2 - ab + b^2)$ 1

ii. Prove that $\cos^6\theta + \sin^6\theta = \frac{1}{4} + \frac{3}{4}\cos^2 2\theta$ 3

Q1.

a) $\dfrac{1+\frac{\sqrt{3}}{3}}{1-\frac{\sqrt{3}}{3}} = \dfrac{\frac{3+\sqrt{3}}{3}}{\frac{3-\sqrt{3}}{3}} = \dfrac{3+\sqrt{3}}{3-\sqrt{3}} \checkmark \times \dfrac{3+\sqrt{3}}{3+\sqrt{3}}$

$= \dfrac{9+6\sqrt{3}+3}{9-3} = \dfrac{12+6\sqrt{3}}{6} = 2+\sqrt{3}$ ✓

b) i $f(x) = \sqrt{4-x^2}$ $\quad f(-x) = \sqrt{4-(-x)^2} = f(x)$

$\therefore$ even ✓

ii $f(x) = \dfrac{x}{1+x^2}$ $\quad f(-x) = \dfrac{-x}{1+(x)^2} = -\dfrac{x}{1+x^2} = -f(x)$

$\therefore$ odd ✓

iii) $f(x) = x^2-4x+5$ $\quad f(-x) = (-x)^2-4(-x)+5$

$= x^2+4x+5$

$\therefore$ neither. ✓

students must show reasoning, they can not just state answer.

c. LHS $= \dfrac{\cos\theta}{\sqrt{\sec^2\theta}} + \dfrac{\sin\theta}{\sqrt{\operatorname{cosec}^2\theta}}$ ✓

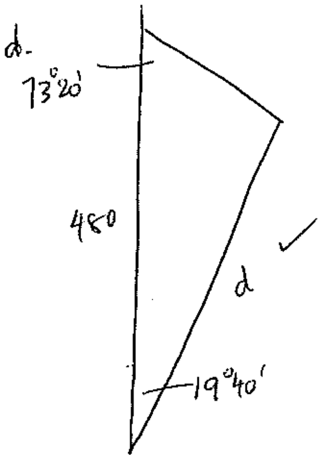
$= \dfrac{\cos\theta}{1/\cos\theta} + \dfrac{\sin\theta}{1/\sin\theta}$

$= \cos^2\theta + \sin^2\theta$ ✓

$= 1$

$=$ RHS

d.



1 mark for correct diagram ✓

remaining angle $= 87°$ ✓
(angle sum of triangle)

$\therefore \dfrac{d}{\sin 73°20'} = \dfrac{480}{\sin 87}$

$d = \dfrac{480}{\sin 87} \times \sin 73°20'$

$d = 460.47$

e. $x^2+y^2-2x-14y+25=0$ $\qquad y = mx$

$x^2+(mx)^2-12x-14(mx)+25=0$ ✓

$(1+m^2)x^2-2(1+7m)x+25=0$

for 2 solutions $\quad \Delta > 0$

$[-2(1+7m)]^2 - 4.(1+m^2).25 > 0$ ✓

$(1+7m)^2 - 25(1+m^2) > 0$

$(1+7m)^2 - 25(1+m^2) = 0$

$1 + 14m + 49m^2 - 25 - 25m^2 = 0$

$24m^2 + 14m - 24 = 0$ ✓

$12m^2 + 7m - 12 = 0$

$m = \dfrac{-7 \pm \sqrt{7^2 - 4.12.(-12)}}{24}$

$m = -\dfrac{32}{24}, \dfrac{18}{24}$ ✓

$= -4/3$ or $3/4$

f. $P(-3,8)$ $\quad m:n = k:1$

$A(6,-4)$ $\quad B(0,4)$

$x = \dfrac{mx_2 + nx_1}{m+n}$

$-3 = \dfrac{k(0) + 1.(6)}{k+1}$

$k+1 = -2$

$k = -3$ ✓

If the ratio was chosen to be either $-k:1$ or $k:-1$ then k should be 3.

Comments

1 a) Well done.

b) A few students did not show reasoning.

c) Some students failed to show sufficient steps.

e) i) If $b = -2(1+7m)$

then $b^2 = 4(1+7m)^2$

Many students left out the 4.

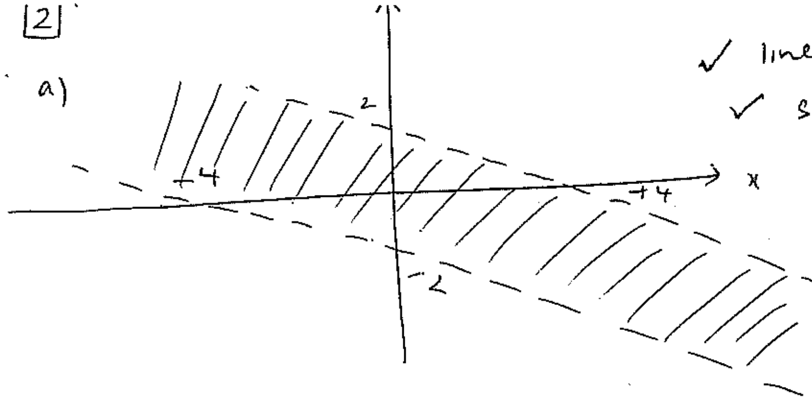
d) Major problem was incorrect positioning of angle $73°20'$ on diagram.

e) ii) Many students could not even make a start with this question.

f) Some students don't know the correct formula but mostly well done.

2

a)



✓ lines

✓ shading

$y < 2 - \frac{x}{2}$

$y > -2 - \frac{x}{2}$

1 mark correct lines

1 mark correct shading

b) i $\tan 75 = \tan(45+30)$

$= \dfrac{\tan 45 + \tan 30}{1 - \tan 45 . \tan 30}$ ✓

$= \dfrac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}}$

$= \dfrac{\sqrt{3}+1}{\sqrt{3}-1} \times \dfrac{\sqrt{3}+1}{\sqrt{3}+1}$

$= \dfrac{4 + 2\sqrt{3}}{2}$ ✓

$= 2 + \sqrt{3}$

ii) $y = mx \quad x = y\sqrt{3}$

$y = \frac{1}{\sqrt{3}} x$

$m_1 = \frac{1}{\sqrt{3}}$

$\tan 75° = \dfrac{m - \frac{1}{\sqrt{3}}}{1 + m.\frac{1}{\sqrt{3}}}$

$2 + \sqrt{3} = \dfrac{\sqrt{3}m - 1}{\sqrt{3} + m}$ ✓

$2\sqrt{3} + 2m + 3 + \cancel{\sqrt{3}m} = \cancel{\sqrt{3}m} - 1$

$2m = -2\sqrt{3} - 4$

$m = -\sqrt{3} - 2$ ✓

also $2+\sqrt{3} = \dfrac{\frac{1}{\sqrt{3}} - m}{1 + \frac{m}{\sqrt{3}}}$

$m = -1$ ✓

c. $\sqrt{(x-6)^2 + y^2} = 2\sqrt{(x-3)^2 + y^2}$

$x^2 - 12x + 36 + y^2 = 4(x^2 - 6x + 9) + 4y^2$ ✓

$0 = 4x^2 - 24x + 36 + 4y^2 - x^2 + 12x - 36 - y^2$

$0 = 3x^2 - 12x + 3y^2$

$4 = x^2 - 4x + 4 + y^2$

$4 = (x-2)^2 + y^2$ ✓

d. $y = 4x + 6 \qquad x^2 + y^2 = 8y$

$x^2 + (4x+6)^2 = 8(4x+6)$ ✓

$x^2 + 16x^2 + 48x + 36 = 32x + 48$

$17x^2 + 16x - 12 = 0$

$x_1 + x_2 = \dfrac{-16}{17}$

$\dfrac{x_1 + x_2}{2} = \dfrac{-8}{17}$ ✓

$y = 4\left(\frac{-8}{17}\right) + 6$

$y = \dfrac{70}{17}$ ✓

e. $y = x^2 \quad y = (x+4)^2$

$x^2 = (x+4)^2$

$x^2 = x^2 + 8x + 16$

$8x = -16$

$x = -2 \qquad y = 4. \qquad P(-2, 4)$ ✓

ii $y = x$ $\quad$ $y = x^2 + 8x + 16$

$y' = 2x$ $\quad$ $f'(x) = 2x + 8$

$f'(-2) = -4$ $\quad$ $f'(-2) = 4.$

$m_1 = -4$ $\quad$ $m_2 = 4.$ ✓

$$\tan\theta = \frac{|4+4|}{1+4 \cdot 4} = \frac{8}{15}$$

$\theta = 28^\circ$ ✓

Comments:

2(a) Very badly done. Most students had no idea.

2(b) Mostly well done. Many students forgot to get both answers.
Also by inspection is not good enough.

2(c) Many students put the 2 on the wrong side.
- Students forgot to square the 2.
- Students did not multiply every term by 4

2(d)
- students got tied up with the algebra
- students found the x-intercepts but not the midpoint

2(e) Well done

2(f) Well done.

$\boxed{3}$ a. i $\alpha + \beta = -p$ ✓

ii $\alpha\beta = q$ ✓

iii $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= (-p)^2 - 2q$

$= p^2 - 2q$ ✓

b. $x^4 - 2x^3 - 44x^2 + 18x + 314 = 0$

i) $a + b + c + d = 2$

ii) $ab + ac + ad + bc + bd + cd = -44$

iii) $(a + b + c + d)^2$ ✓

$= a^2 + b^2 + c^2 + d^2 + (ab + ac + ad + \dots + cd)$

$a^2 + b^2 + c^2 + d^2 = (a+b+c+d)^2 - 2(ab + ac + \dots + cd)$

$= 2^2 + \cancel{44}\ 88$

$= \cancel{48}\ 92$ ✓

c. If a polynomial $P(x)$ is divided by $(x-a)$ then the remainder is given by $P(a)$ ✓

$P(x) = x^3 + 3x^2 - 9x + 5$

$P(1) = 1 + 3 - 9 + 5 = 0$

$\therefore (x-1)$ is a factor ✓

$$
\begin{array}{r}
x^2 + 4x - 5 \\
x - 1 \ \big) \ x^3 + 3x^2 - 9x + 5 \\
-(x^3 - x^2) \\
\hline
4x^2 - 9x + 5 \\
-(4x^2 - 4x) \\
\hline
-5x + 5 \\
-(-5x + 5) \\
\hline
/
\end{array}
$$

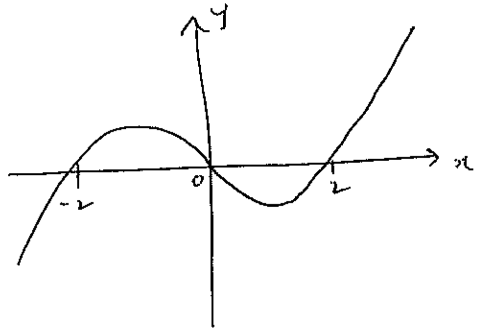
$\therefore P(x) = (x-1)(x^2 + 4x - 5)$

$= (x-1)(x+5)(x-1)$ ✓

d. $\dfrac{x^2 - 4}{x} > 0$

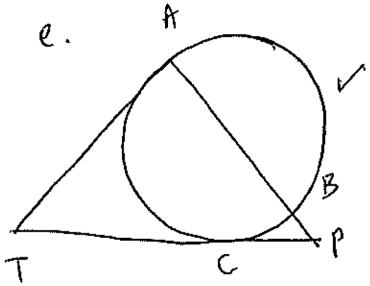
$x^2 \cdot \dfrac{x^2 - 4}{x} > 0$

$x(x+2)(x-2) > 0$



$-2 < x < 0$ ✓ , $x > 2$ ✓

e.



✓ $\angle TAP = 90°$ (diameter is perp. to tangent)
$\angle BCA = 90°$ (angle at circ. subtended b diameter)

$\angle PCB = \angle BAC$ (alt. segment) ✓

$\angle TAC = 90 - \angle BAC$

$= 90 - \angle PCB$

i

$\angle TAC = 90 - \angle PCB$

$TA = TC$ (tangents from a points)

$\angle TAC = \angle TCA$ (isosceles triangle)

In ΔTAC $\angle TAC + 2(90 - \angle PCB) = 180$ ✓

$\angle TAC + \cancel{180} - 2\angle PCB = \cancel{180}$

$\angle TAC = 2\angle PCB$

3 (a) Well Done

(b) (ii) Many students carelessly had.

$ab + ac + ad + bc + bd + cd = 44$

instead of -44.

(iii) Some did not correctly write

$a^2 + b^2 + c^2 + d^2 = (a+b+c+d)^2 - 2(ab + ac + \dots + cd)$

(c) i) Many missed stating remainder theorem or quoted factor theorem by mistake.

(d) Some multiplied by x^2 to incorrectly produce

$$x^2 \frac{(x^2-4)}{x} > x^2.$$

(e) Although many students could understand how to do the circle geometry proofs, often marks were lost as reasons were not quoted or wrong reasons quoted.

Comments

$\boxed{4}$ a. $y = x^{-2}$

$y' = -\frac{1}{2} x^{-3/2}$ ✓

$y'' = \frac{3}{4} x^{-5/2}$ ✓

b. i) $\lim_{x\to 2} \frac{x^3-8}{x-2} = \lim_{x\to 2} \frac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{(x-2)}}$

$= 4+4+4$

$= 12$ ✓

ii) $f(x) = \frac{1}{x}$ $\quad f(x+h) = \frac{1}{x+h}$

$f(x+h) - f(x) = \frac{1}{x+h} - \frac{1}{x}$ ✓

$= \frac{\cancel{x} - \cancel{x} - h}{x(x+h)} = \frac{-h}{x(x+h)}$

$f'(x) = \lim_{h\to 0} \frac{\frac{-h}{x(x+h)}}{h}$

$= \lim_{h\to 0} \frac{-\cancel{h}}{x(x+h)} \cdot \frac{1}{\cancel{h}} = -\frac{1}{x^2}$ ✓

c. $f(x) = \frac{1-x^2}{x^4}$

i) $1-x^2 = 0$

$(1+x)(1-x) = 0$

$x = 1, -1$ ← zeros ✓

ii) $f'(x) = \frac{x^4(-2x) - (1-x^2)4x^3}{x^8}$

$= \frac{-2x^5 - 4x^3 + 4x^5}{x^8}$

$= \frac{2x^5 - 4x^3}{x^8}$ ✓

$f'(x) = 0$, $2x^3(x^2-2) = 0$

$x = 0$, $x = \sqrt{2}$, $x = -\sqrt{2}$ ✓

but $x \neq 0$

x	-2	$-\sqrt{2}$	-1
y'	$-$ve	0	+ve

✓

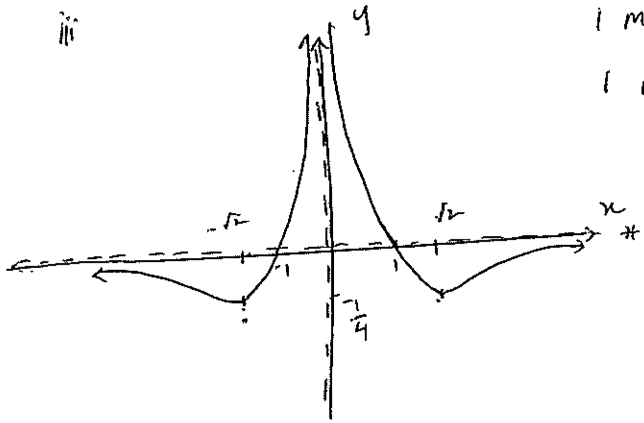
x	1	$\sqrt{2}$	2
y'	$-$ve	0	+ve

↑ they must show some reasoning

$(\sqrt{2}, -\frac{1}{4})$ is a min ✓

$(-\sqrt{2}, -\frac{1}{4})$ is a min ✓

iii)



1 mark for correct shape

1 mark for correct asymptotes

d. $y = x^4 + 1$

$y' = 4x^3$ ✓

$\therefore m = f'(1) = 4$ $(1,2)$

$(y-2) = 4(x-1)$

$y - 2 = 4x - 4$

$y = 4x - 2$ ✓

2. $C = 36 + \frac{v^2}{100}$ per hour

i) $v = \frac{d}{t}$ $t = \frac{d}{v}$

$t = \frac{100}{v}$ hrs. ✓

ii) $\left(36 + \frac{v^2}{100}\right)\frac{100}{v}$

$= \frac{3600}{v} + v$ ✓

iii) $C = 3600v^{-1} + v$

$C' = -3600v^{-2} + 1$ ✓

$C' = 0$ $\frac{3600}{v^2} = 1$

$v = \pm 60$ km/h ✓

$C'' = 7200v^{-3} = \frac{7200}{v^3}$ ✓

$C''(60) > 0 \therefore$ C is minimum at $v = 60$ km/h

4 b (ii) many students lost a mark for not writing:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

correctly, which is finding the derivative as a limit using "first principles".

c ii) Some students did not take the $\pm$ values for x^2.

$\therefore x^2 = 2$

$x = \pm\sqrt{2}$

e (iii) many students did not take the $\pm$ values for V^2

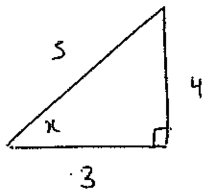
$\therefore V^2 = 3600$

$V = \pm 60$ km/h

Students must state $C' = 0$ for min or test that $C'' < 0$ for 60km/h to be min.

These signs are very important as they indicate where a particle is in a displacement related question. Since this is a speed related question, it did not affect the final answer

$\boxed{5}$ a. $\cos x = \frac{4}{5}$



$\sin x = \frac{3}{5}$ ✓

$\cos 2x = \cos^2 x - \sin^2 x$

$= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$

$= \frac{16-9}{25}$

$= \frac{7}{25}$ ✓

b. $\cos(a+b) = \cos a \cos b - \sin a \sin b$ ✓

$\cos 2q = \cos^2 q - \sin^2 q$

$= 1 - \sin^2 q - \sin^2 q$

$= 1 - 2\sin^2 q$ ✓

c. $LHS = \dfrac{\cos y - [\cos y \cos 2q - \sin y \sin 2q]}{2 \sin q}$

$= \dfrac{\cos y - [\cos y\,(1-2\sin^2 q) - \sin y \cdot 2 \sin q \cos q]}{2 \sin q}$

✓ $= \dfrac{\cancel{\cos y} - \cancel{\cos y} + 2\cos y \sin^2 q + 2 \sin y \sin q \cos q}{2 \sin q}$

$= \dfrac{\cancel{2 \sin q}\,[\sin q \cos y + \sin y \cos q]}{\cancel{2 \sin q}}$ ✓

$= \sin(q+y)$

d.

$LHS = \frac{1}{\tan\theta} + \tan\frac{\theta}{2}$

$= \frac{1-t^2}{2t} + t$ ✓

$= \frac{1-t^2+2t^2}{2t}$

$= \frac{1+t^2}{2t}$

$= \dfrac{1}{\frac{2t}{1+t^2}}$ ✓

$= \frac{1}{\sin\theta}$

$= \operatorname{cosec}\theta$

OR

$LHS = \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta/2}{\cos\theta/2}$

$= \frac{\cos\theta\cos\theta/2 + \sin\theta \sin\theta/2}{\sin\theta\cos\theta/2}$

$= \frac{\cos(\theta - \theta/2)}{\sin\theta\cos\theta/2}$

$= \frac{\cancel{\cos\theta/2}\ 1}{\sin\theta\cancel{\cos\theta/2}}$

$= \operatorname{cosec}\theta$

$= RHS$

e.

$\sin\theta + \cos\theta = 1$

$\therefore \sqrt{2}\sin\left(x + \frac{\pi}{4}\right) = 1$

$x + \frac{\pi}{4} = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}$

$x = 0, \frac{\pi}{2}, 2\pi$ ✓

f. $\tan 14° = \frac{200}{SP}$

$SP = \frac{200}{\tan 14°}$ ✓

$d^2 = \left(\frac{200}{\tan 14°}\right)^2 + 500^2 - 2\left(\frac{200}{\tan 14°}\right) 500 \times \cos 75$ ✓

$d^2 = 685841.2456$

$d = 828m$ ✓

g.

$$\cos 2\theta = \cos\theta$$

$$2\cos^2\theta - 1 = \cos\theta$$

$$2\cos^2\theta - \cos\theta - 1 = 0 \checkmark$$

$$\cos\theta = \frac{1 \pm \sqrt{1 - 4.2.-1}}{2.2}$$

$$\cos\theta = \frac{1 \mp \sqrt{9}}{4}$$

$$\cos\theta = 1 \,, \; -\tfrac{1}{2}$$

$$\theta = 0, 2\pi \checkmark \qquad \theta = \frac{2\pi}{3}, \frac{4\pi}{3} \checkmark$$

h. i $a^3 + b^3$ ✓

ii $\cos^6\theta + \sin^6\theta$

$$= (\cos^2\theta)^3 + (\sin^2\theta)^3$$

$$= [(\cos^2\theta + \sin^2\theta)(\cos^4\theta - \sin^2\theta\cos^2\theta + \sin^4\theta)] \checkmark$$

$\sin^2\theta + \cos^2\theta = 1.$

$$= \cos^4\theta - \sin^2\theta\cos^2\theta + \sin^4\theta$$

$$= (\cos^2\theta + \sin^2\theta)^2 - 3\sin^2\theta\cos^2\theta \checkmark$$

$$= 1 - \frac{3}{4}(2\sin\theta\cos\theta)^2$$

$$= 1 - \frac{3}{4}\sin^2 2\theta$$

$$= 1 - \frac{3}{4}[1 - \cos^2 2\theta] \checkmark$$

$$= \frac{1}{4} + \frac{3}{4}\cos^2 2\theta$$

Comments

5b for 'show' question you must show the substitution $\cos^2 q = 1 - \sin^2 q$.

5c A number of errors ~~due~~ due to poor writing (mixing up 'y' and 'q')

5d alternate soln on previous page. (use proper LHS/RHS setting out)

5f rounding errors - use exact value of SP rather than rounded value.

5e / 5g

Note domain is radians ∴ answers need to be given in radians (not degrees and not general soln!) (some used 't' method)

Check all solns!